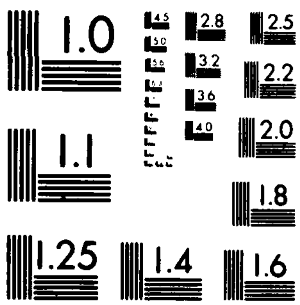
AD-A109 927 MOTOROLA INC TEMPE AZ GOVERNMENT ELECTRONICS DIV F/G 20/14
BATCH COVARIANCE RELAXATION (BCR) ADAPTIVE PROCESSING.(U)
AUG 81 S M DANIEL, I KERTESZ F30602-80-C-0031
UNCLASSIFIED 8512-F RADC-TR-81-212 NL

1 of 7



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

[illegible]

RADC-TR-81-213 [illegible]

APPROVED:

VINCENT [illegible]
Project Engineer

APPROVED:

FRANK J. [illegible]
Technical Director
Surveillance Division

FOR THE COMMANDER:

[illegible]

[illegible]

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER RADC-TR-81-212	2. GOVT ACCESSION NO. AD-A20	3. RECIPIENT'S CATALOG NUMBER 7727
4. TITLE (and Subtitle) BATCH COVARIANCE RELAXATION (BCR) ADAPTIVE PROCESSING		5. TYPE OF REPORT & PERIOD COVERED Final Technical Report December 79 - March 81
		6. PERFORMING ORG. REPORT NUMBER 8512-F
7. AUTHOR(s) Dr. S.M. Daniel I. Kertesz		8. CONTRACT OR GRANT NUMBER(s) F30602-80-C-0031
9. PERFORMING ORGANIZATION NAME AND ADDRESS Motorola Inc. (Gov't Electronics Division) 7402 S. Price Road Tempe AZ 85282		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62702F 45061152
11. CONTROLLING OFFICE NAME AND ADDRESS Rome Air Development Center (OCTS) Griffiss AFB NY 13441		12. REPORT DATE August 1981
		13. NUMBER OF PAGES 686
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Same		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) Same		
18. SUPPLEMENTARY NOTES RADC Project Engineer: Vincent C. Vannicola (OCTS)		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Null Steering Adaptive Processing Side Lobe Cancellation		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The objective of this program was to develop a systematic method of generating adaptive weights through the application of the Batch Covariance Relaxation (BCR) algorithm and assess the performance of such weights in and adaptive sidelobe canceller system. The following items were developed: (a) A mathematical model representing an adaptive radar antenna operating in a wideband noise jamming (over)		

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED 411117

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) environment. The model was formulated as an equivalent baseband transfer function. (b) A complete description of the weight computation procedure for jammer nulling based on the BCR algorithm, including several examples which illustrate its superior numerical characteristics compared to certain other algorithms. (c) A library of modular FORTRAN IV computer programs which implement multiple interference sources, the adaptive radar antenna model, and the BCR algorithm. These were used to extensively evaluate the capability of BCR processing in realistic jamming scenarios.

ACKNOWLEDGEMENT

A number of individuals must be recognized for their direct and indirect support which has contributed to a successful completion of this BCR adaptive processing study sponored by RADC/OCTS, Griffiss Air Force Base. The authors wish to express their appreciation to a number of colleagues for reviewing the technical memoranda that document the results of this study. These individuals who spent sufficient time to thoroughly review and make important suggestions are Dr. J. R. Roman, Dr. R. Hecht-Nielsen, J. L. Bauck, C. R. Champlin, and S. K. Baker of the Advanced Technology and Systems Analysis Section of Motorola's Radar Systems Office, Radar Operations, Tempe, Arizona. Additionally, we would like to thank G. V. Morris, Manager of the Advanced Technology and Systems Analysis Section, J. D. Doggett, Manager of the Radar Systems Section, D. L. Howell, Chief Engineer of the Radar Systems Office and R. R. Yost, Chief Engineer of the Radar Products Office, for reviewing and approving the technical memoranda that have been submitted.

The authors are indebted to a number of secretaries, S. M. McGovern, C. L. Karseboom, E. A. Shick, K. L. Herslow, and M. J. Morton, who helped with the typing of the manuscript. Our deep appreciation goes to Mrs. Karseboom and Mrs. Herslow whose dedication to perfection in organizing and typing the bulk of the material resulted in the presentable style of the final report.

Finally, on behalf of Motorola, the authors wish to extend their appreciation to Russell Brown and Vincent Vanicola of RADC/OCTS who supported this work and displayed an active interest throughout the duration of the program.

BCR ADAPTIVE PROCESSING

- AN OVERVIEW -

Prepared by: Sam M Daniel
S. M. Daniel
Project Leader
Advanced Technology and Systems Analysis

Reviewed by: S. K. Baker
S. K. Baker
Program Development Manager
Advanced Technology and Systems Analysis

Approved by: Guy V. Morris
G. V. Morris
Section Manager
Advanced Technology and Systems Analysis

D. L Howell
D. L. Howell
Chief Engineer
Radar Systems Office
Radar Operations

PREFACE

The work carried out by the Advanced Technology and Systems Analysis Section of the Radar Systems Office at Motorola's Government Electronics Division under contract to RADC/OCTS is presented in a total of six separate technical memoranda written at various stages of the effort.

Although to a large extent these memoranda are self-sufficient, the interested reader may be advised to go through Project Memorandum 8512-06, an overview of the whole effort, before proceeding with the remaining in numerical sequence. As such, if presented in a single volume, the memoranda will be arranged as listed below:

I. BCR Adaptive Processing - An Overview -
Project Memorandum 8512-06
March 31, 1981

II. Definition of Mathematical Model
Project Memorandum 8512-01
March 29, 1980

III. Signal Generation Program
Project Memorandum 8512-02
July 20, 1980

IV. Batch Adaptive Processing and the BCR Process
Project Memorandum 8512-03
December 15, 1980

V. Computer Simulation of BCR Adaptive Process
Project Memorandum 8512-04
March 15, 1981

VI. BCR Adaptive Processing Implementation and Its Performance Evaluation Via Computer Emulation
Project Memorandum 8512-05
March 26, 1981

TABLE OF CONTENTS

1.0 INTRODUCTION

In the midst of a strong interference environment, radar and communications equipment could degrade in performance to the point where their intended operation may be rendered practically ineffective. To insure operational survival, future systems must incorporate appropriate interference suppression measures. Adaptive array processing provides a means for suppressing directionally distinct interferences by creating nulls in these directions through a weight adjustment of at least as many antenna elements of a receiving antenna array.

Although cost-effective analog implementation of gradient-type control have been realized, the attention has turned increasingly towards more sophisticated techniques capable of more rapid convergence rates. The increased sophistication of such techniques dictates the need for processing flexibility which may be met most easily by a digital mechanization.

The effort conducted addresses the essential aspects of Batch Covariance Relaxation (BCR) adaptive processing applied to a digital adaptive array processing. In contrast to dynamic algorithms, a batch approach, such as BCR, circumvents the effects of signal dynamics on nulling performance by operating on a single signal batch at one time, thereby diminishing the requirement for fast-convergence.

BCR adaptive processing constitutes an architecturally more efficient alternative to Sample Matrix Inversion. Based on the conjugate gradients algorithm, BCR solves a linear system of equations by means of a finite-step relaxation procedure without the requirement that the inverse of the matrix involved exist. More importantly, its iterative nature lends itself to an architecturally efficient implementation which is particularly suitable to large-scale applications.

The main purpose of the present memorandum is to provide a brief overview of the work accomplished under RADC Contract F30602-80-C-0031. Presented first is a statement of the problem which provided the vehicle for investigating the performance of BCR adaptive processing. Following this, the specific objective, approach and accomplishments are reviewed with reference to the five project memoranda generated at various stages of the effort. Finally, recommendations are given for related future work.

2.0 STATEMENT OF THE PROBLEM

The problem chosen to evaluate BCR adaptive processing relates to the adaptive weight adjustment of a sidelobe nulling subsystem. More specifically, this subsystem involves a main Taylor-weighted large aperture antenna array of 255 omnidirectional elements. Its field pattern is shown in Figure 2-1. In order to suppress undesired sidelobe interference,

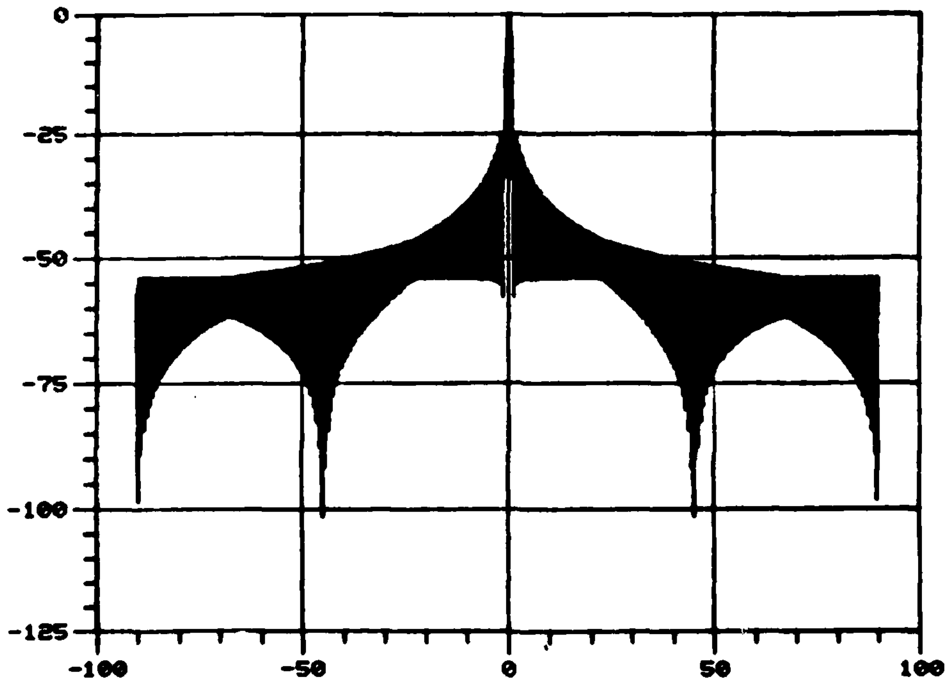


Figure 2-1. Field Pattern of Main Antenna Array

it is assumed that the main antenna may be combined with a number of weight-adjustable omnidirectional auxiliaries located sparsely over the available aperture. It is the purpose of the BCR processor to derive the auxiliary weights adaptively.

The essence of the BCR processor may be described briefly, as follows. Let

$\{s_0(m)\}_{m=1}^{M}$ = M-sample main complex baseband scalar signal batch

$\{\underline{s}(m)\}_{m=1}^{M}$ = M-sample auxiliary complex baseband N-vector signal batch

Then, letting $\underline{w}$ represent the complex weight N-vector applied over the N auxiliaries, the combined complex scalar signal batch is given by

$$\{s_c(m)\}_{m=1}^{M} = \{\underline{s}^T(m)\underline{w} + s_0(m)\}_{m=1}^{M} \tag{2-1}$$

If the signals involved constitute primarily undesired interference, then an appropriate criterion for the underlying adaptive array process is the minimization of the combined power in a given batch with respect to the weight vector $\underline{w}$. In view of (2-1), the combined power is defined by

$$P_c(\underline{w}) = \sum_{m=1}^{M} |s_c(m)|^2 \tag{2-2}$$

clearly, a quadratic function of $\underline{w}$. Geometrically, $P_c(\underline{w})$ represents a hyperparaboloid in (2N+1)-space having a unique minimum value for at least one value of $\underline{w}$. We seek a weight-vector $\underline{w}$ that minimizes $P_c(\underline{w})$. The desired solution satisfies the necessary condition that the complex gradient of $P_c(\underline{w})$ with respect to $\underline{w}$ be zero; that is, a value of $\underline{w}$ which minimizes $P_c(\underline{w})$ satisfies

$$\nabla_{\underline{w}} P_c(\underline{w}) = \underline{0} \tag{2-3}$$

With (2-1) and (2-2) at hand, it may be shown that condition (2-3) leads to the complex linear system

$$C\underline{w} + \underline{b} = \underline{0} \tag{2-4}$$

where C is the NxN complex Hermitian covariance matrix of the auxiliary signal vector $\underline{s}$ computed over a given batch; i.e.,

$$C = \sum_{m=1}^{M} \underline{s}^*(m)\underline{s}^T(m) \qquad (2\text{-}5)$$

and $\underline{b}$ is the complex cross-correlation N-vector over the same batch, given by

$$\underline{b} = \sum_{m=1}^{M} \underline{s}^*(m)s_0(m) \qquad (2\text{-}6)$$

Hence, the adaptive array process has been reduced to solving (2-4) for $\underline{w}$. Formally, the solution is given by

$$\underline{w}^0 = -C^{-1}\underline{b} \qquad (2\text{-}7)$$

often referred to as the Sample Matrix Inversion (SMI) solution. Of course, in using (2-7), the inverse, C^{-1}, is assumed to exist. In contrast, the CG algorithm employed in the BCR approach is a numerically robust iterative procedure which begins with an initial estimate $\underline{w}^0$ and produces the desired solution to (2-4) in a finite number of steps which does not exceed the rank of the matrix C. At each step, the current estimate, $\underline{w}^k$, is updated to an improved estimate, $\underline{w}^{k+1} = \underline{w}^k - \alpha_k\underline{p}^k$, by optimally relaxing $P_C(\underline{w})$ along the search vector $\underline{p}^k$. The process terminates when the gradient, $\underline{r}^{k+1} = C\underline{w}^{k+1} + \underline{b}$, is sufficiently small in length with respect to the forcing vector $\underline{b}$. When the initial estimate $\underline{w}^0$ is taken to be $\underline{0}$, the final weight estimate is the so-called minimum-norm solution, the solution $\underline{w}$ closest to the origin.

As the weight-vector solution is derived for each batch of signals, it is applied to the same batch and thus forms the corresponding combined signal batch. Since signals are practically frozen within the batch and essentially isolated from neighboring batches, signal dynamics present no particular difficulty to a BCR processor. As such, the convergence-rate requirements of dynamic algorithms is not strictly applicable in batch processing.

3.0 OVERVIEW OF INVESTIGATION

The BCR Adaptive Processing study conducted over the previous several months has been documented in great detail in five separate technical memoranda written in the course of the investigation. Reviewed below is the overall objective, general approach and accomplishments of the effort.

3.1 Objective

The essential objective of the study was to introduce the concept of BCR adaptive processing not only in a mathematically appealing but also in a tangible and practical way. As such, the BCR process was to be formulated in the context of the specific adaptive nulling subsystem discussed in the previous section.

3.2 Approach

A multistaged approach was adopted for accomplishing the stated objective. The important elements constituting the basic approach are as follows:

1. Definition of a physically-credible mathematical model of the scenario and system description.
2. Preparation of a modular signal generation computer simulation program incorporating the mathematical model. Main and auxiliary signals could thus be generated for a variety of scenario and system parameter specifications.
3. Mathematical background and formulation of the BCR adaptive process. Pertinent supporting mathematical theory of the CG algorithm and its possible variations could thus be understood from first principles.
4. Preparation of modular BCR simulation program to use signal generation data and provide a detailed numerical summary of the BCR process including nulling performance.
5. Preparation of modular BCR performance and plotting program to afford the user the visualization of certain performance observables.
6. Discussion of a BCR processor implementation and its performance evaluation via precise computer emulation, an exact model of the hardware mechanization.

3.3 Accomplishments

Following the approach outlined above, the investigation resulted in a number of important accomplishments, such as the following:

1. A mathematical model describing the important physical aspects of the scenario and pertinent parameters of the adaptive array system was defined. The detailed mathematical development was reported in Project Memorandum 8512-01. One of the most important contributions of this particular work was the derivation of the equivalent baseband transfer function of the antenna system which describes the transformation of the complex envelope of a given source from incidence on the array, through the receiver and all the way down to baseband. As such, each incident source is viewed as passing through a distinct channel determined by its angle of arrival and the receiver's final IF filter. The received baseband signal at a given port is simply a linear superposition of individual source envelope signals transmitted through their associated equivalent baseband channels.

2. The scenario and system description defined in PM 8512-01 was incorporated into a disc file in the form of a clearly descriptive menu from which the user may specify pertinent parameters. Included among these parameters are choices of percent IF and RF bandwidths, specification of IF filter in terms of its low-pass prototype pole locations, the number of incident sources, their associated amplitudes and angles of arrival, specification of main antenna array, locations of auxiliary antenna elements, etc. This input data file has been referred to as SIGGEN:D.

   Project Memorandum 8512-02 introduces a "library-based" modular signal generation program, SIGGEN, which uses the system description specified via SIGGEN:D to produce sampled main and auxiliary baseband port signals in a separate output file, SIGGEN:O.

3. A complete mathematical development of the BCR process was reported in Project Memorandum 8512-03. Included there is the batch processing concept, the detailed derivation of the CG algorithm and several numerical examples illustrating its numerical characteristics. Presented also is a new constrained formulation of the CG algorithm which may be applicable not only to fully adaptive array systems but also to partially adaptive ones.

4. Project Memorandum 8512-04 was devoted to the evaluation of the BCR process via computer simulation. It includes a detailed description of a refined version of the signal generation program, SIGGEN, a BCR simulation program, BCRS, and a BCR performance evaluation and plotting program, BCRP. All three of these program are modular in construction and share modules from a common libarary, RADAR:LIB.

   An extensive explanation as to how to use these programs is given. It is shown how the output of each is used as part of the input for the next program. As such, SIGGEN:O is used to construct BCRS:D, the input data file to BCRS. In turn, its output file, BCRS:O, is used to construct BCRP:D, the input data file to BCRP.

   As mentioned before, SIGGEN:O includes sampled main and auxiliary complex baseband signals. Output file BCRS:O includes the convergence or relaxation characteristics in terms of gradient metric and combined power reduction at each iteration, as well as the associated evolution of adaptive weight iterates. Also computed is the combined-port signal along with the achieved nulling performance.

Although an output file BCRP:O is created, it is most interesting in graphical form. The TEKTRONIX graphical output provided by BCRP includes a graph of main and combined signal amplitudes, source-related channel amplitude responses before and after adaptation, field-patterns at the vicinity of source-angles of arrival before and after adaptation and, finally, mainbeam pattern before and after adaptation.

Results from a large number of examples are presented graphically. Included among these examples are cases involving scenarios with continuous, blinking, wideband, multiplath equal and unequal power sources. System configurations with singly or multiply-tapped ports are also considered.

5. A practical TTL implementation of a BCR processor has been included in Project Memorandum 8512-05. Although the design given applies specifically to a 4-port system, the general approach applies to other dimensionalities.

   A modular computer emulation program, BCRM, incorporating the exact hardware design is also included. Using an input data file, BCRM:D, which includes SIGGEN:O, BCRM carries out the detailed adaptive fixed-point arithmetic employed by the hardware implementation. The entire arithmetic activity of the BCR processor is written into an output file BCRM:O. Three examples demonstrate the effectiveness of the implementation.

6. Probably, the most important accomplishment of the BCR adaptive processing study is the library of software modules and the four major programs made of these library modules. With these at hand, the user may examine the performance of the BCR processor for any number of examples other than the ones presented. Following the convention of the library approach, the user may add to the sophistication of the software already developed.

   For purposes of easy reference, the next few pages contain a complete catalog of the modular library, RADAR:LIB. Listed there are source module names and special JCL programs indicating authors, date of origin and revision.

Table 3-1. Modular Program Library, RADAR:LIB

RADAR PROGRAM LIBRARY CATALOG

REVISION : APRIL 2, 1981

FILF	AUTHOR(S)	ORIGINATED	REVISED
ADOTB:S	S. M. DANIEL I. KERTESZ	9/12/79	3/19/81
AGC:S	S. M. DANIEL I. KERTESZ	1/15/81	2/02/81
ALLPLOT:S	S. M. DANIEL I. KERTESZ	8/23/80	2/03/81
AMPH:S	S. M. DANIEL I. KERTESZ	4/19/80	7/01/80
ANTWT:S	S. M. DANIEL I. KERTESZ	4/19/80	9/01/80
BCREML:S	S. M. DANIEL I. KERTESZ	4/15/79	3/25/81
BCREXEC:S	S. M. DANIEL I. KERTESZ	8/10/80	1/06/81
BCRMAIN:S	S. M. DANIEL I. KERTESZ	7/23/80	8/12/80
BCRMMAIN:S	S. M. DANIEL I. KERTESZ	8/ /79	3/11/81
BCRMEXEC:S	S. M. DANIEL I. KERTESZ	1/15/81	3/19/81

BCHMSET:S	S. M. DANIEL	7/23/80	3/25/81
	I. KERTESZ		
BCRSET:S	S. M. DANIEL	7/23/80	1/24/81
	I. KERTESZ		
BCRSIM:S	S. M. DANIEL	4/15/78	9/17/80
	I. KERTESZ		
BCHPMAIN:S	S. M. DANIEL	7/23/80	2/ 3/81
	I. KERTESZ		
BCRPSET:S	S. M. DANIEL	7/23/80	2/13/81
	I. KERTESZ		
BIGSGN:S	S. M. DANIEL	1/27/81	3/11/81
	I. KERTESZ		
BLINK:S	I. KERTESZ	6/12/80	6/16/80
	S. M. DANIEL		
CANDB:S	S. M. DANIEL	9/23/80	1/06/81
	I. KERTESZ		
CREML:S	S. M. DANIEL	8/15/79	3/25/81
	I. KERTESZ		
CHANNEL:S	S. M. DANIEL	5/04/80	1/25/81
	I. KERTESZ		
CMBSGNL:S	S. M. DANIEL	9/06/80	9/13/80
	I. KERTESZ		
CMPCHNL:S	S. M. DANIEL	8/04/80	1/30/81
	I. KERTESZ		
CSEML:S	S. M. DANIEL	8/15/79	3/25/81
	I. KERTESZ		
FFT2:S	PROF. P. RANSOM	7/15/69	9/01/80
	S. M. DANIEL		
FIELD:S	S. M. DANIEL	8/04/80	1/31/81

		I. KERTESZ		
	FILTER:S	S. M. DANIEL	4/19/80	6/23/80
		I. KERTESZ		
	FORM:S	S. M. DANIEL	4/15/78	9/13/80
		I. KERTESZ		
	FORMAT:S	S. M. DANIEL	4/15/78	3/17/81
		I. KERTESZ		
	IMPULS:S	S. M. DANIEL	5/11/80	6/07/80
		I. KERTESZ		
	JCL:B	S. M. DANIEL	6/20/80	7/01/80
		G. C. WANG		
	JCL:X	S. M. DANIEL	6/22/80	6/23/80
		G. C. WANG		
	JCLBCRP:RL	S. M. DANIEL	1/06/81	2/26/81
	JCLBCRS:RL	S. M. DANIEL	1/06/81	2/26/81
	JCLSIG:RL	S. M. DANIEL	6/21/80	2/26/81
		G. C. WANG		
	LIMIT:S	S. M. DANIEL	8/15/79	3/25/81
		I. KERTESZ		
	PLOTS:S	S. M. DANIEL	9/ 6/80	2/ 2/81
		I. KERTESZ		
	PWRSUP:S	S. M. DANIEL	9/11/80	9/12/80
		I. KERTESZ		
	RCNORM:S	S. M. DANIEL	8/31/78	1/26/81
		I. KERTESZ		
	ROUNDC:S	S. M. DANIEL	9/ /79	3/11/81
		I. KERTESZ		
	ROUNDR:S	S. M. DANIEL	9/ /79	3/11/81

	I. KERTESZ		
RW:S	S. M. DANIEL	4/15/78	3/17/81
	I. KERTESZ		
RWIRC:S	S. M. DANIEL	4/19/80	8/13/80
	I. KERTESZ		
PRTSGNL:S	S. M. DANIEL	6/01/80	1/05/81
	I. KERTESZ		
SCALE:S	S. M. DANIEL	8/15/79	3/19/81
	I. KERTESZ		
SCATD:S	S. M. DANIEL	8/15/79	3/17/81
	I. KERTESZ		
SIGMAIN:S	S. M. DANIEL	4/19/80	1/05/81
	I. KERTESZ		
SIGNAL:S	S. M. DANIEL	6/08/80	9/01/80
	I. KERTESZ		
SIGSET:S	S. M. DANIEL	6/01/80	1/15/81
	I. KERTESZ		
SKIPR:S	S. M. DANIEL	2/23/81	2/23/81
	I. KERTESZ		
TTEKS:S	S. M. DANIEL	8/01/80	2/23/81
	I. KERTESZ		
	G. C. WANG		
UPDATE:S	S. M. DANIEL	8/15/79	3/19/81
	I. KERTESZ		
WITER:S	S. M. DANIEL	9/13/80	9/13/80
	I. KERTESZ		

4.0 RECOMMENDATIONS

Based on the results of the present study, there are two possible avenues for future productive work involving BCR adaptive processing. A software effort may be defined to increase the capability of the modular library in order to address adaptive performance in more realistic scenarios and operational system configurations. A companion effort must address increasingly more sophisticated, efficient and flexible architectures of BCR adaptive processors.

The modular library already developed is amenable to a systematic improvement and growth. As such, realistic scenarios involving desired signals, interfering sources and ambient noise may be incorporated via modifications to existing modules or creation of new such modules. The constrained BCR formulation, which may be applicable in certain operational situations, must be accommodated in the existing BCRSIM module. The resulting module may now be applicable to solving both unconstrained and constrained minimum mean square problems which arise in radar signal processing. Besides adaptive nulling, applications such as adaptive clutter cancelling, spectrum estimation and spatial or temporal adaptive filtering come to mind.

Since BCR adaptive processing has a wide applicability, it makes sense to exploit promising architectural options that further take advantage of its underlying algorithmic structure and lead to cost-effective, technically reliable and flexible implementations. For example, a serialized BCR processor design involving a single complex multiplier/adder combination would lend itself to a variable dimensionality application. Implied here, of course, is the need for appropriately flexible timing, control and storage architectures and certainly programmability. Finally, future BCR implementations should tend toward distributed, self-diagnosing and fault-tolerant processing architectures with the aim of enhancing computational capability and operational survivability.

DEFINITION OF MATHEMATICAL MODEL

Prepared by: Sam M. Daniel

S. M. Daniel
Project Leader
Systems Analysis Group

Approved by: I. Newton Durboraw

I. N. Durboraw
Manager
Systems Analysis Group

J. G. Doggett

J. G. Doggett
Section Manager
Radar Section

R. R. Yost

R. R. Yost
Chief Engineer
Radar Operations

TABLE OF CONTENTS

1.0 INTRODUCTION

Consider a surveillance radar adaptive array system employing a large-aperture narrow-beam low-sidelobe main linear antenna array in an adjustable weighted combination with a number of omnidirectional auxiliary antenna elements with the aim of suppressing sidelobe interference. A credible evaluation of the dynamic performance of such an adaptive system cannot be conducted unless the analysis employs a sufficiently faithful mathematical model that describes the pertinent physical phenomena involved.

Given an arbitrary signal arriving from a certain azimuth direction, a reliable mathematical model must describe the received form of the signal at each antenna port incorporating the dependence on bandwidth. Beyond this point, the main and auxiliary signals must be down-converted to some convenient intermediate frequency (IF) and band-limited there by appropriate filtering. Expected mismatch and misalignment between the different IF filters must be included in the model, thus reflecting the description of the received signals up to that stage.

In order to analyze the performance of subsequent digital processing, which is usually accomplished at baseband, the mathematical model must describe the signals down to baseband, taking into account any baseband filter mismatch if it is deemed significant. In effect, the signal at each receiver baseband port constitutes the complex envelope of the original signal at the source transmitted through an equivalent "channel" that incorporates the antenna and receiver characteristics already mentioned.

An equivalent channel may be defined for each combination of source and receiver. Clearly, the cancellation performance of a baseband adaptive array process will depend directly on the quality of match between these channels. The discussion that follows develops the mathematical model that describes the general channel and defines the received baseband signal. Specific considerations are given to receiver mismatch, wideband operation, and the multipath phenomenon.

2.0 MATHEMATICAL MODEL

The specific aim in this section is to define the mathematical model that describes the propagation of an arbitrary signal from its source through an arbitrary linear antenna array, through its accompanying receiver down to baseband. To begin with, an appropriate wideband signal is first defined. Subsequently, a linear antenna array configuration is defined appropriate for adaptive array processing. Finally, the equivalent baseband channel description is developed in a clear step-by-step process constituting the "transfer function" from the complex envelope of the source signal to the complex envelope that comprises the desired baseband signal. The actual representation of the baseband signal then follows.

2.1 Transmitted Signal

At its source, a given transmitted signal may be defined by the general expression

$$x(t) = P(t) \cos [2\pi f_1 t + \phi_1(t) + \phi_0] \tag{2-1}$$

where P(t) is a noise-like amplitude-modulation waveform, $\phi_1(t)$ is an arbitrary phase modulation and ϕ_0 is a fixed arbitrary initial phase.

For the purpose of analyzing the performance of an adaptive array system, it is not strictly necessary that x(t) be of the complexity (2-1). It would suffice, for example, to define an appropriate transmitted signal by

$$x(t) = P(t) \cos 2\pi f_1 t \tag{2-2}$$

where P(t) is chosen to be a biphase noiselike sequence with a chip-rate high enough to yield a nearly uniform line spectrum over the band of interest about f_1. It is important to note that intrinsic channel characteristics associated with this band will come into play and imprint their effect upon the ultimate signal at the receiver's baseband. Since phase modulation does not add any further observability of channel characteristics, its use is only academic. Finally, the initial fixed phase ϕ_0 may be taken to be zero without loss of generality, except perhaps in the most degenerate of cases in which received baseband signals might be strictly real or imaginary thus occupying only one of the quadrature channels. It is not expected that this situation will be of concern in the sequel.

In view of the above argument and considering the relative simplicity in the subsequent analysis and computer simulation, the transmitted signal will be assumed to be of form (2-2). It should be noted, however, that a phase shift will be induced at the antenna upon reception of (2-2).

2.2 Antenna Configuration

An antenna configuration appropriate for adaptive radar processing is one involving a multielement main linear array in combination with a number of omnidirectional auxiliary elements.

2.2.1 Main Linear Array

More specifically, the main linear array will be assumed to be comprised of at least 250 equally-spaced omnidirectional elements weighted according to a Taylor window and phased broadside. For convenience, the 25 dB Taylor window [1] to be used will be approximated by a simpler expression that defines the ℓ-th weight by

$$a(\ell) = 1.0 + 0.5 \cos \frac{2\pi(\ell-(L+1)/2)}{L} \qquad (2\text{-}3)$$

where $\ell \in [1,L]$, L is the total number of elements comprising the linear array.

Although the distance between elements is ususally taken to be equal to the half-wavelength of the center frequency of operation f_0 , it is an easy matter to choose the element separation to be some fraction of $\frac{\lambda_0}{2}$, namely,

$$d = d_0 \frac{\lambda_0}{2} \qquad (2\text{-}4)$$

where, typically, $d_0 \leq 1$. This flexibility is needed in analyzing system performance over a large operating frequency range.

It should also be noted that even though the uniform-spacing assumption may be restrictive from thepoint of view of analyzing the practical beamforming capability of the linear array, it poses no such restriction in the investigation of its performance in an adaptive array system as envisioned here.

2.2.2 Auxiliary Elements

The weight-adjustable auxiliary antenna elements will be assumed to be omnidirectional, although, in general, they may be chosen to have a more complex field pattern. However, more important than this is the fact that the auxiliary elements need to be randomly emplaced over the entire aperture of the main array if they are to be effective in the adaptive process.

In choosing a particular emplacement of auxiliary elements over the aperture of the main array, two important issues are involved. First, the need for spatial resolution, that is at least equivalent to that of the main array, dictates that the available auxiliaries span the main antenna aperture. Second, in view of the relatively small number of auxiliaries and their consequent large physical separations ($\gg \lambda_0$) over the main antenna aperture, the emplacement of the auxiliaries must be sufficiently random in order to circumvent undesirable blind regions.

Referring to geometries of poor cancellation, the blind region phenomenon is most pronounced when the auxiliary elements form a uniformly spaced thinned array. Most commonly explained as the grating lobe effect, this undesirable feature is a manifestation of spatial aliasing. A remedy for minimizing spatial aliasing is random spatial sampling, which results in improved performance of the underlying adaptive array process. The validity of random spatial sampling (random emplacement of auxiliary elements) in dealing with the aliasing problem is supported by a considerable amount of investigation in the area of random array theory [2] and, specifically, with the synthesis of directive low-sidelobe random thinned arrays [3], [4].

Recent investigations at Motorola Government Electronics Division have addressed the blind region problem in connection with sidelobe cancelling [5]. A linear dependence hardware test provided direct verification that the blind regions are due to the aliasing structure of the auxiliary subarray.† Based on further measurements, it was concluded that it may not be possible to completely eliminate blind regions in adaptive arrays, even with random emplacement of auxiliaries. It may be that the ultimate way around this problem is the use of an excess of degrees of freedom, namely, more auxiliary elements than strictly necessary.

2.3 Equivalent Baseband Channel

Given a source signal of the form (2-2) incident onto a uniform linear array attached to a given receiver, the resulting baseband signal happens to be the complex envelope output of a properly defined baseband channel whose input, in general, is the complex envelope of the original signal at the source. Of course, in view of (2-2), the input complex envelope is strictly real. Following a discussion of a complex bandpass signal representation, two forms of the so-called equivalent baseband channel are derived, a general form and a simpler alternate one.

---

† An experiment conducted by D. Fraser and S. M. Daniel at Motorola's Government Electronics Division, Scottsdale, Arizona, in 1977 predicted the existence and precise location of blind regions of an adaptive array system by exercising only the auxiliary subarray.

2.3.1 Complex Signal Representation

Before attempting to characterize the equivalent baseband channel, it is necessary to introduce the notion of complex envelope. To begin with, consider the general bandpass signal

$$x(t) = P(t) \cos (2\pi f_1 t + \phi) \tag{2-5}$$

where P(t) is a lowpass pseudonoise function whose bandwidth is a sufficiently small fraction of the carrier frequency f_0, [6] and ϕ is an arbitrary fixed phase induced by a receiving antenna. Since x(t) may be written as

$$x(t) = \frac{P(t)}{2}\left[e^{j(2\pi f_1 t + \phi)} + e^{-j(2\pi f_1 t + \phi)}\right]$$

$$= \frac{e^{j\phi}}{2} P(t) e^{j2\pi f_1 t} + \frac{e^{-j\phi}}{2} P(t) e^{-j2\pi f_1 t} \tag{2-6}$$

Letting $P(f) = \mathfrak{F}\{P(t)\}$, the Fourier Transform of x(t) is given by

$$X(f) = \frac{e^{j\phi}}{2} P(f-f_1) + \frac{e^{-j\phi}}{2} P(f+f_1) \tag{2-7}$$

clearly, a conjugate symmetric function about the origin, f=0. Figure 2-1 shows the corresponding amplitude spectrum, $|X(f)|$, indicating positive and negative-frequency portions by $X^+(f)$ and $X^-(f)$, respectively.

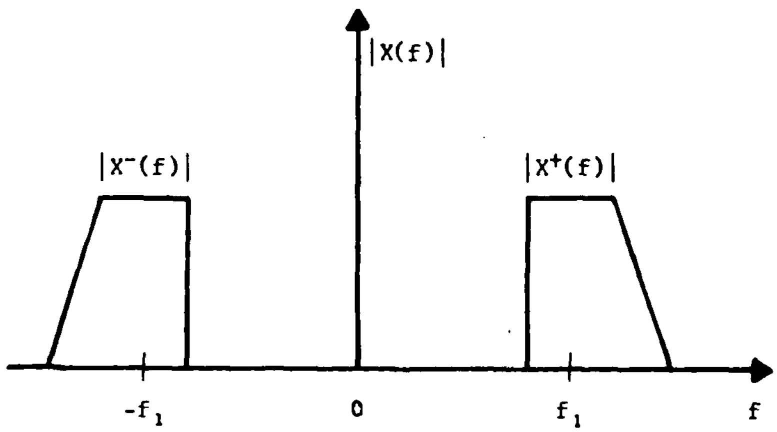


Figure 2-1. Symmetric Amplitude Spectrum of Real Bandpass Signal x(t).

Consider now the downconversion of $x(t)$ to baseband by mixing it with $\cos 2\pi f_1 t$ and its quadrature, $-\sin 2\pi f_1 t$, followed by ideal lowpass filtering to reject the $2f_1$ components. Figure 2-2 depicts this downconversion process applied to $x(t)$ and resulting to the inphase and quadrature components, I and Q, which comprise the complex baseband signal

$$\zeta(t) = P(t)e^{j\phi} \tag{2-8}$$

In view of (2-7), the Fourier Transform of (2-8)

$$Z(f) = e^{j\phi}P(f) \tag{2-9}$$

is, clearly, twice the positive-frequency half of the bandpass complex spectrum of $x(t)$, $X^+(f)$, translated toward the origin by a displacement of f_1. Its amplitude spectrum, $|Z(f)|$, is shown in Figure 2-3.

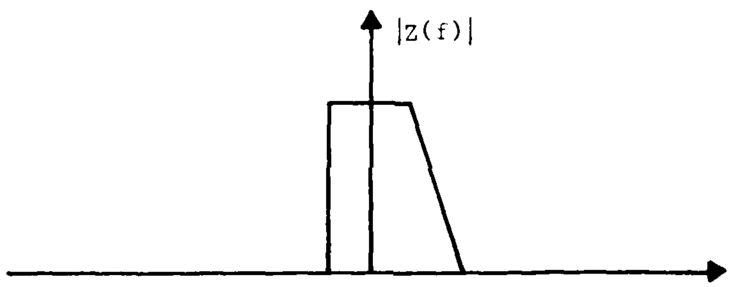


Figure 2-3. Amplitude Spectrum of Complex Baseband Signal $\zeta(t)$

The complex baseband signal, $\zeta(t)$, is also known as the complex envelope of the bandpass signal $x(t)$. Given an arbitrary bandpass signal $x(t)$, its complex envelope may be determined in a way analogous to the above example.

In general, the desired complex envelope is the inverse Fourier transform of twice the positive frequency half of the complex spectrum of $x(t)$, $X^+(f)$, translated toward the origin by an amount equal to the oscillator frequency f_o, which may not necessarily be equal to f_1. As such, in view of (2-6) and (2-7), the determination of the complex envelope is facilitated by dealing directly with the complex representation of $x(t)$; namely,

$$z(t) = P(t)e^{j(2\pi f_1 t + \phi)} \tag{2-10}$$

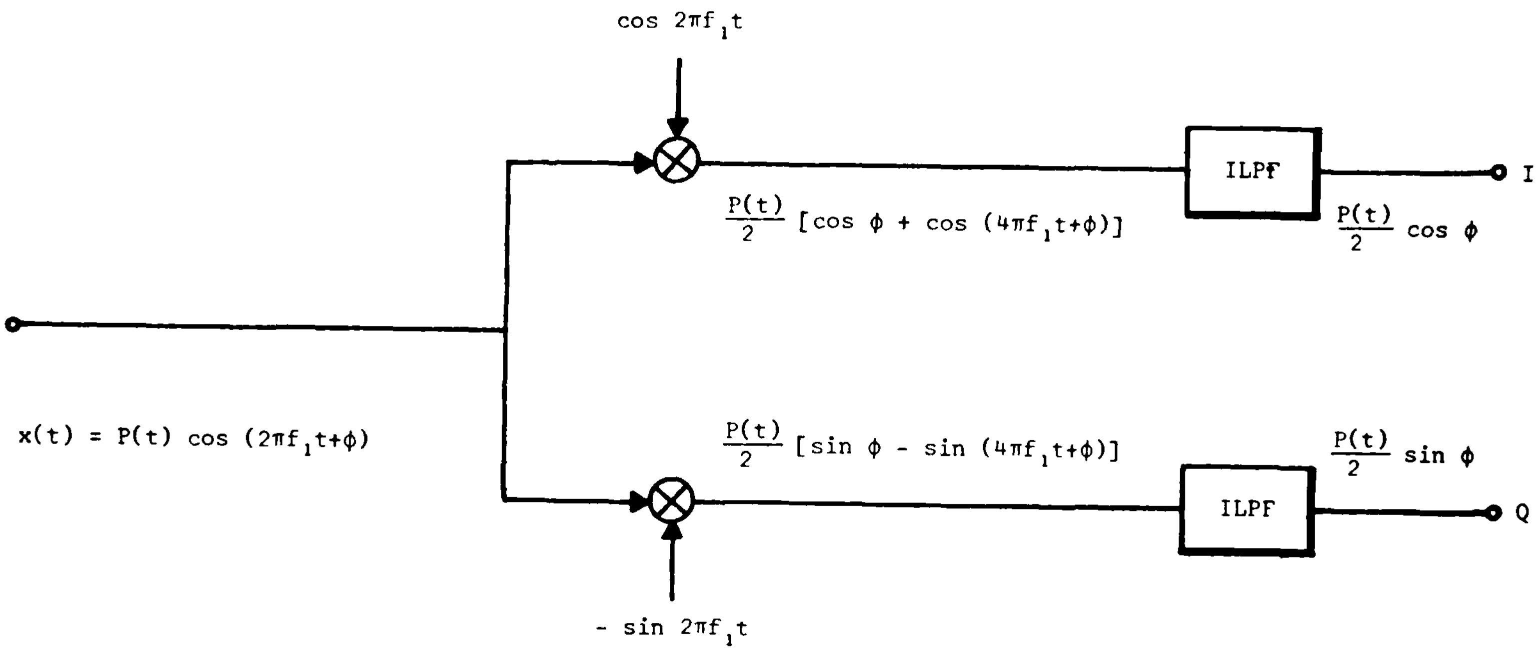


Figure 2-2. A Baseband Downconversion of the Bandpass Signal x(t).

whose Fourier transform is exactly

$$Z(f) = 2X^{+}(f) = e^{j\phi}P(f-f_1) \tag{2-11}$$

Then, the spectrum of the complex envelope associated with a downconversion of f_0 is given by

$$Z(f) = 2X^{+}(f+f_0) = e^{j\phi}P(f+f_0-f_1) \tag{2-12}$$

and the complex envelope by

$$\zeta(t) = P(t)e^{j\phi}e^{j2\pi(f_1-f_0)t} \tag{2-13}$$

a simple frequency translation of (2-9), as expected. Figure 2-4 summarizes the relationship between $x(t)$, $z(t)$ and $\zeta(t)$, in terms of their respective amplitude spectra.

It should be noted here that the complex signal, $z(t)$, is sometimes referred to as the complex pre-envelope of $x(t)$. Clearly, $x(t)$ happens to be the real part of $z(t)$; that is,

$$x(t) = \mathrm{Re}\{z(t)\} \tag{2-14}$$

It is also noteworthy that a phase shift of θ on $x(t)$ may also be written as

$$\tilde{x}(t) = \mathrm{Re}\left\{e^{j\theta}z(t)\right\} \tag{2-15}$$

involving the complex multiplication of $z(t)$ by $e^{j\theta}$. Equivalently, by (2-10) and (2-13)

$$x(t) = \mathrm{Re}\left\{\zeta(t)e^{j2\pi f_0 t}\right\} \tag{2-16}$$

and

$$\tilde{x}(t) = \mathrm{Re}\left\{\zeta(t)e^{j\theta}e^{j2\pi f_0 t}\right\} \tag{2-17}$$

which implies that the phaseshift θ may be effected at baseband, involving, again, complex multiplication. Obviously, the phaseshifted complex envelope

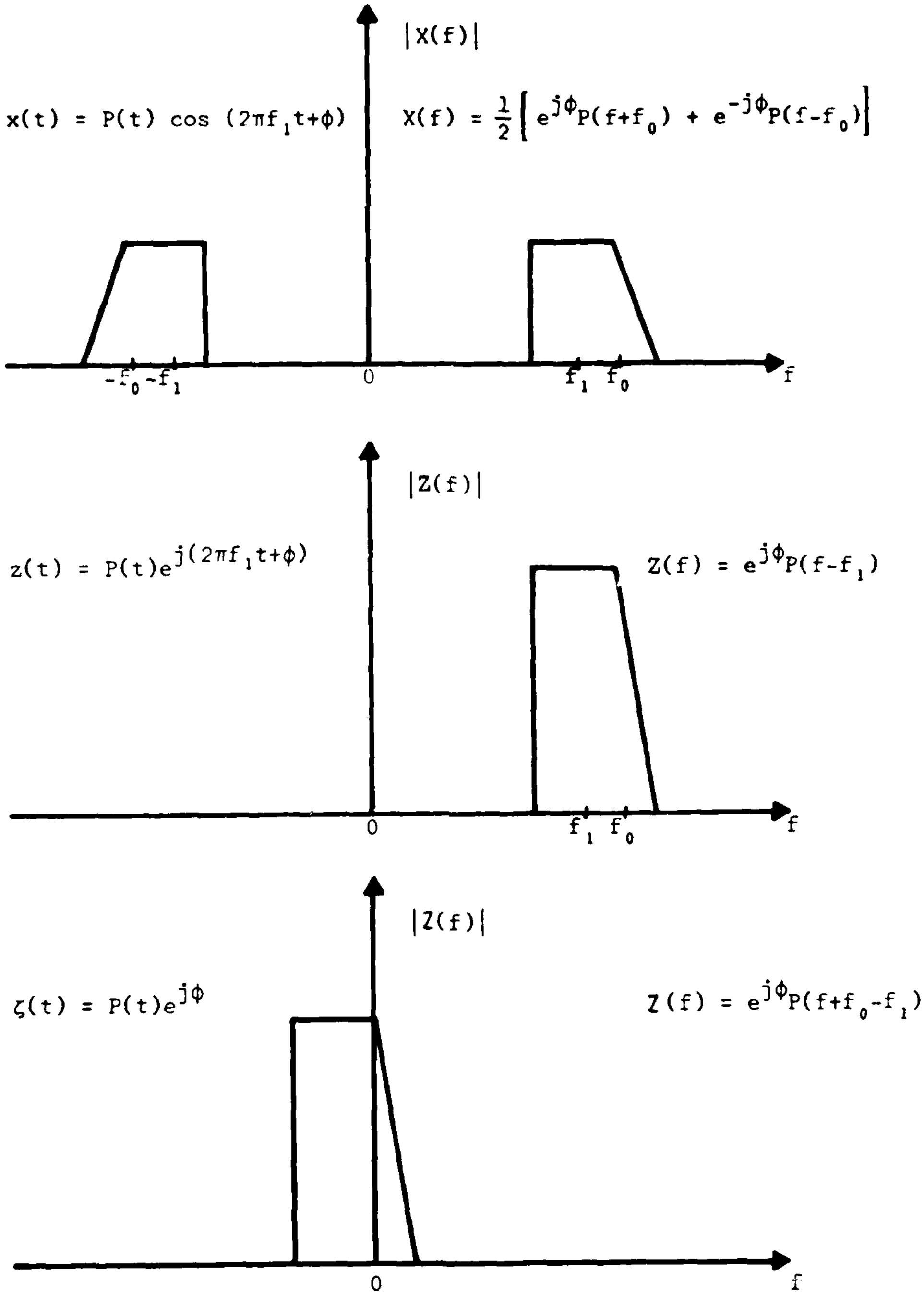


Figure 2-4. Relationships between amplitude spectra of $x(t)$, $z(t)$, and $\zeta(t)$, the bandpass signal, the complex pre-envelope and the complex envelope, respectively. Note that the baseband downconversion shift is $f_0 \neq f_1$.

is given by

$$\tilde{\zeta}(t) = e^{j\theta}\zeta(t) \qquad (2\text{-}18)$$

It should be emphasized at this point that although (2-8) seems to be an intuitively appealing and logical expression for the complex envelope of x(t), in general, f_1 is not known or is not even meaningful. Typically, the bandpass character of x(t) is established via appropriate filtering and f_0 is chosen to be somewhere at the center of the band. The important criterion for downconversion is that x(t) be limited within a band that does not exceed $2f_0$, so as to allow for effective filtering out of the $2f_0$ spectrum that is produced in the underlying mixing process. Only then does the notion of complex envelope discussed above make sense.

2.3.2 General Form

Assuming the source signal (2-2), the general form of the equivalent baseband channel is derived in three stages; that is, the part from source through antenna, through the final IF stage, and finally, through to baseband, as suggested in Figure 2-5.

2.3.2.1 Source through Antenna

According to the discussion in Section 2.3, the complex pre-envelope of the source signal (2-2) is given by

$$z(t) = P(t)e^{j2\pi f_1 t} \qquad (2\text{-}19)$$

Given a linear array of uniformly-spaced omnidirectional elements with weighting $\{a_\ell\}_{\ell=1}^{L}$ and spacing d given by (2-4), let signal (2-19) be incident onto this antenna at an angle θ measured clockwise from broadside. According to the geometry of Figure 2-6, the complex pre-envelope at the antenna terminal is simply

$$z_A(t) = \sum_{\ell=1}^{L} a_\ell P(t+\tau_\ell)e^{j2\pi f_1(t+\tau_\ell)} \qquad (2\text{-}20)$$

where τ_ℓ is the time advance in the received signal at the ℓ-th element with respect to reference at the origin of the x-y coordinate system in Figure 2-6.

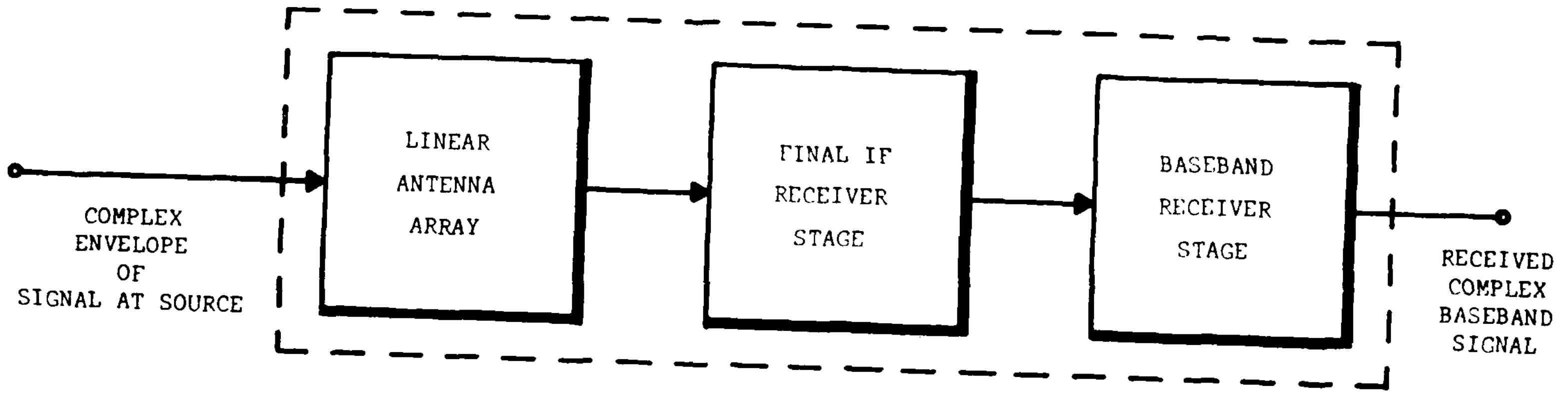


Figure 2-5. Three Stages of the Equivalent Baseband Channel.

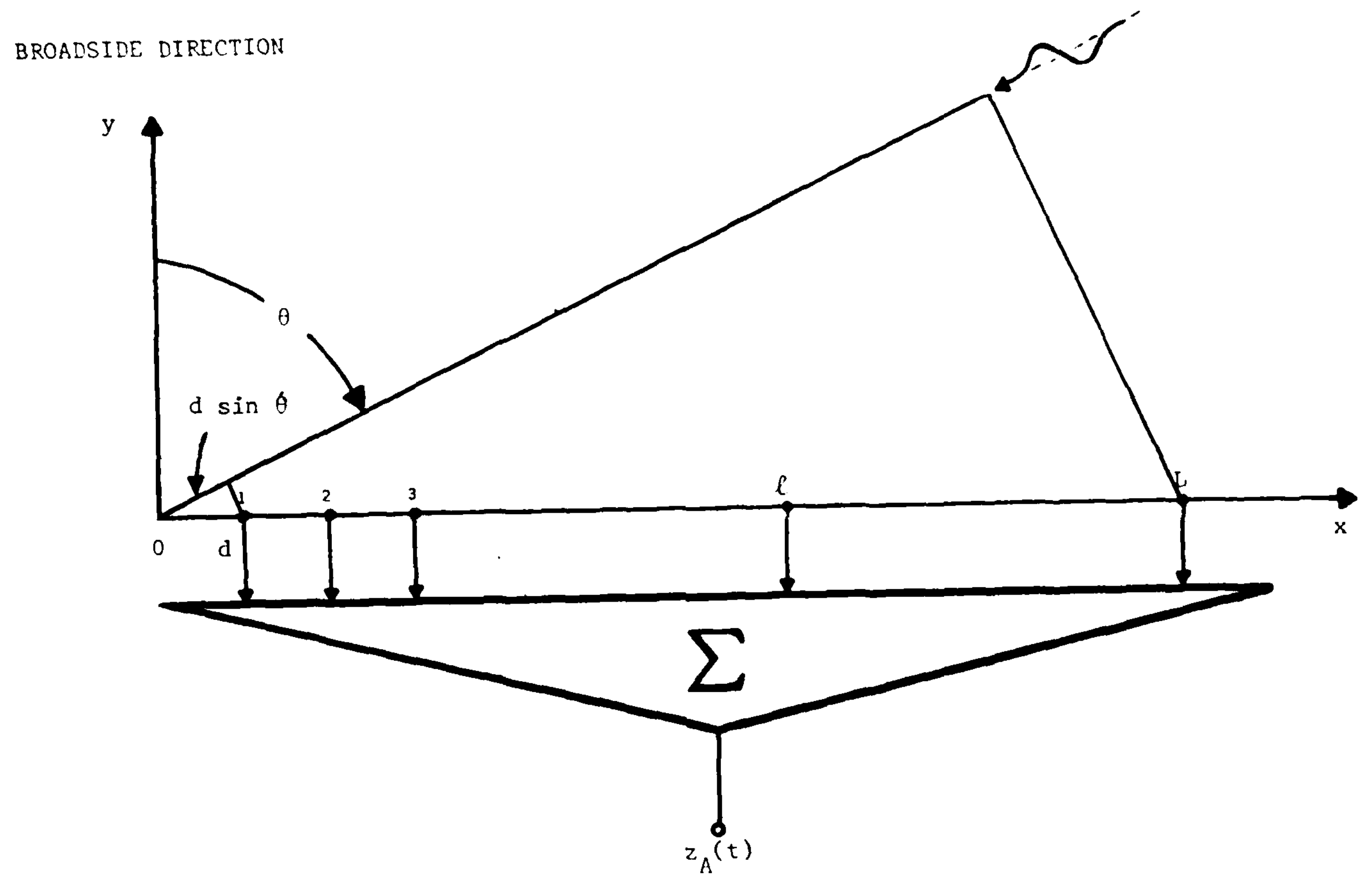


Figure 2-6. Source Signal and Antenna Geometry.

In view of (2-4), τ_ℓ is given by

$$\tau_\ell = \frac{\ell d \sin\theta}{c}$$

$$= \ell d_0 \frac{\lambda_0}{2c} \sin\theta$$

$$= \frac{\ell d_0 \sin\theta}{2f_0} \qquad (2\text{-}21)$$

where, as mentioned previously, d_0 is a dimensionless quantity less than or equal to unity.

For simplicity, consider the general ℓ-th element of the summation in (2-20); namely,

$$z_\ell(t) = a_\ell P(t+\tau_\ell)e^{j2\pi f_1(t+\tau_\ell)} \qquad (2\text{-}22)$$

Its Fourier transform is given by

$$Z_\ell(f) = a_\ell \int_{-\infty}^{\infty} P(t+\tau_\ell)e^{j2\pi f_1(t+\tau_\ell)} \; e^{-j2\pi ft} \, dt$$

$$= a_\ell e^{j2\pi f\tau_\ell} \int_{-\infty}^{\infty} P(t+\tau_\ell) \; e^{-j2\pi(f-f_1)(t+\tau_\ell)} \; d(t+\tau_\ell)$$

$$= a_\ell P(f-f_1)e^{j2\pi f\tau_\ell} \qquad (2\text{-}23)$$

which, in view of the previous discussion, is a positive-sided transform. The corresponding transform of the complex envelope of (2-22) is simply the translation of (2-23) by the baseband local oscillator frequency f_0; that is,

$$Z_\ell(f) = a_\ell e^{j2\pi(f_0+f)\tau_\ell} \; P(f+f_0-f_1) \qquad (2\text{-}24)$$

whence, the complex envelope is

$$\zeta_\ell(t) = a_\ell \, \mathfrak{F}^{-1}\{Z_\ell(f)\}$$

$$= a_\ell \, \mathfrak{F}^{-1}\{F(f+f_0-f_1)\} \;\ast\; \mathfrak{F}^{-1}\{e^{j2\pi(f_0+f)\tau_\ell}\}$$

$$= a_\ell \; P(t)e^{j2\pi(f_0-f_1)t} \;\ast\; e^{j2\pi f_0\tau_\ell} \;\; \delta(t+\tau_\ell) \qquad (2\text{-}25)$$

which, by the sifting property of the δ-function, reduces to

$$\zeta_\ell(t) = a_\ell \; e^{j2\pi f_0\tau_\ell} \;\; P(t+\tau_\ell)e^{j2\pi(f_1-f_0)(t+\tau_\ell)} \qquad (2\text{-}26)$$

a simple f_0 translation of (2-22), as it should be. The significance of (2-25), however, is that it characterizes the transfer function from the signal source to the ℓ-th antenna element of the array in Figure 2-6. Of course, the transfer function up to the antenna terminal consists of a summation of such terms over all the antenna elements. As such,

$$\zeta_A(t) = P(t)e^{j2\pi(f_1-f_0)t} \;\ast \sum_{\ell=1}^{L} a_\ell e^{j2\pi f_0\tau_\ell} \;\; \delta(t+\tau_\ell) \qquad (2\text{-}27)$$

which suggests the structure of the channel from signal source to antenna terminal, as shown in Figure 2-7. Indicated there is the augmented transmitted envelope,

$$\zeta(t) = P(t)e^{j2\pi(f_1-f_0)t} \qquad (2\text{-}28)$$

presented at the input of the antenna array transfer function

$$A(f) = \sum_{\ell=1}^{L} a_\ell e^{j2\pi(f_0+f)\tau_\ell} \qquad (2\text{-}29)$$

and resulting in the desired received complex envelope (2-27).

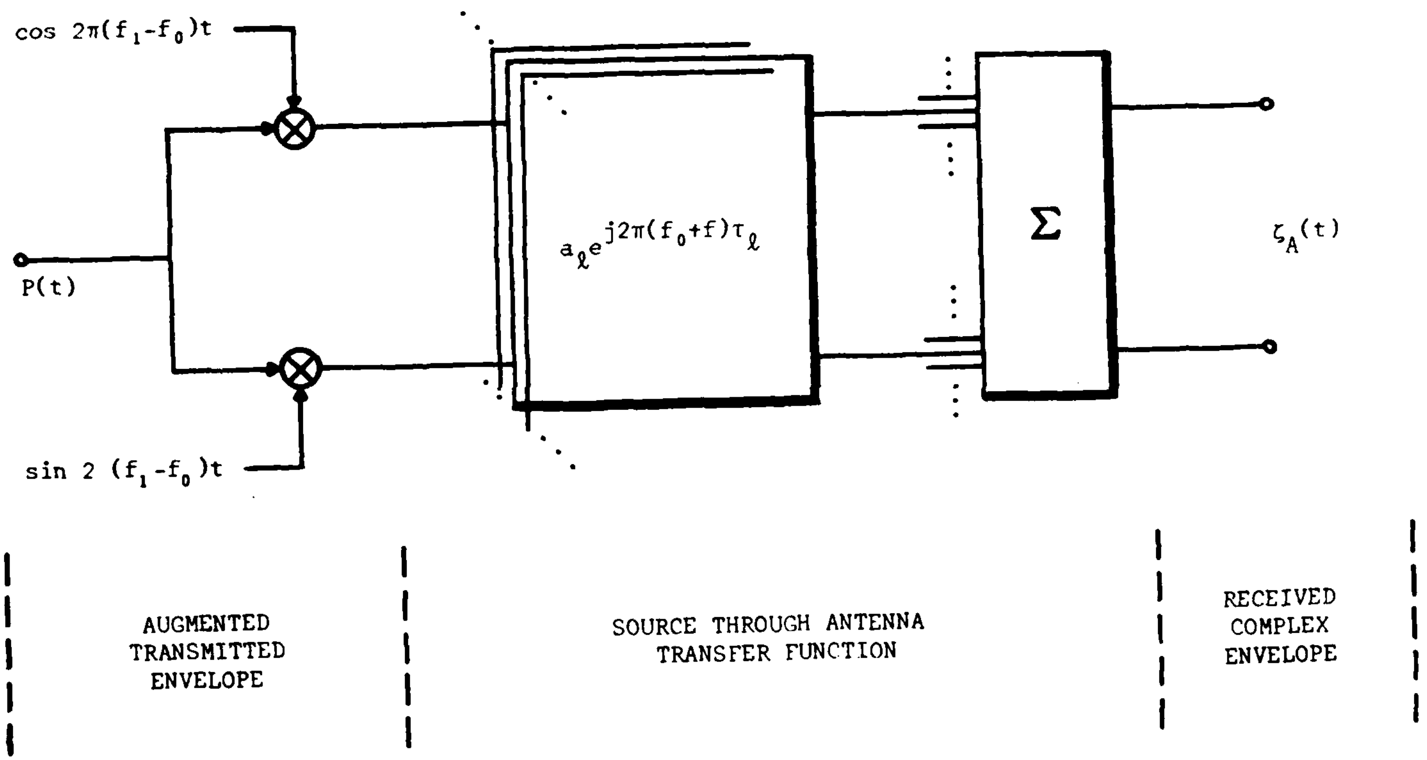


Figure 2-7. Mathematical Model of Equivalent Baseband Channel from Signal Source to Antenna Array Terminal.

2.3.2.2 Source through Final IF

Assume that the processing bandwidth of the receiver connected to the antenna terminal is essentially established by the final IF filter whose transfer function is given by $H(f)$. Then, the Fourier transform of the complex pre-envelope for the ℓ-th element may be written as

$$Z_\ell(f) = a_\ell P(f+f_0-f_I-f_1)e^{j2\pi(f+f_0-f_I)\tau_\ell} \; H^+(f) \qquad (2\text{-}30)$$

whence, the Fourier transform of the envelope becomes

$$Z_\ell(f) = a_\ell P(f+f_0-f_1)e^{j2\pi(f+f_0)\tau_\ell} \; H^+(f+f_I) \qquad (2\text{-}31)$$

and the actual complex envelope is found, as before, to be

$$\zeta_\ell(t) = a_\ell P(t)e^{j2\pi(f_1-f_0)t} * \mathfrak{F}^{-1}\left[e^{j2\pi(f_0+f)\tau_\ell} \; H^+(f+f_I)\right] \qquad (2\text{-}32)$$

Figure 2-8 suggests the derivation of the received envelope of the ℓ-th element carried to the output of a final IF filter, showing amplitude spectrum of the pre-envelope, $|Z(f)|$, the positive-frequency IF-filter amplitude response, $|H^+(f)|$, and, finally, the spectrum magnitude of the received baseband complex envelope, $|Z(f)|$.

The received envelope by the entire antenna array at the final IF filter output is given by summation of (2-32) over the elements of the array; namely,

$$\zeta_{IF}(t) = P(t)e^{j2\pi(f_1-f_0)t} * \mathfrak{F}^{-1}\left\{\sum_{\ell=1}^{L} a_\ell e^{j2\pi(f_0+f)\tau_\ell} \; H^+(f+f_I)\right\} \qquad (2\text{-}33)$$

The equivalent baseband channel from signal source through the antenna array and the final IF filter is shown in Figure 2-9.

2.3.2.3 Source through Baseband

At the last downconversion stage from final IF to baseband, individual I and Q filters must be used to reject the 2f spectrum that results from the

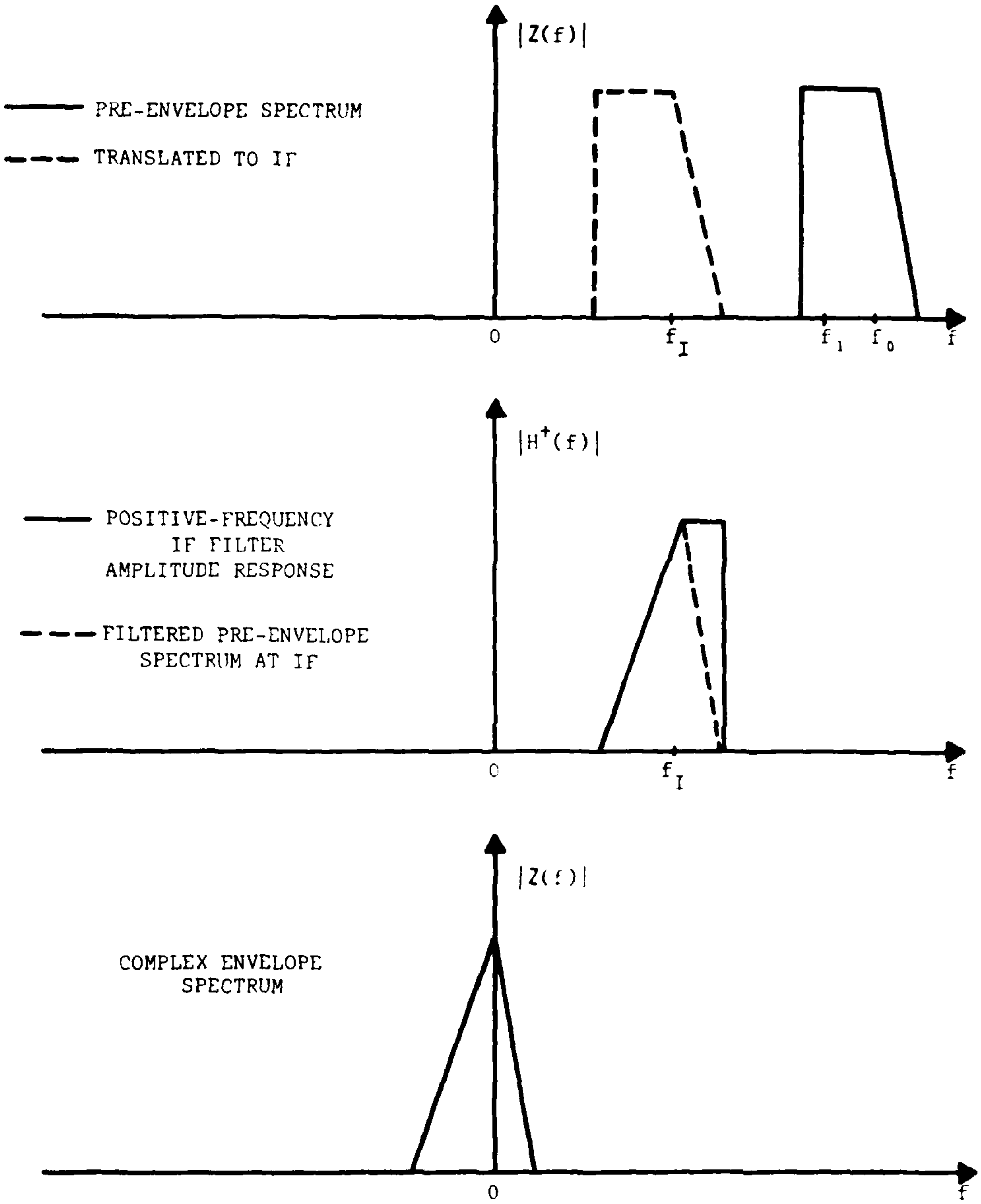


Figure 2-8. Evolution of Complex Envelope Amplitude Spectrum from Original Source Signal Pre-Envelope and Final IF Filter Response.

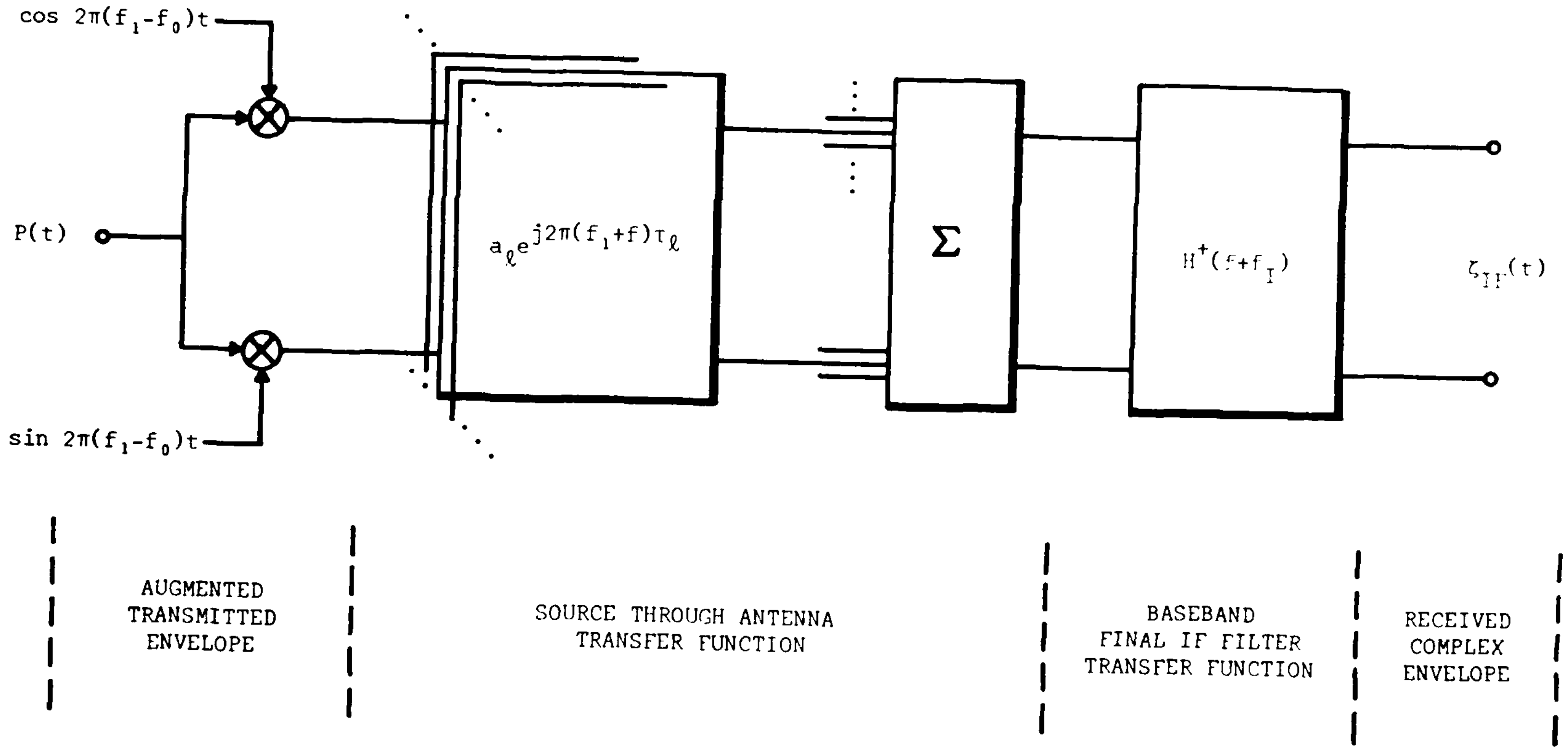


Figure 2-9. Mathematical Model of Equivalent Baseband Channel from Signal Source through Antenna Array, to Final IF Filter Output.

underlying mixing process. Because of limited physical tolerances these two filters may not, in general, be assumed identical and thus should be distinguished by transfer functions $H_I(f)$ and $H_Q(f)$, respectively. Figure 2-10 gives the mathematical model of the general form of the equivalent baseband channel from signal source to baseband. The received envelope in this case may be specified by defining the real and imaginary parts separately; that is,

$$\mathrm{Re}\ \zeta_B(t) = P(t)e^{j2\pi(f_1-f_0)t} * \mathfrak{F}^{-1}\left\{\sum_{\ell=1}^{L} a_\ell e^{j2\pi(f_0+f)\tau_\ell}\ H^{+}(f+f_I)H_I(f)\right\} \quad (2\text{-}34.1)$$

$$\mathrm{Im}\ \zeta_B(t) = P(t)e^{j2\pi(f_1-f_0)t} * \mathfrak{F}^{-1}\left\{\sum_{\ell=1}^{L} a_\ell e^{j2\pi(f_0+f)\tau_\ell}\ H^{+}(f+f_I)H_Q(f)\right\} \quad (2\text{-}34.2)$$

It should be mentioned that although (2-34) characterizes the desired baseband channel quite generally, it remains possible to further simplify the expressions by invoking certain reasonable assumptions. The resulting alternate characterization, although simpler, succeeds in embodying the essential features of the system that account for the substantial source of relative distortion between channels.

2.3.3 Alternate Form

It is common practice in receiver design that the final IF filter establish the operating bandwidth. In downconverting to baseband, the nearly identical baseband filter transfer functions, $H_I(f)$ and $H_Q(f)$, need not be as selective as the IF transfer function, $H(f)$. The function of baseband filtering is to reject the 2f spectrum resulting from the final stage of mixing in the downconversion process. In view of this fact, the equivalent baseband channel as characterized by (2-34) reduces to the simpler form of (2-33).

Another observation to be made at this stage is that from the point of view of assessing the performance of adaptive processing, it is not essential that the spectrum of $P(t)$ be centered about a carrier frequency f_1 different from f_0. Choosing $f_1=f_0$, reduces (2-33) to the simpler received envelope expression

$$\zeta_B(t) = P(t) * \mathfrak{F}^{-1}\left\{\sum_{\ell=1}^{L} a_\ell e^{j2\pi(f_0+f)\tau_\ell}\ H^{+}(\ \ f_I)\right\} \quad (2\text{-}35)$$

The associated equivalent baseband channel is considerably simpler, as shown in Figure 2-11.

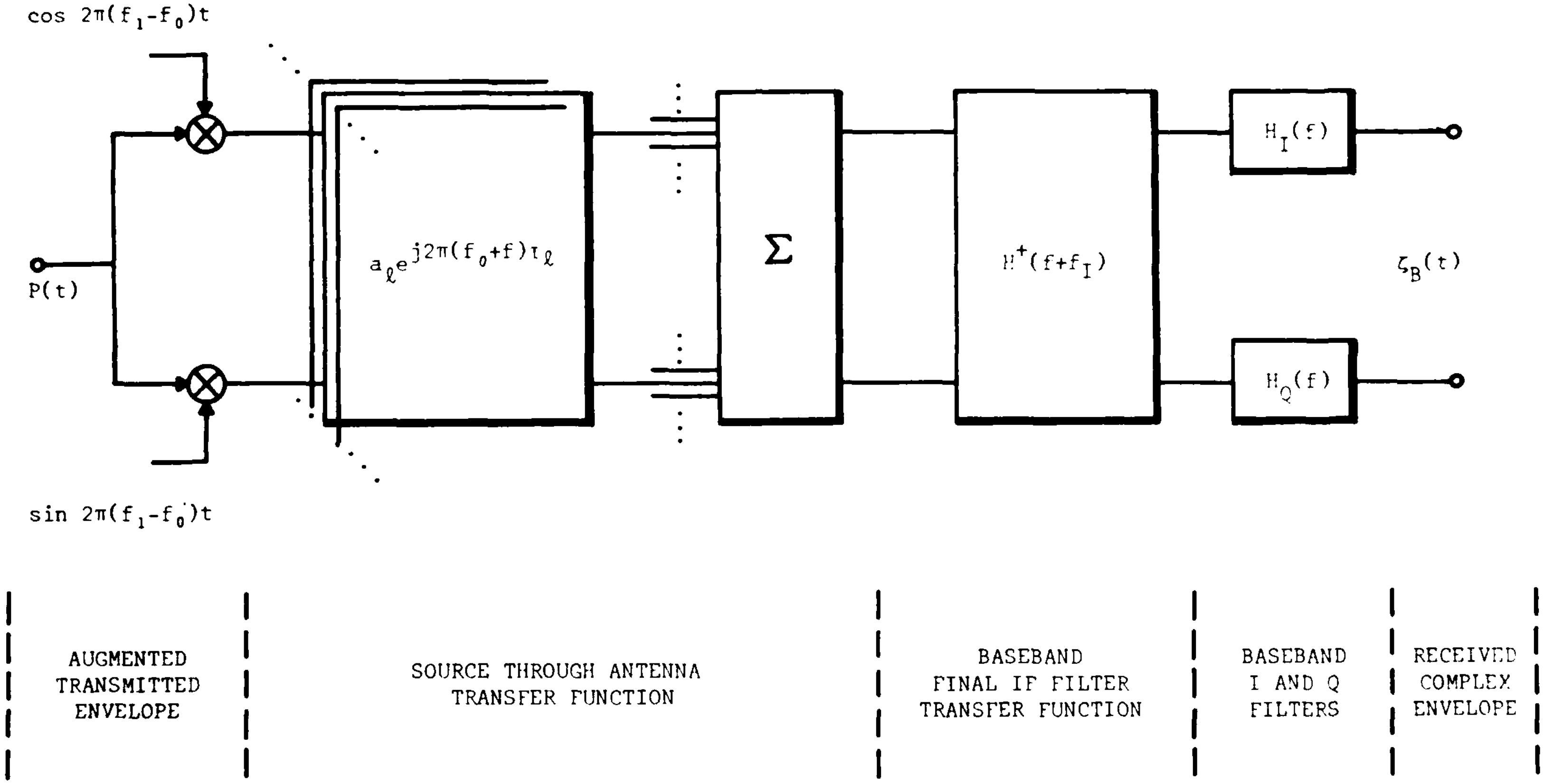


Figure 2-10. Mathematical Model of Equivalent Baseband Channel from Signal Source through Antenna Array, through Final IF Filter, down to Baseband.

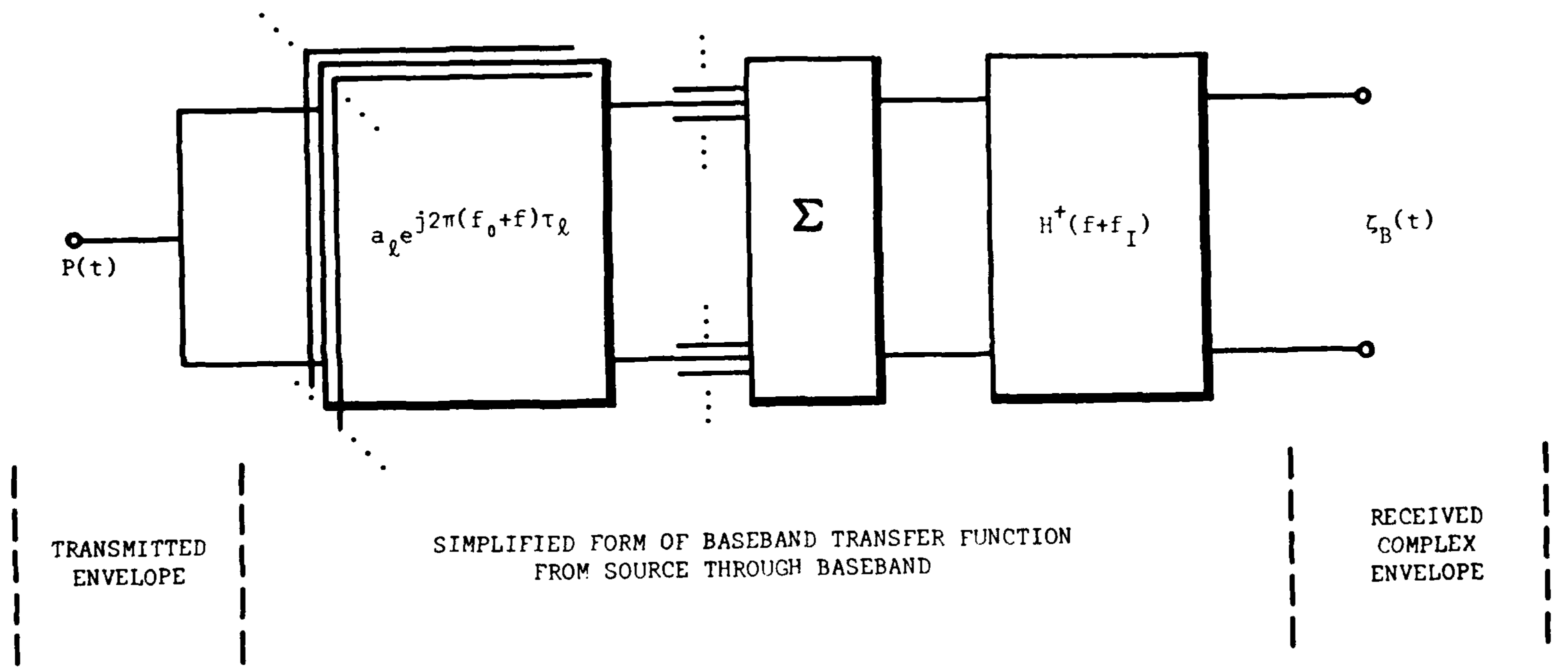


Figure 2-11. Mathematical Model of Alternate Form of Equivalent Baseband Channel from Source through Baseband.

2.4 Received Baseband Signal

The form of the baseband signal to be adapted for subsequent analysis will be (2-35), the simplest expression for the received complex envelope at baseband. Letting the equivalent baseband channel transfer function be given by

$$C(f) = \sum_{\ell=1}^{L} a_\ell e^{j2\pi(f_0+f)\tau_\ell} \; H^+(f+f_I) \tag{2-36}$$

associated with a complex impulse response c(t), the received complex envelope at baseband (2-35) may be rewritten, more explicitly, as

$$\begin{aligned} \zeta_B(t) &= P(t) * c(t) \\ &= \int_{-\infty}^{\infty} P(\tau)c(t-\tau)d\tau \end{aligned} \tag{2-37}$$

This last expression suggests the practical means for computing a sequence of sampled values of the complex envelope (2-37), namely, a discrete convolution of sampled values of P(t) and a finite M-sample impuse response approximating c(t); that is,

$$\zeta_B(n) = \sum_{m=1}^{\min(n,M)} P(n+1-m)c(m) \tag{2-38}$$

Of course, it is understood that these samples must occur at the Nyquist rate, or, more precisely, at twice the cutoff frequency of the equivalent baseband channel.

3.0 SYSTEM CONSIDERATIONS

The operational effectiveness of an adaptive array system depends directly on a number of physical factors which will tend to establish bounds on its ultimate nulling performance. Whereas a dynamic signal environment will bring about a performance degradation that can vary considerably according to algorithm, other pertinent physical factors such as receiver mismatch, operational bandwidth, and multipath effects account for performance limits independent of algorithm.

3.1 Signal Dynamics

Continuous noise interferring sources do not present any particular difficulty to the nulling performance of even the simplest of adaptive algorithms such as the LMS [7], [8]. However, blinking interferences could conceivably degrade the performance of adaptive array systems employing even more sophisticated dynamic algorithms [9].

In the present contract, the study of the Batch Covariance Relaxation (BCR) algorithm [10] is motivated not only by its potentially efficient architectural implementation but just as importantly, by its batch processing operation, which accounts for a certain level of immunity to a dynamic signal environment. This necessary feature of any future adaptive array process will be elaborated further in a future memorandum intended to describe the essential aspects of the BCR process.

3.2 Receiver Mismatch

A very fundamental physical limitation on the performance of an adaptive array system is the relative matching between the main and auxiliary-channel receivers. Mismatch among receivers is due to filter mismatch at a point of highest selectivity in the downconversion chain.

Typically, the receiver operating bandwidth is established at the final IF stage whether or not the process that follows remains at IF or takes place at a subsequent baseband stage. In this situation, the final IF filter mismatch is due to two design inaccuracies; that is, center-frequency alignment and percent-bandwidth deviation. At baseband, the I and Q lowpass filters need only reject the $2f_I$ spectrum and hence may be wideband compared to the IF, thereby not adding appreciably to the overall mismatch.

Alternatively, it may make sense to consider establishing the receiver operating bandwidth at baseband by sufficiently narrow separate I and Q lowpass filters whose transfer functions, $H_I(f)$ and $H_Q(f)$, may deviate in percent-bandwidth.

As already discussed, receiver mismatch may be attributed to deviations in both IF and baseband filter characteristics. For the purposes of the present investigation, consideration will be limited to mismatch that is entirely due to final IF filter deviations, consistent with the typical receiver implementation.

3.3 Wideband Operation

In developing the expression for the transfer function of the equivalent baseband channel in Section 2.0, the corresponding linear array transfer function has been shown to be

$$A(f) = \sum_{\ell=1}^{L} a_\ell e^{j2\pi(f_0+f)\tau_\ell} \tag{3-1}$$

where $\{a_\ell\}_{\ell=1}^{L}$ is the antenna weighting sequence and τ_ℓ is the negative time delay at the ℓ-th element relative to the origin. For a uniform element spacing, the use of (2-21) simplifies (3-1) to

$$A(f) = \sum_{\ell=1}^{L} a_\ell e^{j\ell\pi d_0 \sin\theta \left(1 + \frac{f}{f_0}\right)} \tag{3-2}$$

where θ is the angle of arrival measured from broadside.

For $\theta \approx 0$, $A(f)$ exhibits nearly constant amplitude and linear phase characteristics over a wide variation of f about f_0. However, with increasing aperture (large L), bandwidth (large f) and θ, $A(f)$ will exhibit an increasing measure of amplitude and phase distortion. This trend is manifested by an increased sensitivity in the sidelobe structure of the radiation pattern. More specifically, at a sufficiently large angle θ, a pattern null at some frequency f will not remain such over the operating bandwidth. As a consequence, the transfer function over the band of interest could exhibit a stopband as indicated in Figure 3-1.

The difficulty of cancelling a single interference arriving at a far sidelobe of the large main array by combining with it a singly-weighted omnidirectional auxiliary is clearly seen. In the latter case, the antenna transfer function will exhibit nearly constant amplitude and linear phase over the bandwidth B. The two transfer functions will simply not match with the single degree of freedom provided by one complex auxiliary weight.

3.3.1 Tapped Delayline Option

Achieving a so-called "wideband null" is synonymous to matching the auxiliary channel to the main one. This may be accomplished by using multiple auxiliary taps in the form of a tapped delayline. The number of taps to be used depends directly on the complexity of the main channel response. On the other hand, the time-spread of the taps need not exceed the maximum time delay expected over the main antenna aperture.

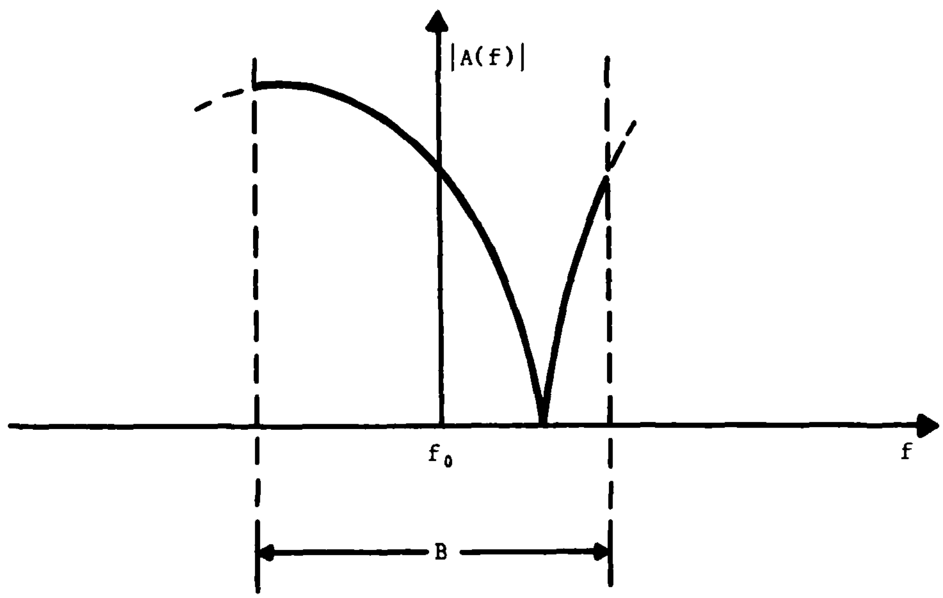


Figure 3-1. Potential Stopband Behavior of Large Antenna Array Transfer Function at a Far Sidelobe With Wide Operational Bandwidth B.

The general problem of matching one channel with another by means of a tapped delayline has been addressed previously in a specific application to equalization [10]. Although the present situation is no different, some care must be exercised that the tap weight adjustment does not deteriorate the quality of the established main beam. Of course, depending on the antenna scheme and adaptive algorithm employed, this concern may or may not be significant. In a fully-adaptive antenna scheme the tap weights must be constrained to maintain a desired beam and low sidelobes while simultaneously suppressing interferences. In a sparsely adaptive antenna arrangement that is being considered presently, the tap weights need to be constrained so as to maintain the integrity of the main beam, something that is most crucial when using a dynamic algorithm and less so when using a batch algorithm.

3.3.2 Single Tap Option

The effect of a delayline tap-weighting may be accomplished equivalently by using a sufficiently large number of singly-tapped auxiliaries. Of course, this approach may be undesirable because of the fact that it would mean a large increase in the number of receivers used. On the other hand, spread randomly over the aperture, the singly-weighted auxiliaries could conceivably act in a more optimal way to simultaneously provide channel matching and interference suppression. Although this alternative may not apply to a space-fed antenna structure, it does apply to a corporate-fed one that is presently being considered.

3.4 Multipath Phenomenon

The mathematical model developed in Section 2.0 is valid when considering an antenna array by itself and ignoring mutual coupling between elements. While it is recognized that interelement coupling will tend to limit the achievement of the desired main array pattern, or hamper adaptive beamforming there is no such concern in the partially adaptive array system considered here.

A phenomenon of much greater concern which could severely limit nulling performance is multipath. In a practical deployment of any antenna system, nearby supporting structures and other scattering bodies within a wide field of view can easily account for alternate paths via which a signal could arrive at the antenna, besides the expected direct path.

3.4.1 Channel Description

The multipath phenomenon may be integrated conveniently in the description of the equivalent baseband channel. Specifically, given the direct and an alternate path, the resulting compound channel is as shown in Figure 3-2. To be noted, there, are the portions of the primary and alternate paths originating from the common source and terminating at the common input to the baseband IF filter transfer function. The distinction in the two paths is attributed not only to the extra relative delay in the alternate path, but, also to its baseband antenna array transfer function. The latter distinction is based on the fact that the alternate path angle of arrival is necessarily different from that of the primary path. More precisely, the primary and secondary path antenna transfer functions are given, respectively, by

$$\left.\begin{aligned} A(f) &= \sum_{\ell=1}^{L} a_\ell e^{j2\pi(f_0+f)\tau_\ell} \\ A'(f) &= \sum_{\ell=1}^{L} a_\ell e^{j2\pi(f_0+f)\tau'_\ell} \end{aligned}\right\} \qquad (3\text{-}3)$$

where τ_ℓ and τ'_ℓ are the distinct negative time delays at the ℓ-th element relative to a convenient origin. These time delays are given by

$$\left.\begin{aligned} \tau_\ell &= \frac{\ell d_0 \sin\theta}{2f_0} \\ \tau'_\ell &= \frac{\ell d_0 \sin\theta'}{2f_0} \end{aligned}\right\} \qquad (3\text{-}4)$$

where θ and θ' are the corresponding angles of arrival measured clockwise from broadside. The delay transfer function associated with the alternate path is given by

$$D(f) = c' e^{-j2\pi(f_0+f)\tau'} \qquad (3\text{-}5)$$

where c' is generally a complex constant representing amplitude and relative phase of the secondary signal in relation to the primary one, and τ' is the associated relative delay.

The extension of the channel description to any number of alternate paths is obvious from the above. The received complex envelope is computed in a

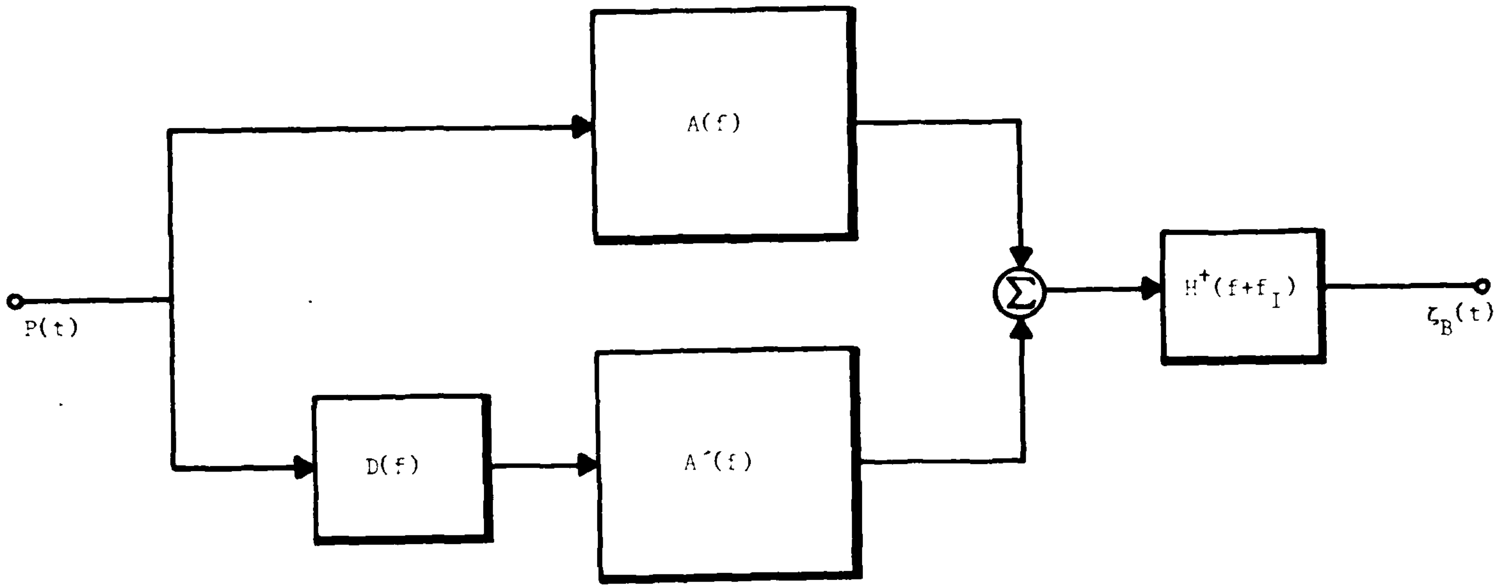


Figure 3-2. Equivalent Baseband Compound Channel Combining the Primary and one Alternate Path from the Source to the Array System.

similar way as already suggested in Section 2.0. That is, with the compound transfer function defined, the desired baseband signal may be expressed as a discrete convolution of the source signal envelope with the channel impulse response according to (2-38).

3.4.2 Effect and Remedies

As shown above, the compound channel transfer function for a two-path situation is of the form

$$C(f) = A(f) + D(f)A'(f) \tag{3-6}$$

If $A'(f) \approx A(f)$, $C(f)$ will be characterized by a rippled amplitude response, in view of (3-5). The amplitude of the ripple will depend directly on c' and its period on τ'. A very similar effect should result in the general case when $A'(f) \neq A(f)$, although the ripple may be somewhat randomized, especially with increased operating bandwidth.

Although rippled amplitude responses will be induced in any antenna transfer function by multipath, it is improbable that the fine structures would match for two different antennas. It is not surprising then that channels corresponding to a multielement main array and a single element auxiliary would differ considerably. As a consequence, it may generally be impossible to provide proper cancellation of a multipath signal incident onto the main array by combining it with a singly-weighted auxiliary. Extra degrees of freedom will be needed to deal with cancellation in a multipath environment, and even more with increasing operating bandwidth.

One possible way to provide effective sidelobe cancellation for a wide operating bandwidth in the presence of multipath is to use a channelized approach. This scheme utilizes a bank of filters to subdivide the operating band and seeks individual adaptive weights for each. Conceivably, if the subdivision is fine enough so that the multipath-induced ripple is not noticeable within an individual filter band, the corresponding single complex weight per filter would suffice. One possible disadvantage in this approach is the potential difficulty in maintaining a certain matching tolerance in the filters comprising the filter bank. More generally, spectral-domain adaptive processing may be used [12].

Two other means for dealing with cancellation in multipath involve the use of tapped delayline auxiliaries or an equivalent multiplicity of singly-weighted auxiliaries, as already mentioned in Section 3.4. The advantage of either of these two approaches over the channelized scheme is that no additional distortion is introduced. Otherwise, the total number of adaptive weights, the degrees of freedom used, are essentially the same in each case.

4.0 REMARKS

The main objective in the present memorandum has been to develop a mathematical model that would reasonably describe the transformation of a source signal through a linear antenna array down to the baseband of its associated receiver. In accomplishing this, an equivalent baseband channel has been defined in terms of a transfer function that relates the source signal envelope at its input to the received complex envelope at its baseband output. Clearly, then, the received complex envelope may be expressed as a convolution of the input source envelope with the channel impulse response. The channel transfer function has been modeled to take into account spatial dispersion depending on the angle of arrival and antenna configuration, incorporating, as well, the associated receiver characteristics. Doing this is crucial in evaluating the performance of an adaptive array system, since it involves combining the main antenna and its receiver to a number of auxiliaries and their associated receivers, with differing characteristics embodied in the basic mathematical model.

A secondary objective of the present memorandum is to address some system issues that pertain to the general performance characteristics of an adaptive array system. Besides inter-receiver mismatch, some attention is focused onto the intrinsic difficulties that may be encountered in a wideband operation, and appropriate means for circumventing a potentially poor performance. The multipath phenomenon is also examined and possible remedies against its effects on performance are discussed. Multiple degrees of freedom are recommended for effective nulling performance both in the case of wideband operation as well as in a multipath situation. Degrees of freedom may be realized via a multiplicity of singly-weighted randomly emplaced auxiliaries, multiply tapped delayline auxiliaries or through receiver channelization using a filter bank or more generally via spectral weighting.

In defining the mathematical model as described in this memorandum, an attempt has been made to incorporate the most prominent limitations in adaptive array performance within the realm of one possible antenna configuration. Antenna aperture dispersion, receiver mismatch, wideband operation, and multipath effects, however, constitute important limitations to general adaptive array performance. Further limitations such as cable reflections and mixer-generated noise were not incorporated in the model. Their effects have been considered of secondary importance assuming, of course, proper design.

BIBLIOGRAPHY

[1] Taylor, T. T., "Design of Line-Source Antennas for Narrow Beam Width and Low Side Lobes," IRE Trans. on Antennas and Propagation, Vol. AP-3, pp. 16-28, January 1955.

[2] Lo, Y. T., "A Mathematical Theory of Antenna Arrays with Randomly Spaced Elements," IRE Trans. on Antennas and Propagation, Vol. AP-12, pp. 257-268, May 1964.

[3] Lo, Y. T., Lee, S. W., Lee, Q. H., "Optimization of Directivity and Signal-to-Noise Ratio of an Arbitrary Antenna Array," Proc. of the IEEE, Vol. 54, pp. 1033-1045, August 1966.

[4] Steinberg, B. D., "The Peak Sidelobe of the Phased Array Having Randomly Located Elements," IEEE Trans. on Antennas and Propagation, Vol. AP-20, No. 2, March 1972.

[5] Peterson, K. M., "A Note on the Occurrence of Blind Regions in Sidelobe Cancelling," Motorola Technical Memorandum No. M-165, October 5, 1977.

[6] Row, H. E., "Signals and Noise in Communications Systems," Van Nostrand, Princeton, N. J., 1965, pp. 60-64.

[7] Widrow, B., et al, "Adaptive Antenna System," Proc. of the IEEE, Vol. 55, No. 12, December 1967.

[8] Daniel, S. M., "Stable Stochastic Steepest Descent Control for Adaptive Array Processing," Motorola Technical Memorandum No. M-172, 1977. Also, Correlations - An Engineering Bulletin from Motorola, Inc., Vol. 1, No. 1, Winter 1978-79.

[9] Daniel, S. M., "Dynamic Performance of a Gradient-Controlled Adaptive Array Process in a Blinking Interference Environment," Motorola Technical Memorandum No. M-197, January 7, 1979 (SECRET).

[10] Daniel, S. M., "Digital Adaptive Array Processing Using Batch Covariance Relaxation (BCR)," Motorola Technical Memorandum No. M-194, September 22, 1978 (SECRET).

[11] Daniel, S. M., "Analysis, Simulation and Realization of a Digital Wire-line Equalizer System," 1973 IEEE Conference on Decision and Control, San Diego, California.

[12] Daniel, S. M., "Adaptive Beamforming in the Spectral Domain," 1973 IEEE G-AP International Symposium, Boulder, Colorado.

SIGNAL GENERATION PROGRAM

Prepared by: Sam M. Daniel
S. M. Daniel
Project Leader
Systems Analysis Group

Imre Kertesz
I. Kertesz
Systems Analysis Group

Reviewed by: Jaime R. Roman
J. R. Roman
Systems Analysis Group

R. Hecht-Nielsen
Systems Analysis Group

Approved by: I. Newton Durboraw
I. N. Durboraw
Manager
Systems Analysis Group

J. G. Doggett
J. . Doggett
Section Manager
Radar Section

R R Yost
R. R. Yost
Chief Engineer
Radar Operations

TABLE OF CONTENTS

1.0 INTRODUCTION

Project Memorandum 8512-01 has presented a fairly authentic mathematical model which can serve to describe the transmission of a noise signal through a linear antenna array and its associated receiver, all the way down to baseband. It has been shown that the complex baseband signal from a given signal source at some azimuth angle of incidence with respect to the antenna array happens to be the envelope of the source signal transmitted through an equivalent baseband channel which incorporates both receiver and antenna characteristics, including an explicit dependence on the angle of arrival. More specifically, the received complex baseband signal may be derived via a convolution of the source signal envelope with the complex impulse response of the equivalent baseband channel. Assuming linearity, the received baseband signal due to a number of source signals arriving from distinct angular directions is the superposition of their corresponding individual channel transmissions.

The present memorandum describes the development of a structured FORTRAN IV program for the purpose of generating received baseband signals at the multiple ports of an antenna system consisting of a Taylor-weighted large-aperture main linear array and a number of omnidirectional auxiliaries sparsely disposed over the aperture. Taking into account the underlying mathematical model already defined, pertinent system parameters are normalized in order to allow for easy generalization. Overall system and scenario description is provided via a carefully constructed input data file, SIGGENO:D, consisting of an easily readable table of user-selectable system parameter values. The actual signal generation program identified by the acronym SIGGENO is modularized into a number of elementary function-oriented subroutines for the sake of clarity, flexibility and adaptability to future growth.

The collection of the elementary subroutines, the source modules, comprise the signal generation source library. The corresponding set of compiled binary modules comprise the associated binary library. Through an appropriate job control language program, the composite binary module, SIGGENO:B, is synthesized from a selected set of binary modules. An executable load module, SIGGENO:L, is subsequently generated. Given the input data, SIGGENO:D, appropriate execution of SIGGENO:L generates the desired baseband port signals and stores them in an output data file, SIGGENO:O.

The validity of the signal generation program, SIGGENO, is ascertained via a successful demonstration of BCR adaptive processing using a specific set of signal data, SIGGENO:O, and an existing BCR simulation program. Actual simulation results are presented.

Keeping in mind the intent of a large-scale applicability of SIGGENO, it has been optimized for minimum-core usage. An attempt to time-optimize the program by means of partitioning turned out to be rather ineffective. Some recommendations for future program additions and modifications are made.

2.0 PARAMETRIC SYSTEM DESCRIPTION

Consider a multiport antenna system consisting of a Taylor-weighted main linear antenna array in conjunction with a number of omnidirectional auxiliaries, emplaced nonuniformly over the main antenna aperture. Given a number of noise signal sources incident onto the antenna system from distinct azimuth angles, we wish to derive received baseband signals at each of the antenna ports by means of a computer simulation program.

A logical development of a clear, efficient and flexible signal generation program must follow a concise parametric system description of the baseline system. With this information at hand, it will be possible to identify elementary system functions, define their mechanization into dedicated subroutine modules and establish a structured computer simulation program that should enhance system understanding, exhibit a high degree of flexibility and allow for a natural adaptability to efficient future growth.

The objective in the present section is to give a parametric system description of the underlying mathematical model and the normalization of some important parameters that will allow for useful generalization of results.

2.1 Mathematical Model

Project Memorandum 8512-01 [1] has already provided a mathematical model that describes the baseband reception of a noise signal incident onto a linear antenna array from a given angular direction. In fact, the received complex baseband signal has been shown to be

$$\zeta_B(t) = P(t) * c(t) \qquad (2\text{-}1)$$

where

$P(t)$ = the real-valued zero-mean envelope of the incident noise-source signal.

$$c(t) = \mathfrak{F}^{-1}\{C(f)\} \qquad (2\text{-}2)$$

= the equivalent baseband channel complex-valued impulse response that characterizes the transmission of the incident noise envelope through the linear array and down to baseband, thus yielding the received baseband signal, $\zeta_B(t)$.

$$C(f) = \sum_{\ell=1}^{L} a_\ell e^{j2\pi(f_0+f)\tau_\ell} H^+(f+f_I) \qquad (2\text{-}3)$$

= the equivalent baseband channel transfer function associated with impulse response $c(\tau)$.

$\{a_\ell\}_{\ell=1}^{L}$ = the L-element linear antenna array weighting sequence.

f_0 = the carrier frequency of the incident noise signal, taken also to be the center frequency of the system's RF band.

$$\tau_\ell = \frac{\ell d_0 \sin\theta}{2f_0} \qquad (2\text{-}4)$$

= the negative time-delay at the ℓ-th antenna element with respect to a reference origin

ℓ = antenna element number

d_0 = the interelement spacing factor, a dimensionless positive constant indicative of the amount of half-wavelength spacing between the uniformly-spaced antenna array elements at the center RF frequency, f_0.

θ = the azimuth angle of incidence of the noise-source signal, onto the linear antenna array, measured clockwise from its broadside direction

$H^+(f+f_I)$ = the positive-frequency transfer function of the receiver's final IF filter translated to baseband.

f_I = the center frequency of the receiver's IF band.

f = baseband frequency ranging around DC.

It should be clear from (2-1), (2-2) and (2-3) that the equivalent baseband channel transfer function, C(f), relates the input incident noise-source signal envelope, P(t), to the output received complex baseband signal, $\zeta_B(t)$. More precisely, (2-1) expresses $\zeta_B(t)$ as a convolution of P(t) with the equivalent baseband channel impulse response. To be noted here is the explicit dependence of the baseband channel transfer function, C(f), on the angle of arrival, θ. Assuming linearity, this implies that, in the general case involving a multiplicity of directionally-distinct noise-source signals, a received baseband port signal is the superposition of individual contributions from each noise-source signal via its associated distinct baseband channel.

Figure 2-1 shows a single-port linear antenna array system receiving three directionally-distinct signals. According to the mathematical model discussed above, the contribution of each incident signal to the actual received baseband port signal is simply its transmission through its associated baseband channel as indicated in Figure 2-2. Note that although the receiver's IF baseband transfer function is common to all channels, they are distinguished by their distinct baseband antenna transfer functions which will generally vary with angle of arrival. Clearly, in order to accommodate a situation involving N_n distinct noise sources and N_p antenna ports, it is necessary to specify $N_n \times N_p$ distinct baseband channels.

2.2 Normalized Model

Although the baseband channel transfer function given in (2-3) could be employed, as is, in a computer simulation program, it would be more desirable to eliminate its dependence on absolute frequency values of f, f_I and f_0. A normalized model would allow for a more meaningful interpretation and generalization of results. As a consequence, the user need not be concerned with specifying actual frequencies and bandwidths, but could instead deal with normalized baseband frequency, f, and normalized bandwidth at IF and RF.

2.2.1 Normalized Baseband Frequency

The basis for normalizing the mathematical model of the equivalent baseband transfer function, C(f), begins with a logical normalization of the baseband frequency, f. In view of existing convention of defining filters in terms of their lowpass prototypes having a 3-dB cutoff radian frequency of unity, $\Omega_0=1$, it makes sense to define a normalized baseband frequency, f, accordingly. As a consequence, normalized radian frequency is understood to be defined by

$$\Omega = \frac{\omega}{\omega_c} \qquad (2\text{-}5)$$

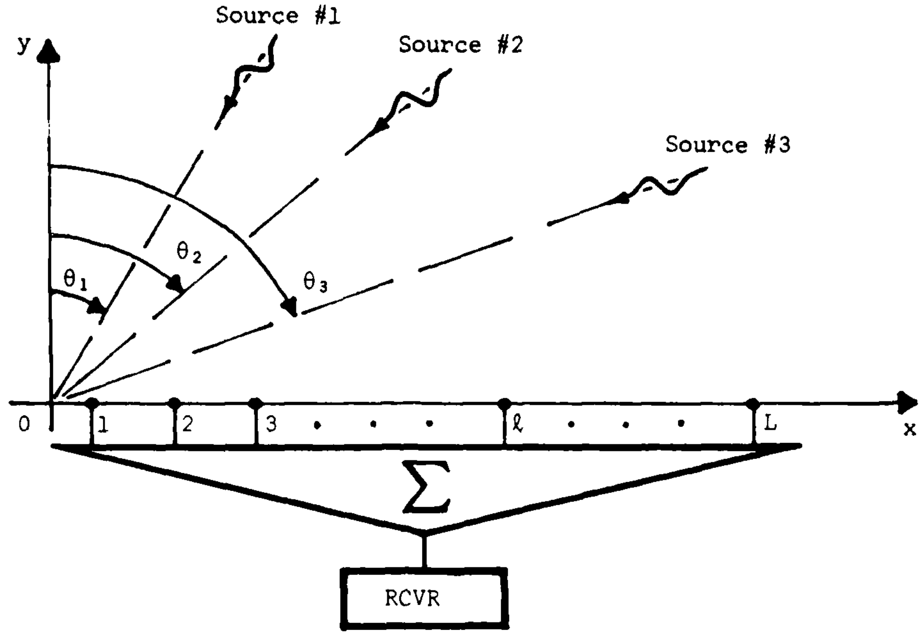


Figure 2-1. Single-Port Linear Antenna Array System with Three Directionally Distinct Incident Sources.

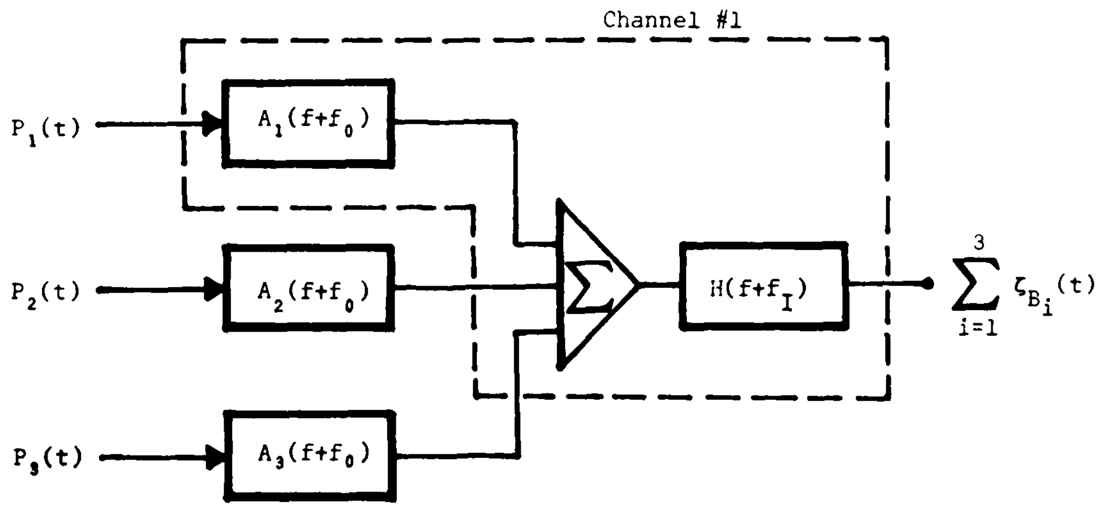


Figure 2-2. Baseband Port Reception of Multiple Directionally-Distinct Noise-Source Signals Via Superposition of Individual Baseband Channel Receptions

where, ω_c is the actual cutoff radian frequency and ω is the corresponding actual radian frequency. It is then logical to normalize the baseband frequency, f, in accordance to (2-5) in the sense that the final IF positive-frequency transfer function, which shapes the baseband spectrum in a generally asymmetric way about $\omega=0$, is attributable to a lowpass prototype filter.

In practice, the 3-dB cutoff frequency $f_c=\omega_c/2\pi$ is often referred to as the lowpass bandwidth of a lowpass filter. Recognizing that the lowpass filter is symmetric about the origin, we will henceforth refer to its baseband bandwidth of $2f_c=\omega_c/\pi$. Accordingly, its normalized radian bandwidth at baseband is equal to 2. Also, in order to eliminate the burden of π-handling we will systematically convert the expression of C(f) to one involving only Ω. In making this transition, the function symbol will remain the same to preserve association with original notation, even though it is recognized that the transformation $f \rightarrow \omega/2\pi$ leads to a function distinctly different from C.

2.2.2 Normalized IF Filter

The transfer function for the final IF filter may be derived via a bandpass transformation of a lowpass prototype filter having desired characteristics. More precisely, given a K-pole lowpass prototype filter transfer function

$$H_{LPP}(s) = \prod_{k=1}^{K} \frac{p_k}{(s-p_k)} \tag{2-6}$$

the transformation [2]

$$s \rightarrow \frac{1}{2}\left(s + \frac{\Omega^2{}_I}{s}\right) \tag{2-7}$$

leads to the equivalent bandpass transfer function

$$H_{BPP}(s) = \prod_{k=1}^{K} \frac{p_k}{\frac{1}{2}\left(s + \frac{\Omega^2{}_I}{s}\right) - p_k} \tag{2-8}$$

where Ω_I is the radian IF frequency normalized by ω_c. Assuming, tacitly, that the IF bandwidth has not changed appreciably from its value of 2 at baseband, we may define the system parameter

$$c_1 \equiv \frac{2}{\Omega_I} \tag{2-9}$$

to be the <u>fractional bandwidth at IF</u>. We may now incorporate c_1 in (2-8) and evaluate $H_{BPP}(s)$ on the $j\Omega$-axis. Letting

$$s = j\Omega = j(\Omega_I + \delta\Omega) \tag{2-10}$$

it is possible to characterize the behavior of $H_{BPP}(\Omega)$ in the neighborhood of Ω_I. Then, using (2-9), it is possible to express H_{BPP} as a function of $\delta\Omega$ which should be recognized as the normalized baseband radian frequency.† To begin with, applying (2-10) into (2-7), we get, in view of (2-9),

$$\begin{aligned}
\frac{1}{2}\left(s + \frac{\Omega_I^2}{s}\right) &= \frac{1}{2}\left[j(\Omega_I + \delta\Omega) + \frac{\Omega_I^2}{j(\Omega_I + \delta\Omega)}\right] \\
&= j\frac{\Omega_I}{2}\left[1 + \frac{\delta\Omega}{\Omega_I} - \frac{1}{1 + \frac{\delta\Omega}{\Omega_I}}\right] \\
&= j\frac{\Omega_I}{2}\left[\frac{\delta\Omega}{\Omega_I} + \frac{\delta\Omega/\Omega_I}{1 + \frac{\delta\Omega}{\Omega_I}}\right] \\
&= j\frac{\delta\Omega}{2}\left[1 + \frac{1}{1 + \frac{\delta\Omega}{\Omega_I}}\right] \\
&= j\frac{\delta\Omega}{2}\left[1 + \frac{1}{1 + c_1\frac{\delta\Omega}{2}}\right]
\end{aligned} \tag{2-11}$$

† It should be noted that $\delta\Omega$ has been used here to represent baseband radian frequency rather than Ω in order to prevent its unavoidable confusion with $Im(s)$ in the present development.

whence, (2-8) becomes†

$$H_{BPP}(\Omega_I + \delta\Omega) = \prod_{k=1}^{K} \frac{p_k}{j\frac{\delta\Omega}{2}\left[1 + \frac{1}{1 + c_1\frac{\delta\Omega}{2}}\right] - p_k} \tag{2-12}$$

clearly independent of IF radian frequency, Ω_I, and only a function of normalized baseband radian frequency $\delta\Omega$. It should be emphasized here that, in accordance to (2-9), Ω_I is understood to be normalized by ω_c, the actual 3-dB lowpass cutoff radian frequency. As a consequence, the positive-frequency IF transfer function in (2-3) is given by

$$H^+(f + f_I) = H^+_{BPP}(\Omega_I + \delta\Omega) \tag{2-13}$$

provided that f and f_I are understood to be normalized by $f_c = \omega_c/2\pi$. In order to minimize notational clutter in what follows, we will assume the following equivalences

$$\left.\begin{aligned} \delta\Omega &\sim \omega \\ \Omega_I &\sim \omega_I \end{aligned}\right\} \tag{2-14}$$

and define the positive-frequency final IF transfer function referred to baseband as

$$H^+(\omega + \omega_I) \equiv \prod_{k=1}^{K} \frac{p_k}{j\frac{\omega}{2}\left[1 + \frac{1}{1 + c_1\frac{\omega}{2}}\right] - p_k} \tag{2-15}$$

† The reader is reminded that H_{BPP}, as used here, serves as a symbol. If taken as a legitimate function, it would be necessary to use different notation with each transformation, which could confuse matters rather than enhance understanding.

In modeling the practically-achievable design of a receiver's final IF filter, two important factors must be taken into account; namely, its bandwidth and center frequency deviations from those desired. Defining parameters

$c_2 \equiv$ bandwidth factor, around unity, equal to the ratio of actual-to-specified IF filter bandwidth

$c_3 \equiv$ center-frequency offset factor, around zero, that relates the IF filter's center frequency deviation as a fraction of the IF bandwidth

it is possible to rewrite (2-15) so as to include effects of design tolerances. It is not difficult to see that the effect of c_2 on ω is the inverse dilation ω/c_2. On the other hand c_3 will give rise to the translation $(\omega-2c_3)$, since the normalized IF bandwidth is 2. To a good first-order approximation, the combined effects of c_1, c_2, and c_3 on (2-15) are embodied in the modified expression that follows, where, again, H^+ is used as a symbol.

$$H^+(\omega + \omega_I) \equiv \prod_{k=1}^{K} \frac{p_k}{j\dfrac{\omega-2c_3}{2c_2}\left[1 + \dfrac{1}{1 + \dfrac{c_1(\omega-2c_3)}{2c_2}}\right] - p_k} \qquad (2\text{-}16)$$

Note that when c_2=1 and c_3=0, (2-16) reduces to (2-15), which indicates no design errors. Typical 1% error implies that c_2 and c_3 would range over (0.99, 1.01) and (-0.01, 0.01), respectively. The consequence of such design variation will be a mismatching of antenna-port receivers, a condition that could tend to limit subsequent processing performance.

2.2.3 Normalization At RF

Another important system parameter is the fractional bandwidth at RF, which may be defined as

$$c_4 = \frac{2}{\omega_0} \qquad (2\text{-}17)$$

where, again, 2 is the normalized bandwidth of the IF filter attributed to its associated lowpass prototype. In view of (2-4), (2-17) may be incorporated into (2-3) to yield an expression for the baseband channel transfer function with no explicit dependence on ω_0. It should be noted here that ω_0, as used in (2-17), is the RF center radian frequency normalized by ω_c, the actual 3-dB cutoff lowpass radar frequency.

Consider the exponential term within the summation of (2-3). Upon substituting (2-4) and (2-17) into this ´ n, we get

$$e^{j2\pi(f+f_0)\tau_\ell} = e^{j\frac{\pi\ell d_0}{f_0}(f + f_0)\sin\theta}$$

$$= e^{j\pi\ell d_0(1 + \frac{f}{f_0})\sin\theta}$$

$$= e^{j\pi\ell d_0(1 + c_4\frac{\omega}{2})\sin\theta} \qquad (2\text{-}18)$$

since $f/f_0 = \omega/\omega_0$, in general. Including the effects of c_2 and c_3 in (2-18), (2-3) may be written, most conveniently, as a function of the normalized baseband radian frequency ω, as follows:

$$C(\omega) = \sum_{\ell=1}^{L} a_\ell e^{j\pi\ell d_0\left[1 + c_4\frac{\omega-2c_3}{2c_2}\right]\sin\theta}\, H^+(\omega+\omega_I) \qquad (2\text{-}19)$$

where, C is understood as a symbol and not a function.

In the above development we introduced four system parameters (in addition to the ones included in Section 2.1), two of which were instrumental in elminating the cumbersome explicit dependence of the baseband channel transfer function on actual IF and RF frequencies, leaving only the desired dependence on normalized radian frequency ω. In fact, we have shown that the baseband channel transfer function reduces to

$$C(\omega) = \sum_{\ell=1}^{L} a_\ell e^{j\pi\ell d_0\left[1 + c_4\frac{\omega-2c_3}{2c_2}\right]\sin\theta} \bullet$$

$$\bullet \prod_{k=1}^{K} \frac{p_k}{j\frac{\omega-2c_3}{2c_2}\left[1 + \frac{1}{1 + \frac{c_1(\omega-2c_3)}{2c_2}}\right] - p_k} \qquad (2\text{-}20)$$

a conceptually-appealing and computationally-desirable form.

3.0 SIGNAL GENERATION PROGRAM

The development of the signal generation program, denoted by the acronym SIGGENO, is based on a discretized form of the normalized baseband channel transfer function, $C(\omega)$, as given in (2-20). An appropriate channel transfer function discretization, $\{C(\omega_i)\}_{i=1}^{I}$, results from sampling $C(\omega)$ at a rate sufficiently higher than the Nyquist rate as dictated by the baseband transfer function of the IF filter, $H^+(\omega+\omega_I)$. Applying an inverse discrete Fourier transform on $\{C(\omega_i)\}_{i=1}^{I}$ gives a good approximation of its corresponding discrete complex-valued impulse response $\{c(t_i)\}_{i=1}^{I}$. Then, a sampled noise-source channel input, $\{P(t_i)\}_{i=1}^{\infty}$ gives rise to a received sampled baseband channel signal $\{\zeta_B(t_i)\}_{i=1}^{\infty}$, via the discrete convolution

$$\zeta_B(t_j) = \sum_{i=1}^{I} P(\tau_i)c(t_j-\tau_i) \tag{3-1}$$

According to previous discussion, a number of directionally-distinct sampled noise-source signals incident onto an antenna array will give rise to a received baseband port signal that is a superposition of sampled baseband signals received from each source through its associated discretized channel. More specifically, the sampled received baseband port signal is given by

$$\begin{aligned}\zeta_{BP}(t_j) &= \sum_{k=1}^{K} \zeta_{B_k}(t_j) \\ &= \sum_{k=1}^{K} \sum_{i=1}^{I} P_k(\tau_i)c_k(t_j-\tau_i)\end{aligned} \tag{3-2}$$

where $\{P_k(t_i)\}_{i=1}^{\infty}$ is the noise sequence into the k-th channel associated with sampled impulse response $\{c_k(t_i)\}_{i=1}^{I}$ and k=1, 2, ..., K, the number of distinct channels.

SIGGENO is a structured FORTRAN IV computer simulation program designed to use (3-2) in order to generate received sampled baseband signals at the main and auxiliary ports of a multiport antenna system mentioned in Section 2.0 and described, in more detail, in Project Memorandum 8512-01. The discussion that follows provides a description of the particular modularized structure of SIGGENO, its execution and subsequent validation by means of a specific numerical example.

3.1 General Program Structure

Program SIGGENO has been developed with an emphasis on a structure which exhibits logical clarity, possesses inherent flexibility and lends itself to a natural and efficient growth. To begin with, all pertinent system specifications of the multiport signal generation problem at hand are relegated to a clearly readable table of parameters constituting an input data file, SIGGENO:D, via which the user may readily describe a variety of system situations of possible interest. In this sense, all pertinent system parameters that could potentially be varied have been removed from the executable part of the program.

Upon examining the [illegible] signal generation problem as a whole, elementary functional components have been identified and corresponding well-commented subroutines [illegible] have been written. The set of these source modules SIGGENO:S, [illegible] source library in the form of distinct uncompiled source files, the name of each bearing the suffix :S, the identifier for source. The compiled binary versions of the source modules form the corresponding binary library, wherein each module is identified by the suffix :B. A concatenation of the set of binary modules results in the composite binary signal generation module, SIGGENO:B. Subsequently, SIGGENO:B is converted to an executable signal generation load module, SIGGENO:L. Given input data file SIGGENO:D, which defines a multiport system situation to be examined, load module SIGGENO:L may be executed to produce an output file, SIGGENO:O, containing the input system specifications, receiver and channel spectral and time-domain characteristics, and, most importantly, the desired set of sampled baseband port signals.†

The simplified block diagram of Figure 3-1 portrays the essence of the modular structure of program SIGGENO. Because of its special structure, the user has the flexibility of editing and augmenting a given source module without needlessly effecting any other. Following this, he may compile the augmented source module into its binary form, again without involving any other module. Finally, SIGGENO:B may be reconstructed and SIGGENO:L regenerated. The inherent efficiency of this approach is clear.

3.2 Specific Program Description

Included below are the specific elements that comprise the signal generation program SIGGENO. Assuming a five-port antenna array system, a complete listing of the associated input data file, SIGGENO:D, is given in the first subsection. The next subsection describes the modules that comprise the source library, gives a functional flow diagram of the program and provides complete listings of the subroutines involved. Following the construction of the library of corresponding binary modules, composite binary and load modules, SIGGENO:B and SIGGENO:L, are formed via appropriate job control language programs. Finally, the execution of load module SIGGENO:L with input data SIGGENO:D results in an output file SIGGENO:O.

---

†In developing the mechanics of the present methodology, the authors have consulted with C. N. Sorg, G. L. Guenthner, and R. T. Short of Motorola's Engineering Computer Center.

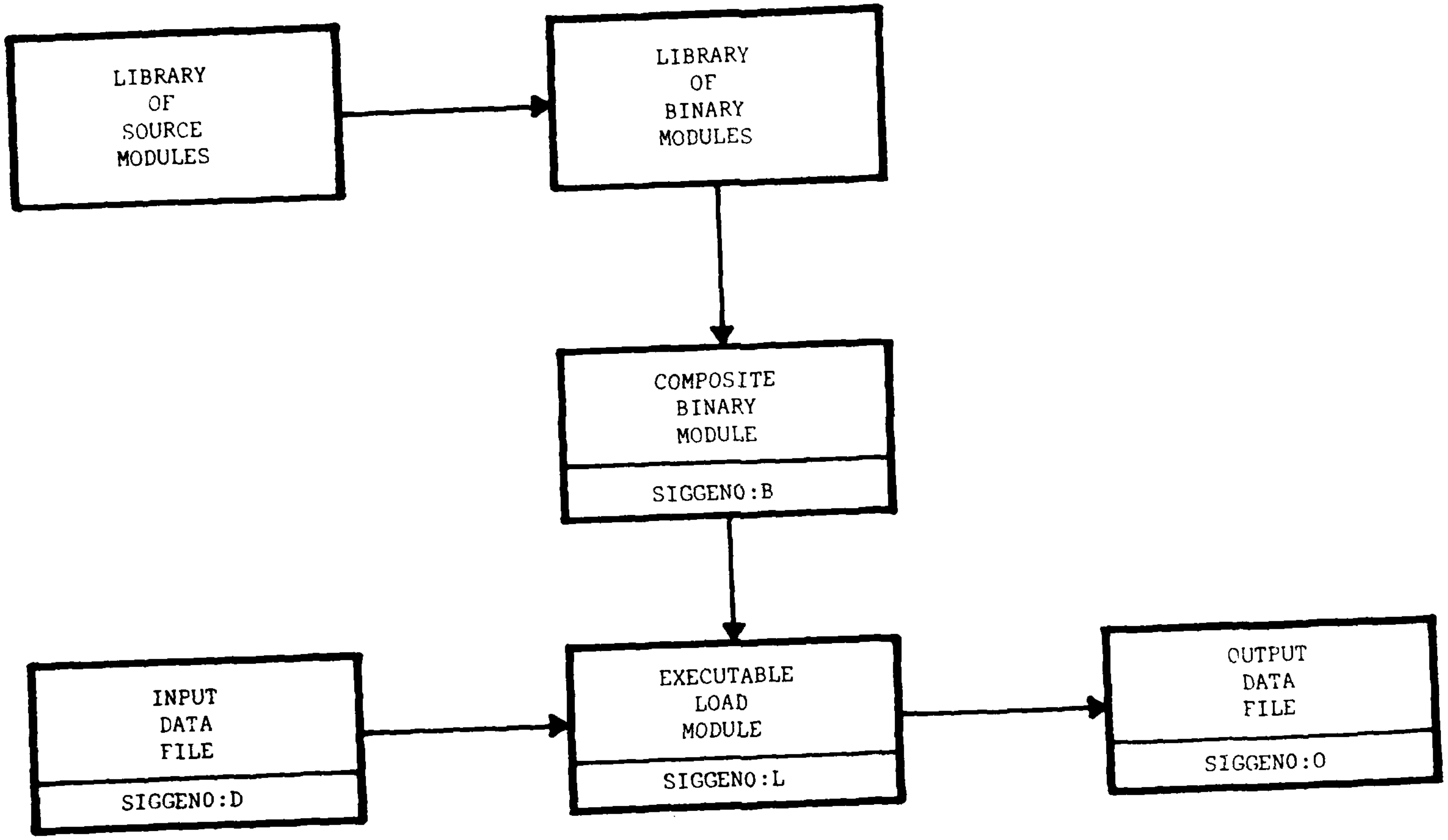


Figure 3-1. General Modular Structure of Signal Generation Program SIGGENO

3.2.1 Input Data File

The input data file that follows gives a complete parametric description of a five-port antenna system in accordance with the discussion of Section 2.0. Excluding print options at the top of the list, the parameters specified fall into three major categories. The filter parameters define the lowpass prototype filter, fractional bandwidths at IF and RF, normalized baseband radian frequency range and an initial radian frequency, number of frequency samples to be used and an optional parameter for choosing a baseband equivalent of a lowpass or a bandpass receiver filter. It should be noted that the latter option is consistent with choosing a receiver whose selectivity is determined by its final IF filter.

The channel parameters relate to the numbers of noise sources, or, equivalently, the channels corresponding to them, with respect to each antenna port. Each channel number is followed by related specifications of noise-source amplitude, initial setting of RANDU noise generator, time sample number at which the source has been turned on, subsequent periodic blink duration in time samples and angle of arrival in degrees. Note that, with RADRNG=4, the normalized time increment between samples is $2/\pi$.

The port parameters include the antenna separation factor d_0, the number of port signal samples desired and, of course, the number of ports, chosen in the present case to be five. Under port "0" are included specifications of the main antenna port. In the present case, NEL=255 refers to the number of elements that comprise the main Taylor-weighted linear array and LOC1=1 indicates the number of the first element of the array with respect to a reference origin as shown in Figure 2-1. The subsequent two parameters are c_1 and c_2, the bandwidth factor and the bandwidth offset factor as defined in subsection 2.2.2. Note that the remaining four ports describe similar parameters associated with the omnidirectional auxiliaries and their receivers. To be noted here is the fact that all receivers are matched. Any distortion among port signals, here, is mainly attributed to the frequency response of the main antenna sidelobe region at the precise angles of arrival specified.

3.2.2 Source Library and Program Architecture

The elementary modules that comprise the source library of the signal generation program SIGGENO are functionally described and classified according to category. Subsequently, the minimum-core architecture of the composite virtual source program, SIGGENO:S, is clearly demonstrated by means of a functional flow diagram. Finally, complete FORTRAN listings of all source modules are included for convenient reference.

3.2.2.1 Functional Description of Source Modules

A functional description of the individual source modules is given in the itemized list below.

SIGMAINO:S	- The main controlling program which calls auxiliary executive programs that perform specific tasks.

```
------------------------------------------------------------------------
                  SPECIFICATION OF SYSTEM PARAMETERS
                  ----------------------------------
------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWR     -  RECEIVER CHARACTERISTICS                  :       1
  IWC     -  CHANNEL CHARACTERISTICS                   :       1
  IWSC    -  INDIVIDUAL CHANNEL SIGNALS                :       1

FILTER PARAMETERS
-----------------
  NPOL    -  NUMBER OF LOWPASS PROTOTYPE POLES         :       2
             POL(1)                                    :   -.70700    -.70700
             POL(2)                                    :   -.70700     .70700
  FBIF    -  FRACTIONAL BANDWIDTH AT FINAL IF          :    .10000
  FBRF    -  FRACTIONAL BANDWIDTH AT RF                :    .00100
  RADRNG  -  NORMALIZED RADIAN FREQUENCY RANGE         :   4.00000
  RADIN   -  NORMALIZED INITIAL RADIAN FREQUENCY       :  -2.00000
  NF      -  NUMBER OF FREQUENCY SAMPLES               :      32
  LPBP    -  LOWPASS/BANDPASS OPTION                   :       1
             0  :   LOWPASS
             1  :   BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS  -  NUMBER OF CHANNELS PER PORT               :       4

CHANNEL 1 :
    AN    -  AMPLITUDE OF NOISE SOURCE                 :   1.00000
    IX0   -  INITIAL RANDU SETTING                     :         1
    N1    -  FIRST-TIME-ON SAMPLE NUMBER               :         1
    NB    -  BLINK DURATION IN SAMPLES                 :     10000
    TH    -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :  45.00000

CHANNEL 2 :
    AN    -  AMPLITUDE OF NOISE SOURCE                 :    .00000
    IX0   -  INITIAL RANDU SETTING                     :        11
    N1    -  FIRST-TIME-ON SAMPLE NUMBER               :         1
    NB    -  BLINK DURATION IN SAMPLES                 :     10000
    TH    -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :  10.00000
```

```
CHANNEL 3 :
   AN      -  AMPLITUDE OF NOISE SOURCE                 :     .00000
   IX0     -  INITIAL RANDU SETTING                     :        111
   N1      -  FIRST-TIME-ON SAMPLE NUMBER               :          1
   NB      -  BLINK DURATION IN SAMPLES                 :      10000
   TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)         : -20.00000

CHANNEL 4 :
   AN      -  AMPLITUDE OF NOISE SOURCE                 :     .00000
   IX0     -  INITIAL RANDU SETTING                     :       1111
   N1      -  FIRST-TIME-ON SAMPLE NUMBER               :          1
   NB      -  BLINK DURATION IN SAMPLES                 :      10000
   TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)         : -35.00000

PORT PARAMETERS
---------------
  D0      -  ANTENNA-ELEMENT SEPARATION FACTOR          :   1.00000
  NS      -  NUMBER OF SIGNAL SAMPLES                   :       128
  NPORTS  -  NUMBER OF PORTS                            :         5

PORT0 :
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :       255
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :         1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :   1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                    :    .00000

PORT1 :
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :         1
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :         1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :   1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                    :    .00000

PORT2 :
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :         1
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :        51
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :   1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                    :    .00000

PORT3 :
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :         1
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :       127
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :   1.00000
```

```
BWOFF  -   BANDWIDTH OFFSET FACTOR                       :      .00000

PORT4 :
  NEL    -  NUMBER OF ANTENNA ELEMENTS                   :           1
  LOC1   -  LOCATION OF THE FIRST ELEMENT                :         255
  BWFCTR -  BANDWIDTH TOLERANCE FACTOR                   :     1.00000
  BWOFF  -  BANDWIDTH OFFSET FACTOR                      :      .00000
----------------------------------------------------------------------
```

SIGSET:S - The first auxiliary executive program designed to read and write from the input data file, SIGGENO:D, by means of special subroutines in accordance to preset formats. All data read is made available to SIGMAINO:S in identified common blocks.

PRTSGNLO:S - The second auxiliary executive program called by SIGMAINO:S. This program uses the input data available in common and generates the desired port signals with the aid of a number of dedicated subroutines. As such, this program may be considered the central executive program in SIGGENO:S.

RW:S - This is a dedicated subroutine designed to read and write an 80-character literal data string. No data is returned to the calling program SIGSETO:S.

RWID:S - This is a dedicated subroutine designed to read and write a 60-character literal data string followed by numerical data within the next 20 spaces which are returned to the calling program, SIGSETO:S. The numerical data may consist of integer, decimal or complex scalar values.

ANTWT:S - This is a dedicated subroutine designed to provide a Taylor weighting sequence for a linear array given the number of elements that comprise it.

FILTER:S - This is a dedicated subroutine designed to generate a discretization of the baseband equivalent transfer function of either the final IF bandpass filter or a lowpass alternative. The number of frequency samples used is specified in SIGGENO:D.

AMPH:S - This is a dedicated but optional subroutine that computes the amplitude and phase counterparts to a given transfer function. This subroutine could be used to graphically assess the frequency dependence of a main-antenna channel and its deviation from an auxiliary one.

IMPULS:S - This is a dedicated subroutine designed to compute a sampled baseband impulse response from a sampled baseband transfer function.

FFT2:S - This is a dedicated subroutine designed to compute the forward or inverse discrete Fourier transform of a complex sequence whose number of samples is a power of 2.

CHANNEL:S - This is a dedicated subroutine which takes the antenna and receiver characteristics to compute a baseband transfer function associated with a particular angle of arrival of a specified noise source.

BLINK:S - This is a dedicated subroutine whose purpose is to generate a discretized on/off switching function consisting of an appropriate sequence of 0's and 1's.

SIGNALO:S - This is a dedicated subroutine which uses the on/off switching sequence in order to initially turn on and subsequently periodically blink an input noise sequence provided by a random number generator, specifically mechanized by the well-known utility subroutine, RANDU. SIGNALO:S subsequently uses this blinked noise sequence in order to produce a desired number of received baseband samples through its associated channel via discrete convolution with its impulse response. Such individual channel baseband signals are returned to PRTSGNLO:S where they are appropriately accumulated into composite baseband port signals.

From their brief descriptions given above, it is easy to see that the source modules may be classified under three prominent categories as indicated by the tree diagram in Figure 3-2. SIGMAINO:S is the main executive control module which delegates specific duties to the auxiliary executive modules, SIGSETO:S and PRTSGNLO:S. In turn, these two modules make use of a number of dedicated subroutines which form the third category of classification. Note that RANDU is only a utility module available from the Honeywell 560 library. As such, it is not considered as a legitimate source module but only shown for the purpose of completeness.

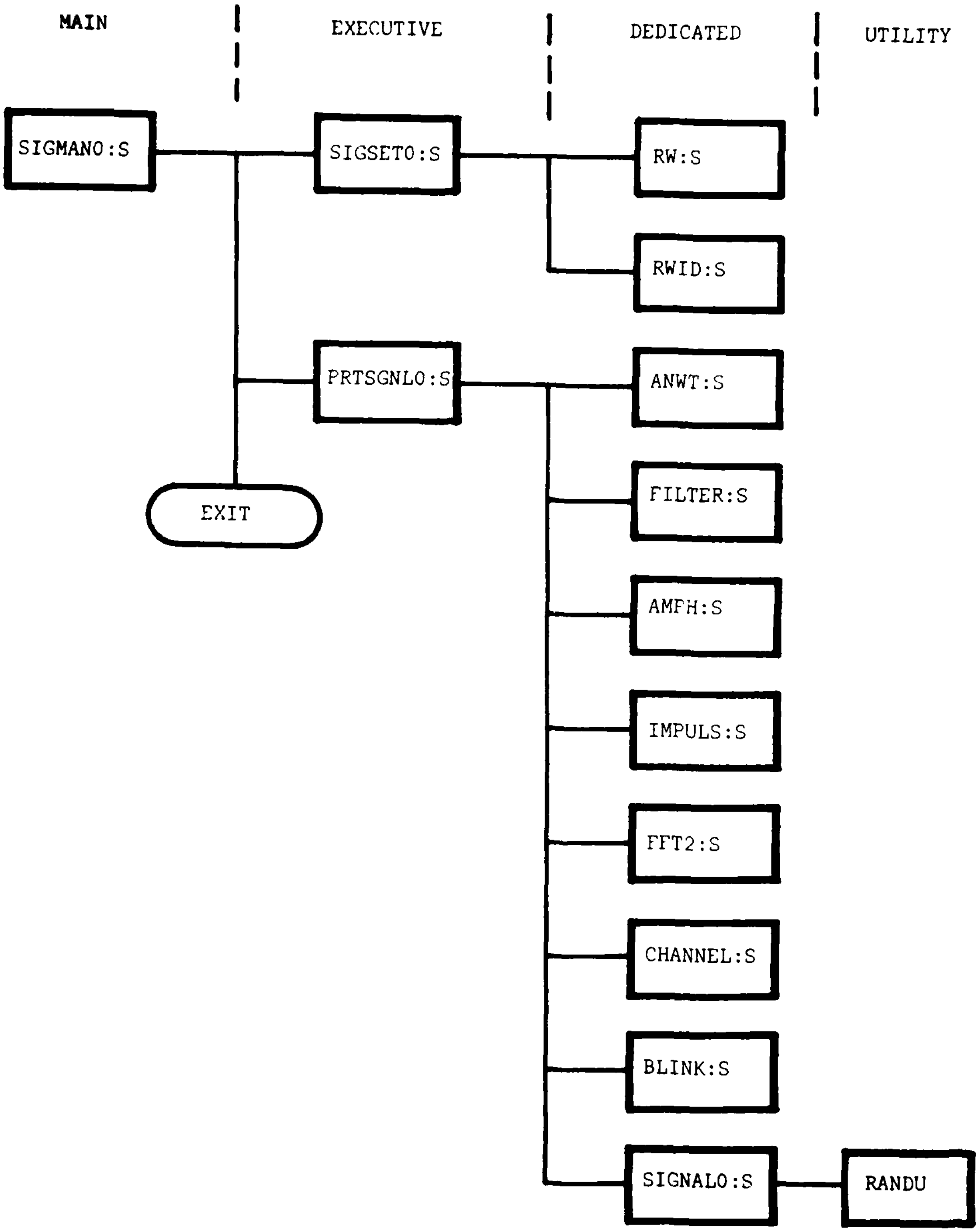


Figure 3-2. Tree-Diagram Category Classification of Modules Comprising the Signal Generation Source Libarary

3.2.2.2 Minimum-Core Program Architecture

The one module in the entire source library that could potentially account for the major part of the composite program's core requirement is PRTSGNLO:S. Having anticipated the need to keep core-requirement to a minimum, PRTSGNLO:S has been constructed appropriately. Given N_n noise sources and N_p ports, it is clear from its description that PRTSGNLO:S has been especially designed to generate one port signal array at a time by accumulating the N_n channel signal contributions. In so doing, the core-requirement is essentially determined by the current channel signal array storage and, of course, the storage of the port-signal array in question. As such, this minimum-core program architecture has the advantage of being independent of system dimensionality $N_n \times N_p$. Of course, the inevitable shortcoming of this approach is the need to recycle through the N_n noise sources and receiver transfer characteristics for each port signal computation of a condition which will tend to increase program execution time by some fraction of $N_n \times N_p$.

This minimum-core signal generation program architecture is clearly demonstrated in the functional flow diagram of Figure 3-3. Note that the key to keeping core at a minimum is the use of single utility arrays for temporarily storing current values of the receiver and channel transfer functions, the channel impulse response, the incident noise sequence, the channel baseband signal and its appropriate accumulation into the baseband port-signal.

3.2.2.3 FORTRAN Listings of Source Modules

Complete FORTRAN listings of the source modules of the signal generation source library are included here for easy reference. In writing each module, a sincere attempt has been made to make it almost self-explantory by including functional description, a complete list of definitions of input and output parameters and appropriate comments throughout the program, as necessary.

In the same sense that SIGGENO is the generic name of the present signal generation program and SIGGENO:S is the composite collection of source modules, the suffix :S has been left out from all subroutine names, as the reader will note. Included in the heading of each subroutine is the name of the source module, its date of origin and latest revision date. This minimal identification along with authors' names allows a general user to remain current with the program development and consult with the informed individuals in case of questions or problems.

3.2.3 Binary Library

Using a general but rather simple job control language program, JCL:B, compiled versions of the signal generation modules have been created. Carrying identical names but distinguished by the identifying suffix :B, the collection of these binary modules forms a binary library which complements the original source library.

Of course, the experienced computer user is well aware that a compiled binary version of a working program avoids the need of undesired repeated compilation during each execution. Clearly, a modular formulation of a large program offers the added advantage that an augmentation of any one source module will mean the recompilation of only that source module and not the entire program.

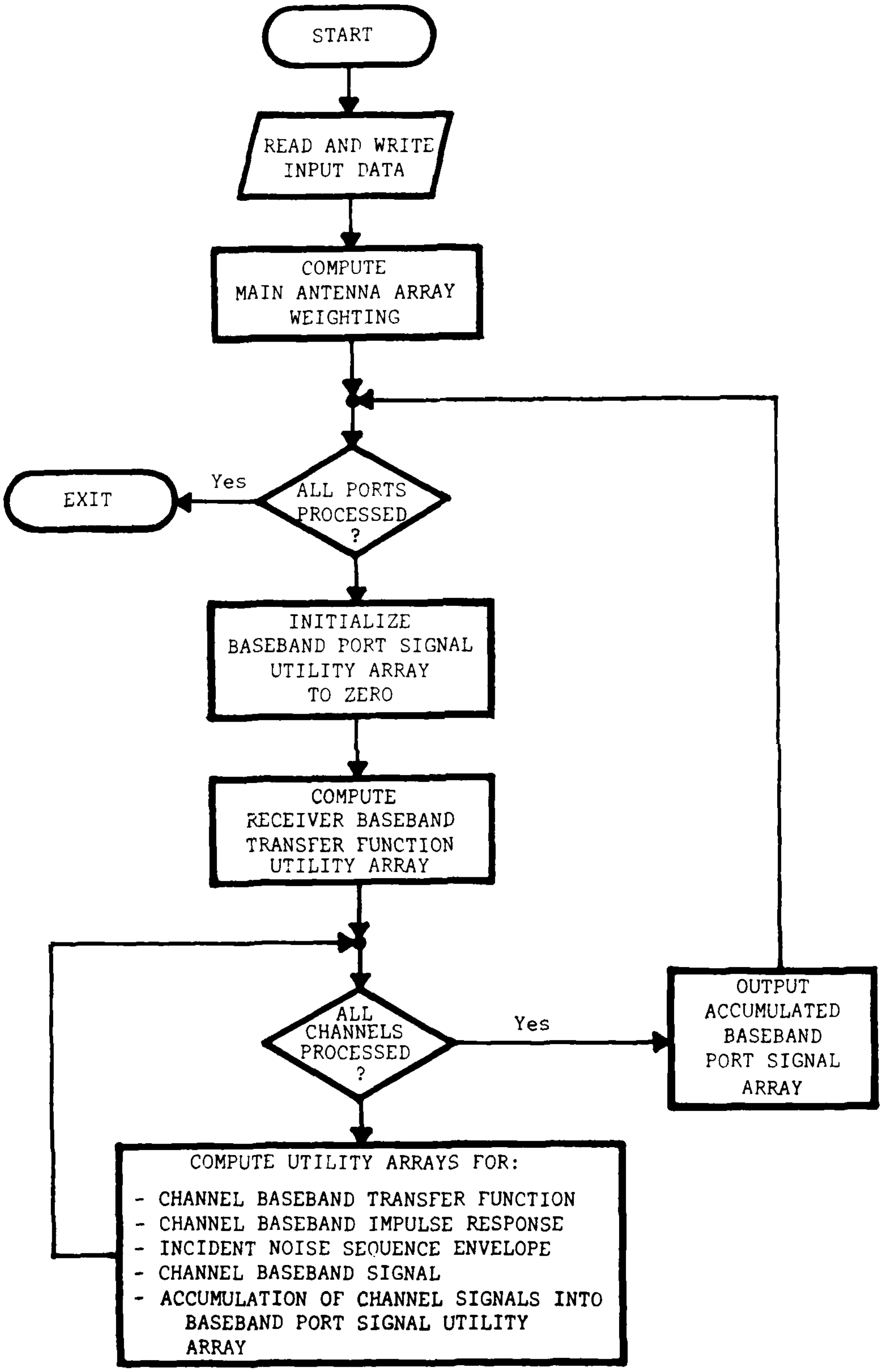


Figure 3-3. Functional Flow Diagram of the Minimum-Core Signal Generation Program SIGGENO.

```
C ============================================================================
C                SIGNAL  GENERATION  PROGRAM  :   SIGGEN0:S
C                ------------------------------------------
C
C                 COMPUTATION OF COMPLEX BASEBAND SIGNALS
C                       AT MULTIPLE RECEIVER PORTS
C                  CONNECTED TO SPECIFIED LINEAR ARRAYS
C
C               PROGRAM        :  SIGMAIN0:S
C               ORIGINAL       :  APRIL 19, 1980
C               REVISION       :  JULY   1, 1980
C
C               PREPARED  BY   :  S. M. DANIEL  &  I. M. KERTESZ
C                                 RADAR SYSTEMS ANALYSIS GROUP
C                                 MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                 TEMPE, ARIZONA  85282
C ============================================================================
      COMPLEX POL
      COMMON /SET0/ IWR,IWC,IWSC
      COMMON /SET1/ POL(6),FBIF,FBRF,RADRNG,RADIN,DRAD,NPOL,LPBP,NF
      COMMON /SET2/ AN(10),TH(10),IX0(10),NI(10),NB(10),NCHNLS
      COMMON /SET3/ BWFCTR(11),BWOFF(11),D0,NEL(11),LOCI(11),NPORTS,NS
      CALL SIGSET0
      CALL PRTSGNL0
      CALL EXIT
      END
```

```
C =====================================================================
      SUBROUTINE SIGSETU
C =====================================================================
C                   SPECIFICATION OF SYSTEM PARAMETERS
C
C              PROGRAM       :  SIGSETIS
C              ORIGINAL      :  JUNE  1, 1980
C              REVISION      :  JULY  1, 1980
C
C              PREPARED  BY  :  S. M. DANIEL  &  I. M. KERTESZ
C                               RADAR SYSTEMS ANALYSIS GROUP
C                               MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                               TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C     INPUT  :    DATA FILE SIGGENO:D
C =====================================================================
      COMPLEX POL,CDUM
      COMMON /SET0/ IWR,IWC,IWSC
      COMMON /SET1/ POL(6),FBIF,FBRF,RADRNG,RADIN,DRAD,NPOL,LPBP,NF
      COMMON /SET2/ AN(10),TH(10),IXO(10),N1(10),NB(10),NCHNLS
      COMMON /SET3/ BWFCTR(11),BWOFF(11),DO,NEL(11),LOCI(11),NPORTS,NS
100   FORMAT(1H1)
      WRITE(108,100)
C ---------------------------------------------------------------------
C     READ IN SYSTEM SPECIFICATIONS FROM  SIGGENO:D
C ---------------------------------------------------------------------
      CALL RW(7)
      CALL RWID(CDUM,RDUM,IWR,0)
      CALL RWID(CDUM,RDUM,IWC,0)
      CALL RWID(CDUM,RDUM,IWSC,0)
      CALL RW(3)
      CALL RWID(CDUM,RDUM,NPOL,0)
      DO 10 I=1,NPOL
10    CALL RWID(POL(I),RDUM,IDUM,2)
      CALL RWID(CDUM,FBIF,IDUM,1)
      CALL RWID(CDUM,FBRF,IDUM,1)
      CALL RWID(CDUM,RADRNG,IDUM,1)
      CALL RWID(CDUM,RADIN,IDUM,1)
      CALL RWID(CDUM,RDUM,NF,0)
      DRAD=RADRNG/NF
      CALL RWID(CDUM,RDUM,LPBP,0)
      CALL RW(5)
      CALL RWID(CDUM,RDUM,NCHNLS,0)
```

```
      DO 20 I=1,NCHNLS
      CALL RW(2)
      CALL RWID(CDUM,AN(I),IDUM,1)
      CALL RWID(CDUM,RDUM,IX0(I),0)
      CALL RWID(CDUM,RDUM,N1(I),0)
      CALL RWID(CDUM,RDUM,NB(I),0)
   20 CALL RWID(CDUM,TH(I),IDUM,1)
      CALL RW(3)
      CALL RWID(CDUM,D0,IDUM,1)
      CALL RWID(CDUM,RDUM,NS,0)
      CALL RWID(CDUM,RDUM,NPORTS,0)
      DO 30 I=1,NPORTS
      CALL RW(2)
      CALL RWID(CDUM,RDUM,NEL(I),0)
      CALL RWID(CDUM,RDUM,LOC1(I),0)
      CALL RWID(CDUM,BWFCTR(I),IDUM,1)
   30 CALL RWID(CDUM,BWOFF(I),IDUM,1)
      CALL RW(1)
C ------------------------------------------------------------------
      RETURN
      END
```

```
C =====================================================================
      SUBROUTINE PRTSGNL0
C =====================================================================
C             GENERATION OF RECEIVED BASEBAND PORT SIGNALS
C
C              PROGRAM        :  PRTSGNL0:S
C              ORIGINAL       :  JUNE  1, 1980
C              REVISION       :  JULY  2, 1980
C
C              PREPARED  BY   :  S. M. DANIEL  &  I. M. KERTESZ
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C     INPUT  :    SYSTEM PARAMETERS IN COMMON
C
C     OUTPUT :    RECEIVED BASEBAND PORT SIGNALS
C =====================================================================
      COMPLEX POL,HR,H,HIMPL,SC,SP
      COMMON /SET0/ IWR,IWC,IWSC
      COMMON /SET1/ POL(6),FBIF,FBRF,RADRNG,RADIN,DRAD,NPOL,LPBP,NF
      COMMON /SET2/ AN(10),TH(10),IX0(10),N1(10),NB(10),NCHNLS
      COMMON /SET3/ BWFCTR(11),BWOFF(11),D0,NEL(11),LOC1(11),NPORTS,NS
      DIMENSION HR(64),H(64),HIMPL(64),SC(2048),SP(2048),AM(64),PH(64)
     -,X(128),AL(255),P(64),IBLNK(2080)
  100 FORMAT(1H1)
  200 FORMAT(1X,79('-'))
  300 FORMAT(37X,'PORT',I3,/,37X,7('-'))
  400 FORMAT(23X,'RECEIVER BASEBAND CHARACTERISTICS')
  500 FORMAT(29X,'PORT',I3,5X,'CHANNEL',I3,/,29X,22('-'))
  600 FORMAT(24X,'CHANNEL BASEBAND CHARACTERISTICS')
  700 FORMAT(24X,'RECEIVED BASEBAND CHANNEL SIGNAL')
  800 FORMAT(8F10.5)
  900 FORMAT(25X,'RECEIVED BASEBAND PORT SIGNAL')
C ---------------------------------------------------------------------
C     PROVIDE MAIN ANTENNA ARRAY ELEMENT WEIGHTING
C ---------------------------------------------------------------------
      CALL ANTWT(NEL(1),LOC1(1),AL)
C ---------------------------------------------------------------------
C     DERIVE BASEBAND PORT SIGNALS
C ---------------------------------------------------------------------
      DO 10 IP=1,NPORTS
      IPM1=IP-1
```

```
      IF(IWR.EQ.0) GOTO 1000
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,300) IPM1
      WRITE(108,200)
      WRITE(108,400)
1000  CONTINUE
C -----------------------------------------------------------------
C     INITIALIZE RECEIVED BASEBAND PORT SIGNAL TO ZERO
C -----------------------------------------------------------------
      DO 20 IS=1,NS
20    SP(IS)=0
      IF(IPM1.GT.0) AL(1)=1
C -----------------------------------------------------------------
C     DERIVE RECEIVER CHARACTERISTICS
C -----------------------------------------------------------------
      CALL FILTER(NF,DRAD,RADIN,FBIF,LPBP,BWFCTR(IP),BWOFF(IP),NPOL,POL
     -,HR)
      IF(IWR.EQ.1) CALL AMPH(HR,NF,AM,PH)
      CALL IMPULS(NF,HR,X,HIMPL,IWR)
C -----------------------------------------------------------------
C     DERIVE CHANNEL BASEBAND CHARACTERISTICS
C -----------------------------------------------------------------
      DO 30 IC=1,NCHNLS
      IF(AN(IC).LE.0.00001) GOTO 2000
      IF(IWC.EQ.0) GOTO 3000
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,500) IPM1,IC
      WRITE(108,200)
      WRITE(108,600)
3000  CONTINUE
      CALL CHANNEL(NEL(IP),LOC1(IP),AL,D0,TH(IC),NF,RADIN,DRAD
     -,BWFCTR(IP),BWOFF(IP),FBRF,HR,H)
      IF(IWC.EQ.1) CALL AMPH(H,NF,AM,PH)
      CALL IMPULS(NF,H,X,HIMPL,IWC)
C -----------------------------------------------------------------
C     DERIVE RECEIVED BASEBAND CHANNEL SIGNALS AND THEIR ACCUMULATION
C     INTO A COMPOSITE PORT SIGNAL
C -----------------------------------------------------------------
      IX=IX0(IC)
      CALL BLINK(N1(IC),NB(IC),NF,NS,IBLNK)
      CALL SIGNAL0(HIMPL,NF,IX,IBLNK,P,SC,NS)
```

```
      IF(I*SC.EQ.0) GOTO 4000
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,500) IPM1,IC
      WRITE(108,200)
      WRITE(108,700)
      WRITE(108,200)
      WRITE(108,800) (SC(I),I=1,NS)
      WRITE(108,200)
4000  CONTINUE
      DO 30 IS=1,NS
30    SP(IS)=SP(IS)+AN(IC)*SC(IS)
2000  CONTINUE
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,300) IPM1
      WRITE(108,200)
      WRITE(108,900)
      WRITE(108,200)
      WRITE(108,800) (SP(I),I=1,NS)
      WRITE(108,200)
10    CONTINUE
C ----------------------------------------------------------------------
      RETURN
      END
```

```
C =================================================================
      SUBROUTINE ANTWT(NEL,LOC1,AL)
C =================================================================
C           COMPUTATION OF LINEAR ANTENNA ARRAY TAYLOR WEIGHTING
C
C                 PROGRAM      :  ANTWT:S
C                 ORIGINAL     :  APRIL 19, 1980
C                 REVISION     :  JUNE  11, 1980
C
C                 PREPARED  BY :  S. M. DANIEL  &  I. M. KERTESZ
C                                 RADAR SYSTEMS ANALYSIS GROUP
C                                 MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                 TEMPE, ARIZONA  85282
C -----------------------------------------------------------------
C     INPUT  :    NEL        -    NUMBER OF ELEMENTS IN LINEAR ARRAY
C                 LOC1       -    LOCATION (NUMBER) OF FIRST ELEMENT
C
C     OUTPUT :    AL         -    ARRAY ELEMENT WEIGHTING
C =================================================================
      DIMENSION AL(1)
100   FORMAT(1H1)
200   FORMAT(1X,79('-'))
300   FORMAT(20X,'TAYLOR WEIGHTING FOR MAIN ANTENNA ARRAY',/,20X
     -,39('-'))
400   FORMAT(8F10.5)
      DATA PI/3.1415965/
      PI2=2*PI
      LOC=LOC1-1
      CON2=0.5*(NEL+1)
      CON1=PI2/NEL
      DO 10 I=1,NEL
      LOC=LOC+1
      ARG=CON1*(LOC-CON2)
10    AL(I)=1+0.5*COS(ARG)
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,300)
      WRITE(108,400) (AL(I),I=1,NEL)
      WRITE(108,200)
      RETURN
      END
```

```
C =======================================================================
      SUBROUTINE FILTER(NF,DRAD,RADIN,FBIF,LPBP,BWFCTR,BWOFF,NPOL,POL
     -,H)
C =======================================================================
C EVALUATION OF NORMALIZED RECEIVER BASEBAND SAMPLED TRANSFER FUNCTION
C                   AT DISCRETE RADIAN FREQUENCIES
C                                USING
C         LOWPASS PROTOTYPE FILTER OR ITS BANDPASS EQUIVALENT
C
C           PROGRAM       :  FILTERIS
C           ORIGINAL      :  APRIL 19, 1980
C           REVISION      :  JUNE  23, 1980
C
C           PREPARED  BY  :  S. M. DANIEL  &  I. M. KERTESZ
C                            RADAR SYSTEMS ANALYSIS GROUP
C                            MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                            TEMPE, ARIZONA  85282
C -----------------------------------------------------------------------
C     INPUT  :     NF         -  NUMBER OF FREQUENCY SAMPLES
C                  DRAD       -  NORMALIZED RADIAN FREQUENCY INCREMENT
C                  RADIN      -  INITIAL RADIAN FREQUENCY
C                  FBIF       -  FRACTIONAL BANDWIDTH AT FINAL IF
C                  LPBP       -  LOWPASS / BANDPASS OPTION
C                                0  :  LOWPASS
C                                1  :  BANDPASS
C                  BWFCTR     -  BANDWIDTH FACTOR
C                  BWOFF      -  BANDWIDTH OFFSET FACTOR
C                  NPOL       -  NUMBER OF POLES
C                  POL        -  POLE LOCATIONS
C
C     OUTPUT :     H          -  SAMPLED TRANSFER FUNCTION
C =======================================================================
      COMPLEX POL,S,DEN,H
      DIMENSION POL(1),H(1)
C -----------------------------------------------------------------------
C     ADJUST INCREMENT AND INITIAL RADIAN FREQUENCY
C     COMPUTE SAMPLED TRANSFER FUNCTION
C -----------------------------------------------------------------------
      DRADL=0.5*DRAD/BWFCTR
      RAD1=(RADIN/2-BWOFF)/BWFCTR-DRADL
      DO 10 I=1,NF
      RAD=RAD1+I*DRADL
      H(I)=1
```

```
      S=CMPLX(0,RAD)
      IF(LPBP.EQ.1) S=S*(1+1/(1+RAD*FBIF))
      DO 10 J=1,NPOL
      DEN=S-POL(J)
10    H(I)=H(I)*POL(J)/DEN
C    ------------------------------------------------------------------
      RETURN
      END
```

```
C ======================================================================
     SUBROUTINE CHANNEL(NEL,LOC1,AL,D0,TH,NF,RADIN,DRAD,BWFCTR,BWOFF
    -,FBRF,HR,H)
C ======================================================================
C EVALUATION OF NORMALIZED CHANNEL BASEBAND SAMPLED TRANSFER FUNCTION
C                    AT DISCRETE RADIAN FREQUENCIES
C                               INCLUDING
C          LINEAR ANTENNA ARRAY AND RECEIVER CHARACTERISTICS
C
C           PROGRAM       :  CHANNELIS
C           ORIGINAL      :  MAY   4, 1980
C           REVISION      :  JUNE 26, 1980
C
C           PREPARED  BY  :  S. M. DANIEL  &  I. M. KERTESZ
C                            RADAR SYSTEMS ANALYSIS GROUP
C                            MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                            TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C    INPUT  :     NEL       -  NUMBER OF ELEMENTS IN LINEAR ARRAY
C                 LOC1      -  LOCATION (NUMBER) OF FIRST ELEMENT
C                 AL        -  ARRAY ELEMENT WEGHTING
C                 TH        -  AZIMUTH ANGLE (DEG) OF INCIDENCE
C                 NF        -  NUMBER OF FREQUENCY SAMPLES
C                 RADIN     -  INITIAL RADIAN FREQUENCY
C                 DRAD      -  NORMALIZED RADIAN FREQUENCY INCREMENT
C                 BWFCTR    -  BANDWIDTH FACTOR
C                 BWOFF     -  BANDWIDTH OFFSET FACTOR
C                 FBRF      -  FRACTIONAL BANDWIDTH AT RF
C                 HR        -  RECEIVER SAMPLED TRANSFER FUNCTION
C                 D0        -  ELEMENT SEPARATION FACTOR
C
C    OUTPUT :     H         -  CHANNEL SAMPLED TRANSFER FUNCTION
C ======================================================================
     COMPLEX HR,H,CH
     DIMENSION HR(1),H(1),AL(1)
     DATA PI/3.14159265/
     RATIO=FBRF/BWFCTR
     DRADL=RATIO*DRAD/2
     RAD1=1+RATIO*(RADIN/2-BWOFF)-DRADL
     PDSTH=PI*D0*SIN(TH*PI/180)
C ----------------------------------------------------------------------
C    EVALUATION OF BASEBAND CHANNEL SAMPLED TRANSFER FUNCTION
C ----------------------------------------------------------------------
```

```
      DO 10 I=1,NF
      RAD=RAD1+I*DRADL
      CH=0
      LOC=LOC1-1
      DO 20 J=1,NEL
      LOC=LOC+1
      ARG=LOC*PDSTH*RAD
   20 CH=CH+AL(J)*CMPLX(COS(ARG),SIN(ARG))
   10 H(I)=CH*HR(I)
C     ----------------------------------------------------------------
      RETURN
      END
```

```
C ================================================================================
      SUBROUTINE AMPH(H,NF,AM,PH)
C ================================================================================
C                    EVALUATION OF AMPLITUDE AND PHASE
C                                  FROM
C                    COMPLEX SAMPLED TRANSFER FUNCTION
C
C             PROGRAM       :  AMPHIS
C             ORIGINAL      :  APRIL 19, 1980
C             REVISION      :  JULY   1, 1980
C
C             PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                              RADAR SYSTEMS ANALYSIS GROUP
C                              MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                              TEMPE, ARIZONA  85282
C --------------------------------------------------------------------------------
C     INPUT  :      H         -  SAMPLED TRANSFER FUNCTION
C                   NF        -  NUMBER OF FREQUENCY SAMPLES
C
C     OUTPUT :      AM        -  SAMPLED AMPLITUDE RESPONSE
C                   PH        -  SAMPLED PHASE RESPONSE
C ================================================================================
      COMPLEX H
      DIMENSION H(1),AM(1),PH(1)
100   FORMAT(1X,79('-'))
200   FORMAT(35X,'AMPLITUDE',/,35X,9('-'),/,(8F10.5))
300   FORMAT(37X,'PHASE',/,37X,5('-'),/,(8F10.5))
C --------------------------------------------------------------------------------
C     COMPUTE AMPLITUDE AND PHASE
C --------------------------------------------------------------------------------
      DO 10 I=1,NF
      AM(I)=CABS(H(I))
      PH(I)=ACOS(REAL(H(I))/AM(I))
10    IF(AIMAG(H(I)).LT.0) PH(I)=-PH(I)
C --------------------------------------------------------------------------------
      WRITE(108,100)
      WRITE(108,200)  (AM(I),I=1,NF)
      WRITE(108,100)
      WRITE(108,300)  (PH(I),I=1,NF)
      WRITE(108,100)
      RETURN
      END
```

```
C ====================================================================
      SUBROUTINE FFT2(X,NX,IF)
C ====================================================================
C             COMPUTATION OF BASE-2 DISCRETE FOURIER TRANSFORM
C                      OF A COMPLEX-VALUED TIME SERIES
C                 REPRESENTED BY REAL AUXILIARY ARRAY X
C
C              PROGRAM       :  FFT2:S
C              ORIGINAL      :  JULY 15, 1969
C              REVISION      :  JUNE  7, 1980
C
C              PREPARED BY   :  PROF. P. RANSOM & S. M. DANIEL
C                               ELECTRICAL ENGINEERING DEPARTMENT
C                               UNIVERSITY OF ILLINOIS
C                               CHAMPAIGN-URBANA, ILLINOIS
C --------------------------------------------------------------------
C     INPUT  :     X         -  AUXILIARY REAL ARRAY WHOSE ODD AND EVEN-
C                             NUMBERED ELEMENTS ARE THE REAL AND THE
C                             IMAGINARY PARTS OF THE ORIGINAL COMPLEX
C                             INPUT ARRAY
C                  NX        -  NUMBER OF COMPLEX SAMPLES
C                  IF        -  FOURIER TRANSFORM OPTION
C                               -1  :  FOREWARD DFT
C                               +1  :  INVERSE  DFT
C
C     OUTPUT :     X         -  TRANSFORMED AUXILIARY ARRAY
C ====================================================================
      DIMENSION X(1)
      DATA PI/3.14159265/
      PI2=2*PI $ NX2=2*NX
      SNXI=1 $ IF(IF.EQ.1) SNXI=1.0/NX
      DO 10 I=1,NX $ IMOD=MOD(I-1,2) $ I2=2*I $ I2M1=I2-1
      FCTR=SNXI $ IF(IMOD.EQ.1) FCTR=-SNXI
      X(I2M1)=FCTR*X(I2M1) $ X(I2)=FCTR*X(I2)
10    CONTINUE
      J=1
      DO 20 I=1,NX2,2 $ IP1=I+1 $ JP1=J+1
      IF(I-J.GE.0) GOTO 1000
      XTR=X(J) $ XTI=X(JP1) $ X(J)=X(I) $ X(JP1)=X(IP1)
      X(I)=XTR $ X(IP1)=XTI
1000  K=NX
2000  IF(J-K.LE.0) GOTO 20
      J=J-K $ K=K/2 $ IF(K-2.GE.0) GOTO 2000
```

```
20    J=J+K
      KMAX=2
3000  IF(KMAX-NX2.GE.0) GOTO 4000
      ISTEP=2*KMAX $ TH=IF*PI2/KMAX $ STH=SIN(TH/2)
      WSR=-2*STH**2 $ WSI=SIN(TH) $ WR=1 $ WI=0
      DO 40 K=1,KMAX,2
      DO 30 I=K,NX2,ISTEP
      J=I+KMAX $ JP1=J+1 $ IP1=I+1
      WXTR=WR*X(J)-WI*X(JP1) $ WXTI=WR*X(JP1)+WI*X(J)
      X(J)=X(I)-WXTR $ X(JP1)=X(IP1)-WXTI
      X(I)=X(I)+WXTR $ X(IP1)=X(IP1)+WXTI
30    CONTINUE
      WXTR=WR $ WR=WR*(1+WSR)-WI*WSI $ WI=WI*(1+WSR)+WXTR*WSI
40    CONTINUE
      KMAX=ISTEP $ GOTO 3000
4000  CONTINUE
      DO 50 I=1,NX $ IMOD=MOD(I-1,2) $ IF(IMOD.EQ.0) GOTO 50
      I1=2*I-1 $ I2=I1+1 $ X(I1)=-X(I1) $ X(I2)=-X(I2)
50    CONTINUE
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE IMPULS(NF,H,X,HIMPL,IW)
C ======================================================================
C                 EVALUATION OF SAMPLED IMPULSE RESPONSE
C                    FROM A SAMPLED TRANSFER FUNCTION
C
C             PROGRAM        :  IMPULSIS
C             ORIGINAL       :  MAY 11, 1980
C             REVISION       :  JUNE 7, 1980
C
C             PREPARED  BY   :  S. M. DANIEL  &  I. M. KERTESZ
C                               RADAR SYSTEMS ANALYSIS GROUP
C                               MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                               TEMPE, ARIZONA 85282
C ----------------------------------------------------------------------
C     INPUT  :     NF        -  NUMBER OF FREQUENCY SAMPLES
C                  H         -  SAMPLED TRANSFER FUNCTION
C
C     OUTPUT :     HIMPL     -  COMPLEX IMPULSE RESPONSE
C                  X         -  AUXILIARY REAL ARRAY
C ======================================================================
      COMPLEX H,HIMPL
      DIMENSION H(1),HIMPL(1),X(1)
  100 FORMAT(1X,79('-'))
  200 FORMAT(32X,'IMPULSE RESPONSE',/,32X,16('-'),/,(8F10.5))
      NF2=2*NF
      X(1)=0
      X(2)=0
      IH=2
      DO 10 I=3,NF2,2
      X(I)=REAL(H(IH))
      X(I+1)=AIMAG(H(IH))
   10 IH=IH+1
      CALL FFT2(X,NF,1)
      DO 20 I=1,NF
      I2=2*I $ I1=I2-1
   20 HIMPL(I)=CMPLX(X(I1),X(I2))
      IF(IW.EQ.0) RETURN
      WRITE(108,100)
      WRITE(108,200) (HIMPL(I),I=1,NF)
      WRITE(108,100)
      RETURN
      END
```

```
C     ======================================================================
      SUBROUTINE BLINK(N1,NB,NF,NS,IBLNK)
C     ======================================================================
C                     ON/OFF PERIODIC BLINK CONTOL GENERATOR
C
C                PROGRAM       :  BLINK:S
C                ORIGINAL      :  JUNE 12, 1980
C                REVISION      :  JUNE 16, 1980
C
C                PREPARED  BY  :  I. M. KERTESZ  &  S. M. DANIEL
C                                 RADAR SYSTEMS ANALYSIS GROUP
C                                 MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                 TEMPE, ARIZONA  85282
C     ----------------------------------------------------------------------
C     INPUT  :     N1         -   FIRST-TIME-ON SAMPLE NUMBER
C                  NB         -   BLINK DURATION IN SAMPLES
C                  NF         -   NUMBER OF FREQUENCY SAMPLES
C                  NS         -   NUMBER OF SIGNAL SAMPLES
C
C     OUTPUT :     IBLNK      -   BLINK CONTROL ARRAY HAVING (NS+NF/2)
C                                 SAMPLES
C     ======================================================================
      DIMENSION IBLNK(1)
      J1=1
      J2=0
      N1M1=N1-1
      DO 10 I=1,N1M1
   10 IBLNK(I)=0
      J=0
      NSPH=NS+NF/2
      DO 20 I=N1,NSPH
      J=J+1
      IBLNK(I)=J1
      IF(MOD(J,NB).NE.0) GOTO 20
      J3=J2
      J2=J1
      J1=J3
   20 CONTINUE
      RETURN
      END
```

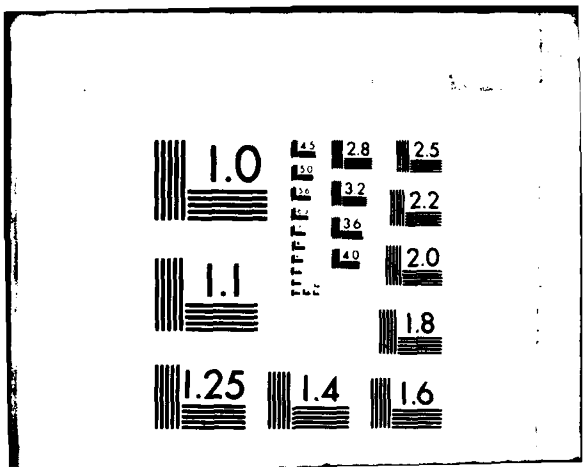
```
C =====================================================================
      SUBROUTINE SIGNAL0(XIMPL,NX,IX,IBLNK,P,S,NS)
C =====================================================================
C              COMPUTATION OF RECEIVED BASEBAND SAMPLED SIGNALS
C                                     FROM
C                    A WIDEBAND NOISE INTERFERENCE SOURCE
C
C               PROGRAM        :  SIGNAL0:S
C               ORIGINAL       :  JUNE  8, 1980
C               REVISION       :  JUNE 23, 1980
C
C               PREPARED  BY   :  S. M. DANIEL  &  I. M. KERTESZ
C                                 RADAR SYSTEMS ANALYSIS GROUP
C                                 MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                 TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C     INPUT  :     XIMPL      -   COMPLEX BASEBAND SAMPLED IMPULSE
C                                 RESPONSE
C                  NX         -   NUMBER OF SAMPLES IN XIMPL
C                  IX         -   INITIAL RANDU SETTING FOR NOISE SOURCE
C                  P          -   NX-ELEMENT SLIDING-WINDOW NOISE ARRAY
C                  NS         -   NUMBER OF SIGNAL SAMPLES
C
C     OUTPUT :     S          -   NS-ELEMENT RECEIVED COMPLEX BASEBAND
C                                 SAMPLED SIGNAL ARRAY
C =====================================================================
      COMPLEX XIMPL,S
      DIMENSION XIMPL(1),P(1),S(1),IBLNK(1)
      NXH=NX/2
      NXHP1=NXH+1
      NXM1=NX-1
      NXP1=NX+1
      DO 10 I=1,NXH
   10 P(I)=0
      IB=1
      DO 20 I=NXHP1,NX
      CALL RANDU(IX,IY,Z)
      P(I)=IBLNK(IB)*(Z-0.5)
   20 IB=IB+1
      DO 30 I=1,NS
      S(I)=0
      DO 40 J=1,NX
   40 S(I)=S(I)+P(J)*XIMPL(NXP1-J)
```

```
      DO 50 J=1,NXM1
   50 P(J)=P(J+1)
      CALL RANDU(IX,IY,Z)
      P(NX)=IBLNK(IB)*(Z-0.5)
   30 IB=IB+1
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE RWID(CX,RX,IX,IND)
C ======================================================================
C                   READING AND WRITING AN 80-CHARACTER LINE
C                               CONSISTING OF
C                          A 60-CHARACTER COMMENT
C                                    AND
C               A COMPLEX, REAL OR INTEGER SCALAR VARIABLE VALUE
C
C                 PROGRAM       :  RWID:S
C                 ORIGINAL      :  APRIL 19, 1980
C                 REVISION      :  JUNE  15, 1980
C
C                 PREPARED  BY  :  S. M. DANIEL  &  I. M. KERTESZ
C                                  RADAR SYSTEMS ANALYSIS GROUP
C                                  MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                  TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     INPUT  :    IND        -  OPTION INDICATOR
C                               0  :  READ & WRITE INTEGER
C                               1  :  READ & WRITE REAL
C                               2  :  READ & WRITE COMPLEX
C
C     OUTPUT :    CX         -  COMPLEX SCALAR EXCLUSIVE OF RX & IX
C                 RX         -  REAL SCALAR EXCLUSIVE OF CX & IX
C                 IX         -  INTEGER SCALAR EXCLUSIVE OF CX & RX
C ======================================================================
      COMPLEX CX
      DIMENSION AR15(15),AR3(3)
  100 FORMAT(15A4,I8,3A4)
  200 FORMAT(15A4,F8.5,3A4)
  300 FORMAT(15A4,2(F8.5,2X))
      IF(IND.NE.0) GOTO 1000
      READ (105,100) AR15,IX,AR3
      WRITE(108,100) AR15,IX,AR3
      RETURN
 1000 IF(IND.NE.1) GOTO 2000
      READ (105,200) AR15,RX,AR3
      WRITE(108,200) AR15,RX,AR3
      RETURN
 2000 READ (105,300) AR15,CX
      WRITE(108,300) AR15,CX
      RETURN
      END
```

AD-A109 927 MOTOROLA INC TEMPE AZ GOVERNMENT ELECTRONICS DIV F/G 20/14
BATCH COVARIANCE RELAXATION (BCR) ADAPTIVE PROCESSING.(U)
AUG 81 S M DANIEL, I KERTESZ F30602-80-C-0031
UNCLASSIFIED 8512-F RADC-TR-81-212 NL

2 of 7


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

```
C ========================================================================
      SUBROUTINE RW(N)
C ========================================================================
C                 READING AND WRITING 80-CHARACTER DATA COMMENTS
C
C                 PROGRAM       :  RWIS
C                 ORIGINAL      :  APRIL 19, 1980
C                 REVISION      :  JUNE  15, 1980
C
C                 PREPARED  BY  :  S. M. DANIEL  &  I. M. KERTESZ
C                                  RADAR SYSTEMS ANALYSIS GROUP
C                                  MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                  TEMPE, ARIZONA  85282
C ------------------------------------------------------------------------
C     INPUT  :    N         -  NUMBER OF LINES TO BE READ AND WRITTEN
C ========================================================================
      DIMENSION AR20(20)
100   FORMAT(20A4)
      DO 10 I=1,N
      READ (105,100) AR20
10    WRITE(108,100) AR20
      RETURN
      END
```

3.2.3.1 Creation of Binary Modules

The specific job control language program, JCL:B, written for the purpose of creating a binary version of a given source module is listed below.

Individual Binary Module Generation Program JCL:B

```
1.000 !JOB 298,DANIEL(8512),7,BLDG90
2.000 !LIMIT (TIME,1),(UO,1),(CO,16),(ACCOUNT)
3.000 !SET M:SI/NAME:S;IN;SAVE
4.000 !SET M:BO/NAME:B;OUT;SAVE
5.000 !FORTRAN LS,NS,BC,SI,BO
```

Given a specific module to be processed, this program is batched with a name substitution command; e.g.,

```
! BATCH JCL:B 'NAME' = SIGMAINO
```

As such, JCL:B is completely general.

3.2.3.2 Composite Binary and Load Modules

Given the set of signal generation binary modules, it is an easy matter to form a composite signal generation binary module which is logically identified by the *name*, SIGGENO:B. The specific JCL program that actually accomplishes this is called JCLSIGO:BL. Clearly, the first letter of the suffix :BL refers to the creation of the composite binary module, SIGGENO:B. The second letter denotes the additional generation of the associated executable load module, SIGGENO:L, which is included here strictly for convenience.

The complete listing of JCLSIGO:BL is given below. Note that by virtue of the special form of this

Composite Binary and Load Module Generation Program JCLSIGO:BL

```
 1.000 !JOB 298,DANIEL(8512),7,BLDG90
 2.000 !LIMIT (TIME,1),(UO,2),(CO,16),(ACCOUNT)
 3.000 !PCL
 4.000 COPY  AMPH:B OVER SIGGENO:B
 5.000 COPY  ANTWT:B
 6.000 COPY  BLINK:B
 7.000 COPY  CHANNEL:B
 8.000 COPY  FFT2:B
 9.000 COPY  FILTER:B
10.000 COPY  IMPULS:B
11.000 COPY  PRTSGNLO:B
12.000 COPY  RW:B
13.000 COPY  RWIO:B
14.000 COPY  SIGMAINO:B
15.000 COPY  SIGNALO:B
16.000 COPY  SIGSETO:B
17.000 !LYNX SIGGENO:B OVER SIGGENO:L
```

JCL program, all binary modules that comprise the composite one may be listed alphabetically, thus forming a convenient list for easy reference. To execute this program, the simple batch command !BATCH JCLSIGO:BL is used.

3.2.4 Program Execution

The execution of the load module, SIGGENO:L, is carried out by means of a specially designed JCL program shown below.

Execution Program JCL:EX

```
1 -    1.000 !JOB 298,DANIEL(8512),7,BLDG90
2 -    2.000 !LIMIT (TIME,1),(UO,50),(CO,16),(ACCOUNT)
3 -    3.000 !SET F:108/NAME:O
4 -    4.000 !RUN (LMN,NAME:L)
5 -    5.000 !DATA
6 -    6.000 !EXEC NAME:D
```

As is the case of JCL:B, the present program is of general use, in the sense that the name of the generic program may be defined in the batch command

! BATCH JCL:EX (E) 'NAME' = SIGGENO

Other important program features of JCL:EX include the reading from the input data file SIGGENO:D and creation of the output file SIGGENO:O. Here, suffices :D and :O denote data and output, respectively. An additional important feature of JCL:EX is that it allows for multiple simultaneous runs with augmented SIGGENO:D data. This is because the current version of SIGGENO:D is buffered at each batching.†

3.2.5 Output File

The output file generation from a specific program execution involving one channel and four ports is listed in the pages that follow. In addition to the parametric system description from SIGGENO:D, SIGGENO:O includes the main antenna array weighting, receiver and channel frequency and time domain characteristics, individual channel and port signals. Figure 3-4 compares representative main and auxiliary baseband channel amplitude responses. Although the asymmetry of the auxiliary response is barely noticeable, that of the main is rather pronounced. Of course, based on previous discussion, this expected effect is due to the aperture dispersion of the main array.

It should be noted that if the three print options in the table of system specifications were set to "0," the printing of receiver and channel characteristics along with associated channel signals could be conveniently suppressed. The BCR adaptive processing that will follow will need only main and auxiliary port signal information.

---

†It should be noted that the JCL programs presented here are peculiar to Motorola's Honeywell CP-V engineering computer. They are included here to clarify fully the methodology behind the signal generation software. Similar programs may be generated for other computers, with appropriate changes, as needed.

```
----------------------------------------------------------------------------
                    SPECIFICATION OF SYSTEM PARAMETERS
                    ----------------------------------

----------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWR     -   RECEIVER CHARACTERISTICS                     :          1
  IWC     -   CHANNEL CHARACTERISTICS                      :          1
  IWSC    -   INDIVIDUAL CHANNEL SIGNALS                   :          1

FILTER PARAMETERS
-----------------
  NPOL    -   NUMBER OF LOWPASS PROTOTYPE POLES            :          2
              POL(1)                                       :   -.70700     -.70700
              POL(2)                                       :   -.70700      .70700
  FBIF    -   FRACTIONAL BANDWIDTH AT FINAL IF             :    .10000
  FBRF    -   FRACTIONAL BANDWIDTH AT RF                   :    .00100
  RADRNG  -   NORMALIZED RADIAN FREQUENCY RANGE            :   4.00000
  RADIN   -   NORMALIZED INITIAL RADIAN FREQUENCY          :  -2.00000
  NF      -   NUMBER OF FREQUENCY SAMPLES                  :         32
  LPBP    -   LOWPASS/BANDPASS OPTION                      :          1
              0  :   LOWPASS
              1  :   BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS  -   NUMBER OF CHANNELS PER PORT                  :          4

CHANNEL 1 :
   AN     -   AMPLITUDE OF NOISE SOURCE                    :   1.00000
   IX0    -   INITIAL RANDU SETTING                        :          1
   N1     -   FIRST-TIME-ON SAMPLE NUMBER                  :          1
   NB     -   BLINK DURATION IN SAMPLES                    :      10000
   TH     -   AZIMUTH ANGLES OF INCIDENCE (DEG)            :  45.00000

CHANNEL 2 :
   AN     -   AMPLITUDE OF NOISE SOURCE                    :    .00000
   IX0    -   INITIAL RANDU SETTING                        :         11
   N1     -   FIRST-TIME-ON SAMPLE NUMBER                  :          1
   NB     -   BLINK DURATION IN SAMPLES                    :      10000
   TH     -   AZIMUTH ANGLES OF INCIDENCE (DEG)            :  10.00000
```

```
CHANNEL 3 :
   AN     -  AMPLITUDE OF NOISE SOURCE                  :      .00000
   IX0    -  INITIAL RANDU SETTING                      :         111
   N1     -  FIRST-TIME-ON SAMPLE NUMBER                :           1
   NB     -  BLINK DURATION IN SAMPLES                  :       10000
   TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)          : -20.00000

CHANNEL 4 :
   AN     -  AMPLITUDE OF NOISE SOURCE                  :      .00000
   IX0    -  INITIAL RANDU SETTING                      :        1111
   N1     -  FIRST-TIME-ON SAMPLE NUMBER                :           1
   NB     -  BLINK DURATION IN SAMPLES                  :       10000
   TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)          : -35.00000

PORT PARAMETERS
---------------
  D0      -  ANTENNA-ELEMENT SEPARATION FACTOR          :     1.00000
  NS      -  NUMBER OF SIGNAL SAMPLES                   :         128
  NPORTS  -  NUMBER OF PORTS                            :           5

PORT0 :
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :         255
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                    :      .00000

PORT1 :
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                    :      .00000

PORT2 :
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :          51
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                    :      .00000

PORT3 :
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :         127
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :     1.00000
```

```
  BWOFF  =   BANDWIDTH OFFSET FACTOR              :    .00000

PORT4 :
  NEL    =   NUMBER OF ANTENNA ELEMENTS           :         1
  LOC1   =   LOCATION OF THE FIRST ELEMENT        :       255
  BWFCTR =   BANDWIDTH TOLERANCE FACTOR           :   1.00000
  BWOFF  =   BANDWIDTH OFFSET FACTOR              :    .00000
-----------------------------------------------------------------------
```

TAYLOR WEIGHTING FOR MAIN ANTENNA ARRAY

.50004	.50034	.50095	.50186	.50307	.50458	.50640	.50851
.51093	.51364	.51664	.51994	.52353	.52741	.53157	.53602
.54076	.54577	.55105	.55661	.56244	.56853	.57489	.58150
.58837	.59549	.60285	.61046	.61830	.62637	.63467	.64319
.65193	.66088	.67004	.67939	.68894	.69868	.70860	.71870
.72897	.73941	.75000	.76074	.77163	.78266	.79382	.80511
.81651	.82802	.83964	.85136	.86317	.87506	.88702	.89906
.91115	.92330	.93550	.94773	.96000	.97229	.98460	.99692
1.00924	1.02155	1.03385	1.04613	1.05838	1.07060	1.08278	1.09490
1.10697	1.11897	1.13090	1.14275	1.15451	1.16618	1.17774	1.18920
1.20055	1.21177	1.22287	1.23383	1.24465	1.25531	1.26583	1.27618
1.28637	1.29638	1.30621	1.31585	1.32531	1.33456	1.34362	1.35246
1.36109	1.36950	1.37769	1.38565	1.39337	1.40086	1.40810	1.41509
1.42183	1.42832	1.43454	1.44051	1.44620	1.45162	1.45677	1.46164
1.46624	1.47054	1.47457	1.47830	1.48175	1.48490	1.48776	1.49032
1.49258	1.49454	1.49621	1.49757	1.49863	1.49939	1.49985	1.50000
1.49985	1.49939	1.49863	1.49757	1.49621	1.49454	1.49258	1.49032
1.48776	1.48490	1.48175	1.47830	1.47457	1.47054	1.46624	1.46164
1.45677	1.45162	1.44620	1.44051	1.43454	1.42832	1.42183	1.41509
1.40810	1.40086	1.39337	1.38565	1.37769	1.36950	1.36109	1.35246
1.34362	1.33456	1.32531	1.31585	1.30621	1.29638	1.28637	1.27618
1.26583	1.25531	1.24465	1.23383	1.22287	1.21177	1.20055	1.18920
1.17774	1.16618	1.15451	1.14275	1.13090	1.11897	1.10697	1.09490
1.08278	1.07060	1.05838	1.04613	1.03385	1.02155	1.00924	.99692
.98460	.97229	.96000	.94773	.93550	.92330	.91115	.89906
.88702	.87506	.86317	.85136	.83964	.82802	.81651	.80511
.79382	.78266	.77163	.76074	.75000	.73941	.72897	.71870
.70860	.69868	.68894	.67939	.67004	.66088	.65193	.64319
.63467	.62637	.61830	.61046	.60285	.59549	.58837	.58150
.57489	.56853	.56244	.55661	.55105	.54577	.54076	.53602
.53157	.52741	.52353	.51994	.51664	.51364	.51093	.50851
.50640	.50458	.50307	.50186	.50095	.50034	.50004	

PORT 0

RECEIVER BASEBAND CHARACTERISTICS

AMPLITUDE

.21887	.24898	.28493	.32799	.37964	.44131	.51399	.59734
.68840	.78038	.86323	.92737	.96864	.98988	.99800	.99988
1.00000	.99988	.99810	.99060	.97150	.93510	.87898	.80611
.72383	.64035	.56184	.49158	.43054	.37834	.33402	.29644

PHASE

2.42933	2.37392	2.30990	2.23532	2.14777	2.04434	1.92177	1.77689
1.60773	1.41510	1.20424	.98457	.76677	.55867	.36306	.17828
.00000	-.17716	-.35838	-.54754	-.74596	-.95120	-1.15697	-1.35477
-1.53694	-1.69888	-1.83942	-1.95978	-2.06240	-2.14996	-2.22503	-2.28976

IMPULSE RESPONSE

.00567	.00075	-.00618	-.00072	.00670	.00069	-.00725	-.00066
.00783	.00063	-.00847	-.00059	.00918	.00056	-.00999	-.00052
.01096	.00047	-.01214	-.00042	.01365	.00035	-.01569	-.00027
.01865	.00015	-.02342	.00002	.03245	-.00031	-.05579	.00098
.23241	-.01276	.66174	.01673	.18167	.00393	-.00713	-.00919
-.03027	-.00002	-.00291	-.00064	-.00038	.00187	.00117	-.00113
.00094	.00103	-.00167	-.00106	.00230	.00097	-.00299	-.00092
.00358	.00088	-.00413	-.00085	.00466	.00081	-.00517	-.00078

PORT 0 CHANNEL 1

CHANNEL BASEBAND CHARACTERISTICS

AMPLITUDE

.02516	.03111	.03819	.04721	.05741	.07175	.08866	.10756
.13021	.15612	.17942	.20275	.21973	.23489	.24453	.25425
.26303	.27158	.27949	.28597	.28855	.28593	.27593	.25933
.23889	.21616	.19387	.17358	.15566	.13909	.12534	.11333

PHASE

-2.54147	-2.58440	-2.62338	-2.67416	-2.75840	-2.83401	-2.93445	-3.07337
3.06784	2.89006	2.69825	2.49650	2.29800	2.10861	1.92829	1.76085
1.60033	1.43937	1.27740	1.10573	.92522	.73514	.54932	.36945
.20368	.06104	-.05964	-.16455	-.25084	-.31677	-.37486	-.42064

IMPULSE RESPONSE

-.00107	.00086	.00104	-.00125	-.00118	.00126	.00101	-.00151
-.00102	.00169	.00094	-.00184	-.00082	.00217	.00063	-.00244
-.00067	.00276	.00068	-.00314	-.00051	.00363	.00044	-.00426
-.00014	.00525	-.00013	-.00681	.00052	.00956	-.00163	-.01688
.03108	.07904	-.01964	.16878	-.02107	.03532	.00277	-.00075
-.00109	-.00883	.00270	.00087	-.00204	-.00131	.00174	.00136
-.00166	-.00074	.00158	.00044	-.00166	-.00013	.00149	.00001
-.00134	.00018	.00121	-.00037	-.00121	.00074	.00129	-.00075

PORT 0 CHANNEL 1

RECEIVED BASEBAND CHANNEL SIGNAL

.00187	.00323	-.01627	-.03101	-.00430	-.12111	.00505	-.13317
.00402	-.13817	.00524	-.12671	.00335	-.13047	.00868	-.11011
.01396	-.07821	.01617	.03235	-.02411	.01303	.01298	-.04663
.01499	.05881	-.00386	.10206	-.02370	.05140	-.00163	-.05167
.00452	-.02603	.00498	-.03625	.01859	.07055	-.02890	.05202
.00696	-.02636	-.00149	.01150	.00726	.00837	-.00539	.05801
-.02007	-.04303	.01769	-.05763	-.00277	-.00879	.01474	.00045
.00338	.09579	-.01962	.04148	.01109	.01480	.00591	.09187
-.02249	.06113	-.00349	-.03437	.01161	-.00481	-.00163	.02496
.00267	.03042	.00131	.05130	.00327	.07865	-.00878	.07727
.00012	.06209	-.01363	.04679	-.00685	-.03073	.00895	-.02676
-.00078	-.00556	-.00355	-.01778	.00576	-.02992	.01404	.03470
-.00600	.07720	-.01020	.04013	-.00086	.00360	.00653	.02997
-.01482	.01094	.01523	-.01241	-.00009	.07345	-.02265	-.00618
.00655	-.06727	.02300	.01137	-.00281	.11030	-.02559	.02531
.00242	-.05410	.01166	-.01629	-.00698	.00341	-.00230	-.04610
.01451	-.00824	.00698	.05159	.00114	.11260	-.03047	.03335
.01399	-.03259	-.00149	.03038	.00142	.00464	.00971	.06811
-.00549	.09189	-.01406	.05179	.00605	.01652	-.01509	.03259
-.01512	-.11005	.01699	-.09279	-.00667	-.10155	.02499	-.05323
-.01211	-.00779	.01906	-.02405	.00944	.10518	-.01271	.09842
-.00034	.08088	.00284	.09991	-.00282	.13020	-.01214	.09993
-.00808	.05787	-.00317	.02078	.00185	.02751	-.01758	-.00557
-.00544	-.10526	.02870	-.05102	-.00046	.05391	.00134	.04251
.00043	.08879	-.01794	.02990	.00654	-.00676	.01286	.04938
-.00426	.10868	-.01377	.04661	.00922	.05311	-.01678	.03809
.00788	.00391	-.01264	.01274	.01129	-.02814	.00613	.06966
-.01626	.02858	-.01009	-.02899	.00325	-.09887	.02942	.01401
-.02598	.04037	.00109	-.06509	.01224	-.00975	-.00453	.00094
.01025	.01370	.00242	.07343	-.01282	.04849	-.00252	.00510
.00748	.01670	.00363	.06073	.00382	.08629	-.00903	.10863
-.01287	.03734	.00560	.03335	-.01634	.00887	.01133	-.03585

PORT 0

RECEIVED BASEBAND PORT SIGNAL

.00187	.00323	-.01627	-.03101	-.00430	-.12111	.00505	-.13317
.00402	-.13817	.00524	-.12671	.00335	-.13047	.00868	-.11011
.01396	-.07821	.01617	.03235	-.02411	.01303	.01298	-.04663
.01499	.05881	-.00386	.10206	-.02370	.05140	-.00163	-.05167
.00452	-.02603	.00498	-.03625	.01859	.07055	-.02890	.05202
.00696	-.02636	-.00149	.01150	.00726	.00837	-.00539	.05801
-.02007	-.04303	.01769	-.05763	-.00277	-.00879	.01474	.00045
.00338	.09579	-.01962	.04148	.01109	.01480	.00591	.09187
-.02249	.06113	-.00349	-.03437	.01161	-.00481	-.00163	.02496
.00267	.03042	.00131	.05130	.00327	.07865	-.00878	.07727
.00012	.06209	-.01363	.04679	-.00685	-.03073	.00895	-.02676
-.00078	-.00556	-.00355	-.01778	.00576	-.02992	.01404	.03470
-.00600	.07720	-.01020	.04013	-.00086	.00360	.00653	.02997
-.01482	.01094	.01523	-.01241	-.00009	.07345	-.02265	-.00618
.00655	-.06727	.02300	.01137	-.00281	.11030	-.02559	.02531
.00242	-.05410	.01166	-.01629	-.00698	.00341	-.00230	-.04610
.01451	-.00824	.00698	.05159	.00114	.11260	-.03047	.03335
.01399	-.03259	-.00149	.03038	.00142	.00464	.00971	.06811
-.00549	.09189	-.01406	.05179	.00605	.01652	-.01509	.03259
-.01512	-.11005	.01699	-.09279	-.00667	-.10155	.02499	-.05323
-.01211	-.00779	.01906	-.02405	.00944	.10518	-.01271	.09842
-.00034	.08088	.00284	.09991	-.00282	.13020	-.01214	.09993
-.00808	.05787	-.00317	.02078	.00185	.02751	-.01758	-.00557
-.00544	-.10526	.02870	-.05102	-.00046	.05391	.00134	.04251
.00043	.08879	-.01794	.02990	.00654	-.00676	.01286	.04938
-.00426	.10868	-.01377	.04661	.00922	.05311	-.01678	.03809
.00788	.00391	-.01264	.01274	.01129	-.02814	.00613	.06966
-.01626	.02858	-.01009	-.02899	.00325	-.09887	.02942	.01401
-.02598	.04037	.00109	-.06509	.01224	-.00975	-.00453	.00094
.01025	.01370	.00242	.07343	-.01282	.04849	-.00252	.00510
.00748	.01670	.00363	.06073	.00382	.08629	-.00903	.10863
-.01287	.03734	.00560	.03335	-.01634	.00887	.01133	-.03585

PORT 1

RECEIVER BASEBAND CHARACTERISTICS

AMPLITUDE

.21887	.24898	.28493	.32799	.37964	.44131	.51399	.59734
.68840	.78038	.86323	.92737	.96864	.98988	.99800	.99988
1.00000	.99988	.99810	.99060	.97150	.93510	.87898	.80611
.72383	.64035	.56184	.49158	.43054	.37834	.33402	.29644

PHASE

2.42933	2.37392	2.30990	2.23532	2.14777	2.04434	1.92177	1.77689
1.60773	1.41510	1.20424	.98457	.76677	.55867	.36306	.17828
.00000	-.17716	-.35838	-.54754	-.74596	-.95120	-1.15697	-1.35477
-1.53694	-1.69888	-1.83942	-1.95978	-2.06240	-2.14996	-2.22503	-2.28976

IMPULSE RESPONSE

.00567	.00075	-.00618	-.00072	.00670	.00069	-.00725	-.00066
.00783	.00063	-.00847	-.00059	.00918	.00056	-.00999	-.00052
.01096	.00047	-.01214	-.00042	.01365	.00035	-.01569	-.00027
.01865	.00015	-.02342	.00002	.03245	-.00031	-.05579	.00098
.23241	-.01276	.66174	.01673	.18167	.00393	-.00713	-.00919
-.03027	-.00002	-.00291	-.00064	-.00038	.00187	.00117	-.00113
.00094	.00103	-.00167	-.00106	.00230	.00097	-.00299	-.00092
.00358	.00088	-.00413	-.00085	.00466	.00081	-.00517	-.00078

PORT 1 CHANNEL 1

CHANNEL BASEBAND CHARACTERISTICS

AMPLITUDE

.21887	.24898	.28492	.32799	.37964	.44131	.51399	.59734
.68840	.78038	.86322	.92737	.96864	.98988	.99800	.99988
1.00000	.99988	.99810	.99060	.97150	.93510	.87898	.80611
.72383	.64035	.56184	.49158	.43054	.37834	.33402	.29644

PHASE

-1.63464	-1.68991	-1.75379	-1.82823	-1.91564	-2.01893	-2.14137	-2.28610
-2.45512	-2.64761	-2.85834	-3.07787	2.98766	2.77969	2.58422	2.39958
2.22144	2.04442	1.86334	1.67432	1.47604	1.27093	1.06531	.86765
.68561	.52380	.38341	.26318	.16071	.07328	-.00158	-.06624

IMPULSE RESPONSE

-.00402	.00405	.00431	-.00447	-.00460	.00491	.00491	-.00536
-.00524	.00584	.00560	-.00637	-.00600	.00696	.00646	-.00764
-.00701	.00843	.00769	-.00941	-.00855	.01065	.00972	-.01233
-.01143	.01476	.01419	-.01866	-.01944	.02603	.03306	-.04504
-.13091	.19303	-.41410	.51634	-.11293	.14190	.01162	-.00010
.01837	-.02410	.00224	-.00189	-.00124	-.00145	.00017	.00164
-.00137	.00011	.00184	-.00067	-.00215	.00123	.00253	-.00181
-.00286	.00230	.00317	-.00276	-.00346	.00320	.00374	-.00363

PORT 1 CHANNEL 1

RECEIVED BASEBAND CHANNEL SIGNAL

-.00340	.00652	.04613	-.07209	.26888	-.34846	.31180	-.40276
.31767	-.41781	.29436	-.38674	.29872	-.39376	.26236	-.34145
.19522	-.25197	-.04857	.06680	-.06972	.06686	.12620	-.14657
-.10984	.15034	-.23979	.30403	-.15213	.18482	.11582	-.14052
.06938	-.08479	.08520	-.11207	-.12997	.17915	-.16552	.18877
.07218	-.07781	-.02659	.03357	-.00973	.01678	-.13809	.17378
.06457	-.09603	.16158	-.18976	.01451	-.02822	.01887	-.01622
-.21095	.27281	-.12630	.15076	-.01459	.03616	-.19797	.26070
-.17397	.20951	.07364	-.08837	.03116	-.02900	-.06111	.07312
-.06552	.08707	-.11482	.15059	-.17280	.22880	-.18895	.24123
-.13898	.18697	-.12492	.15615	.05941	-.07618	.07649	-.08981
.01073	-.01856	.03444	-.04874	.07581	-.09391	-.05686	.08182
-.18580	.23262	-.10630	.13467	-.00844	.01658	-.05584	.07964
-.04875	.05071	.05149	-.05114	-.16490	.21080	-.02295	.01385
.16476	-.20088	.00931	.00171	-.25598	.32161	-.09806	.11167
.13042	-.15579	.05650	-.06504	-.01938	.01444	.09953	-.13040
.04242	-.04414	-.10982	.14154	-.25318	.32856	-.12322	.14141
.09856	-.10502	-.07044	.08748	-.01146	.01458	-.13845	.18694
-.21795	.27815	-.13799	.17444	-.02661	.04711	-.09293	.11096
.22448	-.29827	.24151	-.29789	.21553	-.29541	.15937	-.19265
-.00440	-.01416	.08196	-.09115	-.22331	.29189	-.24615	.30866
-.18169	.24466	-.22148	.29489	-.29891	.38966	-.24474	.31532
-.14103	.18655	-.05076	.07009	-.05808	.07965	-.01371	.00535
.23040	-.29841	.16103	-.18881	-.12567	.15019	-.09768	.12613
-.19760	.26021	-.09578	.11499	.02778	-.02303	-.09256	.12862
-.25307	.32236	-.12913	.16164	-.10205	.14851	-.11267	.13418
.00456	.00631	-.04750	.05164	.07945	-.09208	-.14672	.19189
-.09239	.10657	.05449	-.07049	.22810	-.29076	.01560	-.00230
-.13516	.14515	.14936	-.18552	.04410	-.04715	-.01199	.00633
-.01621	.02869	-.16242	.21076	-.13045	.16141	-.01416	.02333
-.02548	.04122	-.13247	.17274	-.19119	.25135	-.25898	.33251
-.10444	.13347	-.06328	.09438	-.04536	.04705	.09816	-.11456

PORT 1

RECEIVED BASEBAND PORT SIGNAL

-.00340	.00652	.04613	-.07209	.26888	-.34846	.31180	-.40276
.31767	-.41781	.29436	-.36674	.29872	-.39376	.26236	-.34145
.19522	-.25197	-.04857	.06680	-.06972	.06686	.12620	-.14657
-.10984	.15034	-.23979	.30403	-.15213	.18482	.11582	-.14052
.06938	-.08479	.08520	-.11207	-.12997	.17915	-.16552	.18877
.07218	-.07781	-.02659	.03357	-.00973	.01678	-.13809	.17378
.06457	-.09603	.16158	-.18976	.01451	-.02822	.01887	-.01622
-.21095	.27281	-.12630	.15076	-.01459	.03616	-.19797	.26070
-.17397	.20951	.07364	-.08837	.03116	-.02900	-.06111	.07312
-.06552	.08707	-.11482	.15059	-.17280	.22880	-.18895	.24123
-.13898	.18697	-.12492	.15615	.05941	-.07618	.07649	-.08981
.01073	-.01856	.03444	-.04874	.07581	-.09391	-.05686	.08182
-.18580	.23262	-.10630	.13467	-.00844	.01658	-.05584	.07964
-.04875	.05071	.05149	-.05114	-.16490	.21080	-.02295	.01385
.16476	-.20088	.00931	.00171	-.25598	.32161	-.09806	.11167
.13042	-.15579	.05650	-.06504	-.01938	.01444	.09953	-.13040
.04242	-.04414	-.10982	.14154	-.25318	.32856	-.12322	.14141
.09856	-.10502	-.07044	.08748	-.01146	.01458	-.13845	.18694
-.21795	.27815	-.13799	.17444	-.02661	.04711	-.09293	.11096
.22448	-.29827	.24151	-.29789	.21553	-.29541	.15937	-.19265
-.00440	-.01416	.08196	-.09115	-.22331	.29189	-.24615	.30866
-.18169	.24466	-.22148	.29489	-.29891	.38966	-.24474	.31532
-.14103	.18655	-.05076	.07009	-.05808	.07965	-.01371	.00535
.23040	-.29841	.16103	-.18881	-.12567	.15019	-.09768	.12613
-.19760	.26021	-.09578	.11499	.02778	-.02303	-.09256	.12862
-.25307	.32236	-.12913	.16164	-.10205	.14851	-.11267	.13418
.00456	.00631	-.04750	.05164	.07945	-.09208	-.14672	.19189
-.09239	.10657	.05449	-.07049	.22810	-.29076	.01560	-.00230
-.13516	.14515	.14936	-.18552	.04410	-.04715	-.01199	.00633
-.01621	.02869	-.16242	.21076	-.13045	.16141	-.01416	.02333
-.02548	.04122	-.13247	.17274	-.19119	.25135	-.25898	.33251
-.10444	.13347	-.06328	.09438	-.04536	.04705	.09816	-.11456

PORT 2

RECEIVER BASEBAND CHARACTERISTICS

AMPLITUDE

.21887	.24898	.28493	.32799	.37964	.44131	.51399	.59734
.68840	.78038	.86323	.92737	.96864	.98988	.99800	.99988
1.00000	.99988	.99810	.99060	.97150	.93510	.87898	.80611
.72383	.64035	.56184	.49158	.43054	.37834	.33402	.29644

PHASE

2.42933	2.37392	2.30990	2.23532	2.14777	2.04434	1.92177	1.77689
1.60773	1.41510	1.20424	.98457	.76677	.55867	.36306	.17828
.00000	-.17716	-.35838	-.54754	-.74596	-.95120	-1.15697	-1.35477
-1.53694	-1.69888	-1.83942	-1.95978	-2.06240	-2.14996	-2.22503	-2.28976

IMPULSE RESPONSE

.00567	.00075	-.00618	-.00072	.00670	.00069	-.00725	-.00066
.00783	.00063	-.00847	-.00059	.00918	.00056	-.00999	-.00052
.01096	.00047	-.01214	-.00042	.01365	.00035	-.01569	-.00027
.01865	.00015	-.02342	.00002	.03245	-.00031	-.05579	.00098
.23241	-.01276	.66174	.01673	.18167	.00393	-.00713	-.00919
-.03027	-.00002	-.00291	-.00064	-.00038	.00187	.00117	-.00113
.00094	.00103	-.00167	-.00106	.00230	.00097	-.00299	-.00092
.00358	.00088	-.00413	-.00085	.00466	.00081	-.00517	-.00078

PORT 2 CHANNEL 1

CHANNEL BASEBAND CHARACTERISTICS

AMPLITUDE

.21887	.24898	.28493	.32799	.37964	.44131	.51399	.59734
.68840	.78038	.86323	.92737	.96864	.98988	.99800	.99988
1.00000	.99988	.99810	.99060	.97150	.93510	.87898	.80611
.72383	.64035	.56184	.49158	.43054	.37834	.33402	.29644

PHASE

2.51224	2.46391	2.40697	2.33948	2.25901	2.16266	2.04716	1.90937
1.74729	1.56174	1.35795	1.14536	.93465	.73364	.54509	.36741
.19621	.02607	-.14802	-.33016	-.52144	-.71967	-.91830	-1.10907
-1.28412	-1.43904	-1.57245	-1.68578	-1.78127	-1.86171	-1.92974	-1.98735

IMPULSE RESPONSE

.00477	.00174	-.00533	-.00182	.00590	.00192	-.00650	-.00202
.00714	.00212	-.00784	-.00223	.00861	.00236	-.00950	-.00251
.01054	.00268	-.01182	-.00290	.01345	.00318	-.01565	-.00356
.01882	.00411	-.02391	-.00500	.03348	.00672	-.05810	-.01119
.25414	.03792	.64092	.14522	.16034	.03464	-.00476	-.00991
-.03118	-.00624	-.00045	-.00057	-.00222	.00133	.00272	-.00054
-.00053	.00088	-.00030	-.00109	.00104	.00117	-.00180	-.00128
.00246	.00138	-.00306	-.00147	.00365	.00156	-.00421	-.00165

PORT 2 CHANNEL 1

RECEIVED BASEBAND CHANNEL SIGNAL

.00899	.00063	-.09615	-.01192	-.43913	-.09070	-.49835	-.10294
-.51525	-.10220	-.47517	-.09424	-.48557	-.09588	-.41791	-.08488
-.30482	-.06371	.09618	.01803	.07588	.03002	-.18283	-.04890
.19675	.03620	.37983	.08225	.21837	.05265	-.18323	-.04432
-.10307	-.02359	-.13722	-.02791	.23507	.04248	.22535	.06217
-.10077	-.03111	.04247	.00995	.02342	.00172	.21722	.04872
-.13138	-.02012	-.23229	-.05988	-.03205	-.00032	-.01419	-.00867
.34549	.07221	.17784	.04403	.04444	-.00188	.32921	.06575
.25167	.06165	-.11818	-.02933	-.03194	-.01320	.09297	.02294
.10940	.02100	.18790	.03750	.28597	.05586	.29572	.06303
.22976	.04287	.18767	.04249	-.10222	-.02200	-.10927	-.02803
-.02116	-.00133	-.06158	-.01015	-.11572	-.02682	.10974	.01777
.29008	.06488	.16126	.03488	.01646	-.00035	.10176	.01675
.05758	.01956	-.06104	-.02282	.26672	.05764	.00614	.01058
-.25253	-.06008	.01358	-.00559	.40589	.09046	.12582	.03487
-.19981	-.04962	-.07554	-.02024	.01852	.01071	-.16516	-.03367
-.04851	-.01646	.18190	.03777	.41209	.08494	.16069	.04402
-.13248	-.04181	.11149	.02616	.01787	.00312	.23817	.04473
.34492	.07356	.20869	.04556	.05628	.00336	.13379	.03440
-.38335	-.07544	-.36396	-.08339	-.36701	-.06582	-.22693	-.05513
-.01675	.00919	-.10797	-.03247	.37257	.07603	.37919	.08390
.30038	.05595	.36587	.07032	.48332	.09821	.38433	.07997
.22434	.04380	.08212	.01405	.09912	.01792	-.00037	.00789
-.37833	-.07857	-.22256	-.05792	.19472	.04852	.15704	.03180
.32461	.06480	.13291	.03258	-.03057	-.01451	.16620	.02889
.40287	.08681	.19179	.04226	.18540	.02834	.15987	.04028
.00739	-.00635	.06045	.01910	-.11351	-.03095	.24582	.05051
.12569	.03347	-.09504	-.01934	-.36499	-.07885	.01338	-.00754
.17626	.05514	-.23671	-.05469	-.05264	-.01632	.00838	.00729
.03956	.00312	.26572	.05487	.19475	.04460	.02416	.00168
.05307	.00584	.21815	.04435	.31428	.06218	.41030	.08670
.15587	.03265	.11668	.01663	.05176	.01807	-.14205	-.03774

PORT 2

RECEIVED BASEBAND PORT SIGNAL

.00899	.00063	-.09615	-.01192	-.43913	-.09070	-.49835	-.10294
-.51525	-.10220	-.47517	-.09424	-.48557	-.09588	-.41791	-.08488
-.30482	-.06371	.09618	.01803	.07588	.03002	-.18283	-.04890
.19675	.03620	.37983	.06225	.21837	.05265	-.18323	-.04432
-.10307	-.02359	-.13722	-.02791	.23507	.04248	.22535	.06217
-.10077	-.03111	.04247	.00995	.02342	.00172	.21722	.04872
-.13138	-.02012	-.23229	-.05988	-.03205	-.00032	-.01419	-.00867
.34549	.07221	.17784	.04403	.04444	-.00188	.32921	.06575
.25167	.06165	-.11818	-.02933	-.03194	-.01320	.09297	.02294
.10940	.02100	.18790	.03750	.28597	.05586	.29572	.06303
.22976	.04287	.18767	.04249	-.10222	-.02200	-.10927	-.02803
-.02116	-.00133	-.06158	-.01015	-.11572	-.02682	.10974	.01777
.29008	.06488	.16126	.03488	.01646	-.00035	.10176	.01675
.05758	.01956	-.06104	-.02282	.26672	.05764	.00614	.01058
-.25253	-.06008	.01358	-.00559	.40589	.09046	.12582	.03487
-.19981	-.04962	-.07554	-.02024	.01852	.01071	-.16516	-.03367
-.04851	-.01646	.18190	.03777	.41209	.08494	.16069	.04402
-.13248	-.04181	.11149	.02616	.01787	.00312	.23817	.04473
.34492	.07356	.20869	.04556	.05628	.00336	.13379	.03440
-.38335	-.07544	-.36396	-.08339	-.36701	-.06582	-.22693	-.05513
-.01675	.00919	-.10797	-.03247	.37257	.07603	.37919	.08390
.30038	.05595	.36587	.07032	.48332	.09821	.38433	.07997
.22434	.04380	.08212	.01405	.09912	.01792	-.00037	.00789
-.37833	-.07857	-.22256	-.05792	.19472	.04852	.15704	.03180
.32461	.06480	.13291	.03258	-.03057	-.01451	.16620	.02889
.40287	.08681	.19179	.04226	.18540	.02834	.15987	.04028
.00739	-.00635	.06045	.01910	-.11351	-.03095	.24582	.05051
.12569	.03347	-.09504	-.01934	-.36499	-.07885	.01338	-.00754
.17626	.05514	-.23671	-.05469	-.05264	-.01632	.00838	.00729
.03956	.00312	.26572	.05487	.19475	.04460	.02416	.00168
.05307	.00584	.21815	.04435	.31428	.06218	.41030	.08670
.15587	.03265	.11668	.01663	.05176	.01807	-.14205	-.03774

PORT 3

RECEIVER BASEBAND CHARACTERISTICS

AMPLITUDE

.21887	.24898	.28493	.32799	.37964	.44131	.51399	.59734
.68840	.78038	.86323	.92737	.96864	.98988	.99800	.99988
1.00000	.99988	.99810	.99060	.97150	.93510	.87898	.80611
.72383	.64035	.56184	.49158	.43054	.37834	.33402	.29644

PHASE

2.42933	2.37392	2.30990	2.23532	2.14777	2.04434	1.92177	1.77689
1.60773	1.41510	1.20424	.98457	.76677	.55867	.36306	.17828
.00000	-.17716	-.35838	-.54754	-.74596	-.95120	-1.15697	-1.35477
-1.53694	-1.69888	-1.83942	-1.95978	-2.06240	-2.14996	-2.22503	-2.28976

IMPULSE RESPONSE

.00567	.00075	-.00618	-.00072	.00670	.00069	-.00725	-.00066
.00783	.00063	-.00847	-.00059	.00918	.00056	-.00999	-.00052
.01096	.00047	-.01214	-.00042	.01365	.00035	-.01569	-.00027
.01865	.00015	-.02342	.00002	.03245	-.00031	-.05579	.00098
.23241	-.01276	.66174	.01673	.18167	.00393	-.00713	-.00919
-.03027	-.00002	-.00291	-.00064	-.00038	.00187	.00117	-.00113
.00094	.00103	-.00167	-.00106	.00230	.00097	-.00299	-.00092
.00358	.00088	-.00413	-.00085	.00466	.00081	-.00517	-.00078

PORT 3 CHANNEL 1

CHANNEL BASEBAND CHARACTERISTICS

AMPLITUDE

.21887	.24898	.28492	.32799	.37964	.44131	.51399	.59734
.68840	.78038	.86322	.92737	.96864	.98988	.99800	.99988
1.00000	.99988	.99810	.99060	.97150	.93510	.87898	.80611
.72383	.64035	.56184	.49158	.43054	.37834	.33402	.29644

PHASE

1.52681	1.48898	1.44253	1.38578	1.31581	1.22996	1.12496	.99791
.84633	.67128	.47799	.27590	.07592	-.11460	-.29263	-.45983
-.62029	-.78012	-.94351	-1.11510	-1.29594	-1.48361	-1.67154	-1.85201
-2.01636	-2.16073	-2.28369	-2.38647	-2.47126	-2.54125	-2.59874	-2.64565

IMPULSE RESPONSE

.00369	-.00163	-.00417	.00204	.00471	-.00238	-.00525	.00281
.00584	-.00324	-.00647	.00370	.00719	-.00420	-.00800	.00482
.00894	-.00550	-.01010	.00636	.01160	-.00743	-.01358	.00886
.01644	-.01093	-.02103	.01423	.02963	-.02038	-.05160	.03605
.23194	-.17995	.53690	-.36096	.11385	-.08018	-.00941	-.00328
-.02757	.01956	.00211	-.00177	-.00248	.00363	.00315	-.00341
-.00133	.00201	.00045	-.00144	.00023	.00091	-.00090	-.00039
.00155	-.00005	-.00209	.00048	.00263	-.00088	-.00315	.00126

PORT 3 CHANNEL 1

RECEIVED BASEBAND CHANNEL SIGNAL

.00869	-.00733	-.08925	.07255	-.37648	.26408	-.41547	.29281
-.42765	.30563	-.39216	.28077	-.40310	.28839	-.34279	.24305
-.24548	.17098	.09757	-.06953	.05395	-.02138	-.15428	.09402
.17818	-.12931	.32124	-.22143	.16866	-.10955	-.16469	.10690
-.08319	.05714	-.11270	.07836	.21344	-.15576	.17688	-.10631
-.08969	.04992	.03690	-.02344	.02294	-.02072	.18403	-.12329
-.12597	.09546	-.19011	.11984	-.02305	.02431	-.00437	-.00482
.29838	-.20735	.13680	-.08862	.03695	-.03957	.28336	-.20078
.19993	-.12919	-.10983	.07026	-.02118	.00785	.08067	-.05228
.09331	-.06775	.15874	-.11327	.24227	-.17399	.24306	-.16872
.18972	-.13899	.14948	-.10010	-.09542	.06522	-.08847	.05619
-.01535	.01460	-.05312	.04033	-.09590	.06338	.10260	-.07746
.24465	-.16590	.12746	-.08809	.00868	-.01096	.08926	-.06712
.04124	-.02003	-.04791	.02060	.23015	-.15703	-.00937	.01662
-.21547	.14160	.02638	-.02820	.34860	-.23599	.08866	-.05287
-.17544	.11295	-.05634	.03480	.01614	-.00271	-.14284	.09991
-.03242	.01561	.16034	-.11280	.35077	-.24591	.11535	-.06909
-.11333	.06186	.09744	-.06331	.01454	-.01220	.20762	-.15051
.28854	-.20017	.16481	-.11313	.04460	-.04173	.10723	-.06554
-.33830	.24021	-.29713	.20083	-.30828	.22782	-.17449	.11336
-.01330	.02472	-.08393	.04542	.32556	-.22835	.31244	-.21398
.24766	-.18155	.30666	-.22194	.40474	-.28606	.31305	-.21983
.17895	-.12879	.06278	-.04790	.08362	-.06133	-.00937	.01645
-.32722	.22839	-.17100	.10650	.17365	-.11127	.13191	-.09489
.27426	-.19436	.09865	-.06406	-.02787	.00963	.14793	-.11031
.34108	-.23468	.14916	-.10310	.15709	-.12159	.12537	-.07962
.00581	-.01380	.04606	-.02373	-.09408	.05579	.21602	-.15033
.09653	-.05995	-.08921	.06366	-.31100	.21312	.03244	-.03338
.14220	-.07746	-.20676	.13679	-.03621	.02030	.00753	.00099
.03821	-.03325	.22802	-.15963	.15585	-.10477	.01420	-.01424
.04715	-.03921	.18767	-.13257	.26628	-.19089	.34073	-.23660
.11801	-.08351	.09756	-.07680	.03485	-.01560	-.11872	.07234

PORT 3

RECEIVED BASEBAND PORT SIGNAL

.00869	-.00733	-.08925	.07255	-.37648	.26408	-.41547	.29281
-.42765	.30563	-.39216	.28077	-.40310	.28839	-.34279	.24305
-.24548	.17098	.09757	-.06953	.05395	-.02138	-.15428	.09402
.17818	-.12931	.32124	-.22143	.16866	-.10955	-.16469	.10690
-.08319	.05714	-.11270	.07836	.21344	-.15576	.17688	-.10631
-.08969	.04992	.03690	-.02344	.02294	-.02072	.18403	-.12329
-.12597	.09546	-.19011	.11984	-.02305	.02431	-.00437	-.00482
.29838	-.20735	.13680	-.08862	.03695	-.03957	.28336	-.20078
.19993	-.12919	-.10983	.07026	-.02118	.00785	.08067	-.05228
.09331	-.06775	.15874	-.11327	.24227	-.17399	.24306	-.16872
.18972	-.13899	.14948	-.10010	-.09542	.06522	-.08847	.05619
-.01535	.01460	-.05312	.04033	-.09590	.06338	.10260	-.07746
.24465	-.16590	.12746	-.08809	.00868	-.01096	.08926	-.06712
.04124	-.02003	-.04791	.02060	.23015	-.15703	-.00937	.01662
-.21547	.14160	.02638	-.02820	.34860	-.23599	.08866	-.05287
-.17544	.11295	-.05634	.03480	.01614	-.00271	-.14284	.09991
-.03242	.01561	.16034	-.11280	.35077	-.24591	.11535	-.06909
-.11333	.06186	.09744	-.06331	.01454	-.01220	.20762	-.15051
.28854	-.20017	.16481	-.11313	.04460	-.04173	.10723	-.06554
-.33830	.24021	-.29713	.20083	-.30828	.22782	-.17449	.11336
-.01330	.02472	-.08393	.04542	.32556	-.22835	.31244	-.21398
.24766	-.18155	.30666	-.22194	.40474	-.28606	.31305	-.21983
.17895	-.12879	.06278	-.04790	.08362	-.06133	-.00937	.01645
-.32722	.22839	-.17100	.10650	.17365	-.11127	.13191	-.09489
.27426	-.19436	.09865	-.06406	-.02787	.00963	.14793	-.11031
.34108	-.23468	.14916	-.10310	.15709	-.12159	.12537	-.07962
.00581	-.01380	.04606	-.02373	-.09408	.05579	.21602	-.15033
.09653	-.05995	-.08921	.06366	-.31100	.21312	.03244	-.03338
.14220	-.07746	-.20676	.13679	-.03621	.02030	.00753	.00099
.03821	-.03325	.22802	-.15963	.15585	-.10477	.01420	-.01424
.04715	-.03921	.18767	-.13257	.26628	-.19089	.34073	-.23660
.11801	-.08351	.09756	-.07680	.03485	-.01560	-.11872	.07234

PORT 4

RECEIVER BASEBAND CHARACTERISTICS

AMPLITUDE

.21887	.24898	.28493	.32799	.37964	.44131	.51399	.59734
.68840	.78038	.86323	.92737	.96864	.98988	.99800	.99988
1.00000	.99988	.99810	.99060	.97150	.93510	.87898	.80611
.72383	.64035	.56184	.49158	.43054	.37834	.33402	.29644

PHASE

2.42933	2.37392	2.30990	2.23532	2.14777	2.04434	1.92177	1.77689
1.60773	1.41510	1.20424	.98457	.76677	.55867	.36306	.17828
.00000	-.17716	-.35838	-.54754	-.74596	-.95120	-1.15697	-1.35477
-1.53694	-1.69888	-1.83942	-1.95978	-2.06240	-2.14996	-2.22503	-2.28976

IMPULSE RESPONSE

.00567	.00075	-.00618	-.00072	.00670	.00069	-.00725	-.00066
.00783	.00063	-.00847	-.00059	.00918	.00056	-.00999	-.00052
.01096	.00047	-.01214	-.00042	.01365	.00035	-.01569	-.00027
.01865	.00015	-.02342	.00002	.03245	-.00031	-.05579	.00098
.23241	-.01276	.66174	.01673	.18167	.00393	-.00713	-.00919
-.03027	-.00002	-.00291	-.00064	-.00038	.00187	.00117	-.00113
.00094	.00103	-.00167	-.00106	.00230	.00097	-.00299	-.00092
.00358	.00088	-.00413	-.00085	.00466	.00081	-.00517	-.00078

PORT 4 CHANNEL 1

CHANNEL BASEBAND CHARACTERISTICS

AMPLITUDE

.21887	.24898	.28492	.32799	.37964	.44131	.51399	.59734
.68840	.78038	.86322	.92737	.96864	.98988	.99800	.99988
1.00000	.99988	.99810	.99060	.97150	.93510	.87898	.80611
.72383	.64035	.56184	.49158	.43054	.37834	.33402	.29644

PHASE

2.84353	2.82352	2.79490	2.75573	2.70358	2.63555	2.54837	2.43890
2.30514	2.14791	1.97244	1.78817	1.60578	1.43332	1.27286	1.12373
.98085	.83861	.69303	.53902	.37625	.20616	.03604	-.12660
-.27313	-.39992	-.50481	-.59002	-.65698	-.70891	-.74881	-.77791

IMPULSE RESPONSE

.00040	.00204	-.00081	-.00260	.00116	.00320	-.00160	-.00389
.00202	.00454	-.00252	-.00530	.00302	.00611	-.00365	-.00705
.00433	.00815	-.00521	-.00948	.00627	.01118	-.00773	-.01346
.00978	.01670	-.01308	-.02189	.01911	.03148	-.03425	-.05570
.20603	.29001	.32965	.52735	.05828	.08366	.00326	-.00757
-.01992	-.03030	.00402	.00648	-.00507	-.00557	.00479	.00596
-.00341	-.00397	.00280	.00290	-.00225	-.00205	.00169	.00125
-.00123	-.00050	.00080	-.00013	-.00038	.00080	.00001	-.00138

PORT 4 CHANNEL 1

RECEIVED BASEBAND CHANNEL SIGNAL

.00847	.01193	-.08798	-.11875	-.26076	-.39839	-.28052	-.42325
-.29210	-.43659	-.26767	-.39807	-.27563	-.41208	-.22731	-.34139
-.14944	-.23127	.08289	.12801	-.00231	.01881	-.07703	-.14070
.13960	.20892	.21205	.32841	.08130	.13657	-.10511	-.17524
-.05339	-.07959	-.06796	-.10818	.16813	.25137	.07551	.13907
-.04148	-.08386	.01986	.03602	.02943	.03604	.11166	.18218
-.11115	-.16272	-.09977	-.17035	-.02642	-.02732	.02188	.01937
.20313	.31561	.06790	.11112	.04963	.05464	.19959	.30362
.10265	.17197	-.07132	-.12102	.00260	-.00422	.04993	.08214
.06895	.10110	.11181	.16695	.17221	.25675	.15526	.23831
.13479	.19640	.08195	.13295	-.06987	-.11088	-.04663	-.07827
-.01470	-.01637	-.04274	-.06068	-.05478	-.09055	.08929	.12919
.15547	.24535	.07524	.11568	.00989	.00679	.07057	.10274
.00485	.01959	-.00351	-.02626	.14997	.23893	-.03773	-.04550
-.13091	-.21434	.05120	.06457	.22540	.35849	.02663	.05138
-.10793	-.18000	-.02308	-.04028	-.00471	.00646	-.09783	-.15190
-.00118	-.01087	.11824	.17845	.23849	.36589	.03721	.07134
-.04638	-.09740	.05809	.09888	.01606	.01834	.15516	.23089
.18938	.29128	.09543	.14825	.04675	.05481	.04506	.08705
-.24603	-.37554	-.18161	-.28297	-.22544	-.32880	-.08539	-.14134
-.03555	-.03154	-.02204	-.05645	.22957	.35332	.19697	.30483
.17575	.25574	.21849	.32225	.27425	.41530	.20144	.30526
.11558	.17135	.04312	.05925	.06021	.08916	-.03489	-.03772
-.22600	-.34868	-.07481	-.13204	.10777	.18140	.09618	.13967
.18703	.28474	.04507	.07384	-.00320	-.01977	.12053	.17423
.22260	.34763	.08796	.13251	.12539	.17642	.06062	.10371
.02106	.01751	.00915	.02791	-.04048	-.07984	.14975	.23414
.04370	.07528	-.07432	-.10994	-.20154	-.31801	.06007	.08045
.04972	.10826	-.12999	-.21374	-.00950	-.01869	-.00429	.00152
.04352	.05600	.15676	.24077	.08923	.14143	.01105	.00983
.04556	.06050	.13270	.20109	.19017	.28255	.21922	.33888
.06910	.10139	.07820	.10877	-.00166	.01033	-.05730	-.10576

PORT 4

RECEIVED BASEBAND PORT SIGNAL

.00847	.01193	-.08798	-.11875	-.26076	-.39839	-.28052	-.42325
-.29210	-.43659	-.26767	-.39807	-.27563	-.41208	-.22731	-.34139
-.14944	-.23127	.08289	.12801	-.00231	.01881	-.07703	-.14070
.13960	.20892	.21205	.32841	.08130	.13657	-.10511	-.17524
-.05339	-.07959	-.06796	-.10818	.16813	.25137	.07551	.13907
-.04148	-.08386	.01986	.03602	.02943	.03604	.11166	.18218
-.11115	-.16272	-.09977	-.17035	-.02642	-.02732	.02188	.01937
.20313	.31561	.06790	.11112	.04963	.05464	.19959	.30362
.10265	.17197	-.07132	-.12102	.00260	-.00422	.04993	.08214
.06895	.10110	.11181	.16695	.17221	.25675	.15526	.23831
.13479	.19640	.08195	.13295	-.06987	-.11088	-.04663	-.07827
-.01470	-.01637	-.04274	-.06068	-.05478	-.09055	.08929	.12919
.15547	.24535	.07524	.11568	.00989	.00679	.07057	.10274
.00485	.01959	-.00351	-.02626	.14997	.23893	-.03773	-.04550
-.13091	-.21434	.05120	.06457	.22540	.35849	.02663	.05138
-.10793	-.18000	-.02308	-.04028	-.00471	.00646	-.09783	-.15190
-.00118	-.01087	.11824	.17845	.23849	.36589	.03721	.07134
-.04638	-.09740	.05809	.09888	.01606	.01834	.15516	.23089
.18938	.29128	.09543	.14825	.04675	.05481	.04506	.08705
-.24603	-.37554	-.18161	-.28297	-.22544	-.32880	-.08539	-.14134
-.03555	-.03154	-.02204	-.05645	.22957	.35332	.19697	.30483
.17575	.25574	.21849	.32225	.27425	.41530	.20144	.30526
.11558	.17135	.04312	.05925	.06021	.08916	-.03489	-.03772
-.22600	-.34868	-.07481	-.13204	.10777	.18140	.09618	.13967
.18703	.28474	.04507	.07384	-.00320	-.01977	.12053	.17423
.22260	.34763	.08796	.13251	.12539	.17642	.06062	.10371
.02106	.01751	.00915	.02791	-.04048	-.07984	.14975	.23414
.04370	.07528	-.07432	-.10994	-.20154	-.31801	.06007	.08045
.04972	.10826	-.12999	-.21374	-.00950	-.01869	-.00429	.00152
.04352	.05600	.15676	.24077	.08923	.14143	.01105	.00983
.04556	.06050	.13270	.20109	.19017	.28255	.21922	.33888
.06910	.10139	.07820	.10877	-.00166	.01033	-.05730	-.10576

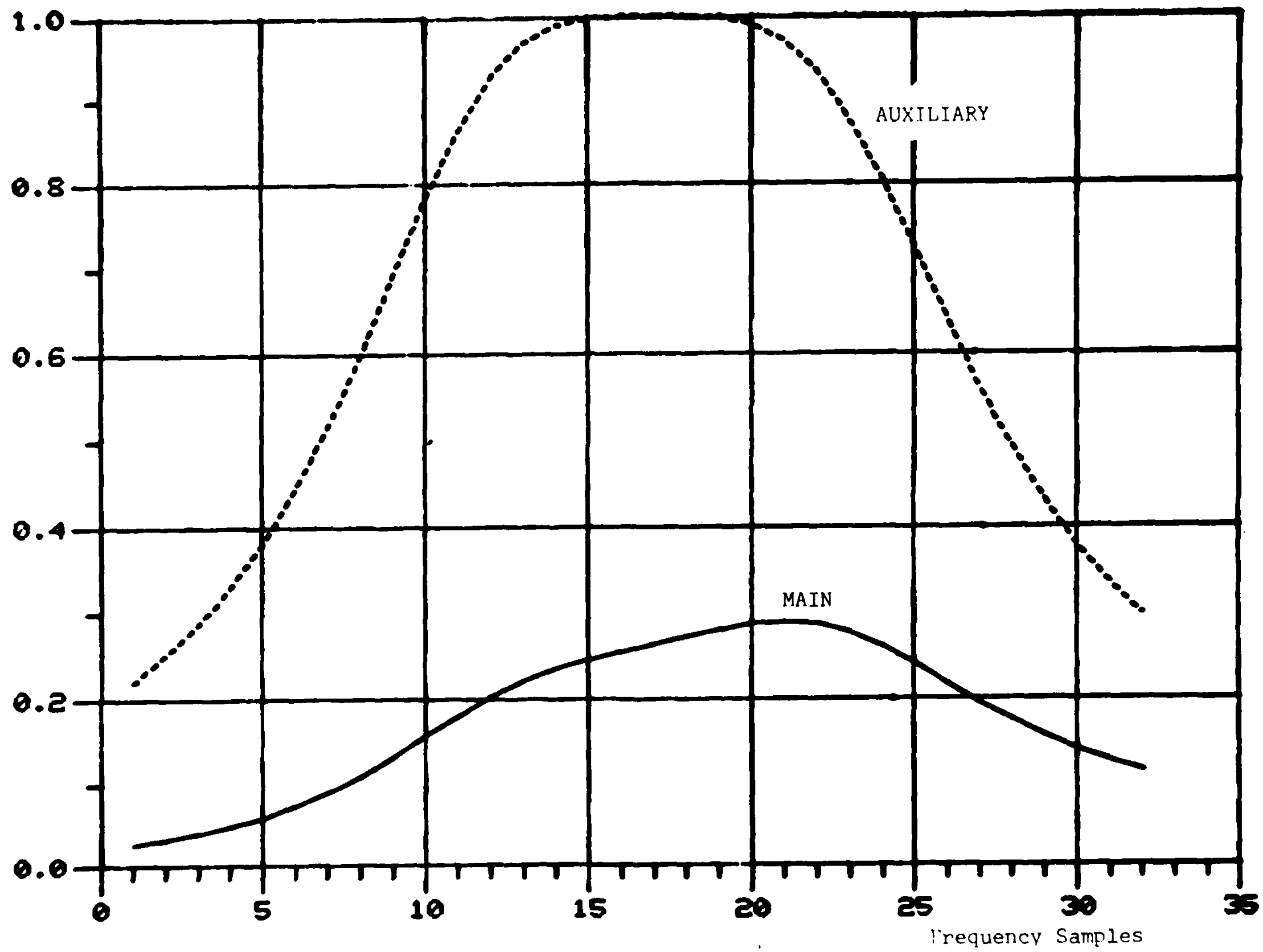


Figure 3-4. Representative Main and Auxiliary Baseband Channel Amplitude Responses

3.3 Program Library Catalog

In order to facilitate the use of the subroutine library being developed under the present contract, it has been useful to create a library catalog for easy user reference. The two pages that follow contain an alphabetized list of programs developed to date including author(s), date originated and revised.

The availability of this library catalog has already been found useful in the present contractual effort. As programs are generated, qualified and subsequently revised, the users of the library become readily aware of the progress being made. When Motorola delivers the software to be developed under the present effort, this cross reference will also be useful to RADC, especially if multiple users are envisioned.

3.4 Program Validation

Although the signal generation program has been developed very methodically leaving very little room for error, it nevertheless makes sense, whenever possible, to provide an independent check before proceeding with the next stage of the effort. In the present case, an existing BCR processing program has been used to assure the validity of the port signals generated by SIGGENO.

Figure 3-5 shows cancellation performance for the single interference example described in the output file in Section 3.2.5. More specifically, Figure 3-5(a) shows the sample-and-held amplitude envelope at the main antenna port while Figure 3-5(b) shows the corresponding envelope of the combined signal after cancellation. A detailed explanation of the BCR adaptive process used to obtain this cancellation will be given in a subsequent project memorandum. For the moment, it suffices to show that the port signals generated by SIGGENO appear to be valid. At least three other cases have been examined and their successful results have served to insure against pathological validation.

The validity of SIGGENO has been further reassured via Figure 3-6 which shows the center-frequency main-antenna and composite-antenna field patterns evaluated over a 0.25° azimuth range about the incident interference at 45°. A 38 dB cancellation may be noted.

In Figure 3-6, the double-null pattern about 45° may justifiably arouse the reader's curiosity. For the moment, it can be said that this phenomenon must be associated with the antenna aperture dispersion over the 0.1% RF bandwidth, since no other channel mismatch condition has been introduced in the present example. Interestingly, the BCR process (which is capable of scanning the eigenspace of the auxiliary-signal 4x4 covariance matrix, C) has revealed the existence of two nontrivial eigenvalues. This fact is demonstrated in Figure 3-7 showing a two-step evolution of the four adaptive weights. Here, the four complex weight-vector components have been initially set to zero, the origin of the common complex plane shown. Two BCR iterations are clearly observable.

To test the existence of a second non-trivial eigenvalue of C, the BCR process was inhibited to a single iteration. A single-null composite pattern resulted with a depth of about 42 dB. An additional 10 dB of suppression was obtained after the second iteration. Clearly, C possesses a second, though relatively small eigenvalue, despite the fact that only one interference is involved. This topic will be discussed further in subsequent investigation.

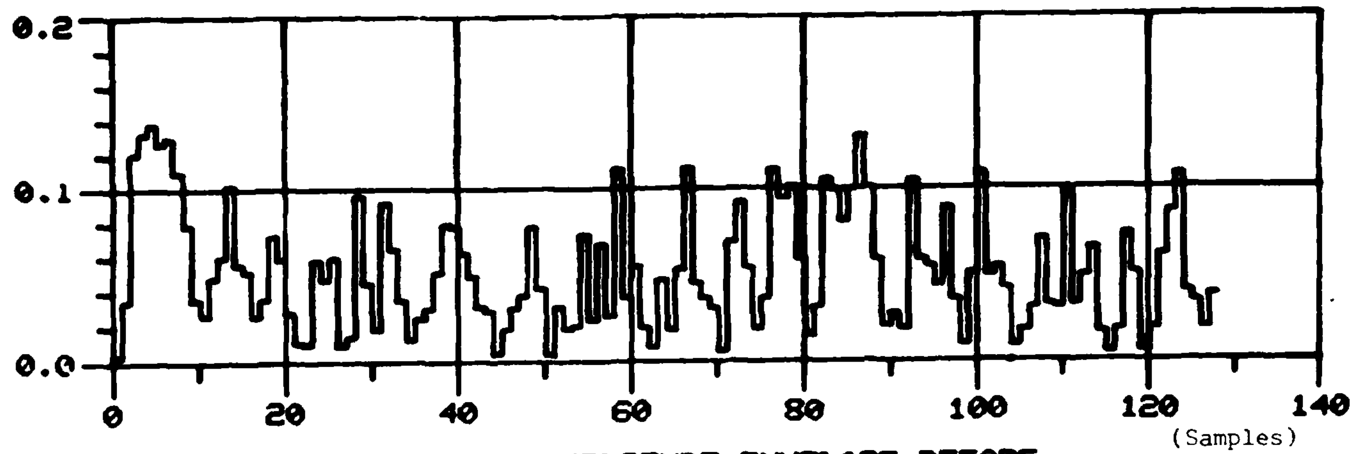
14:34 JUL 22,'80 DC/LIBRARY.298

PROGRAM LIBRARY CATALOG

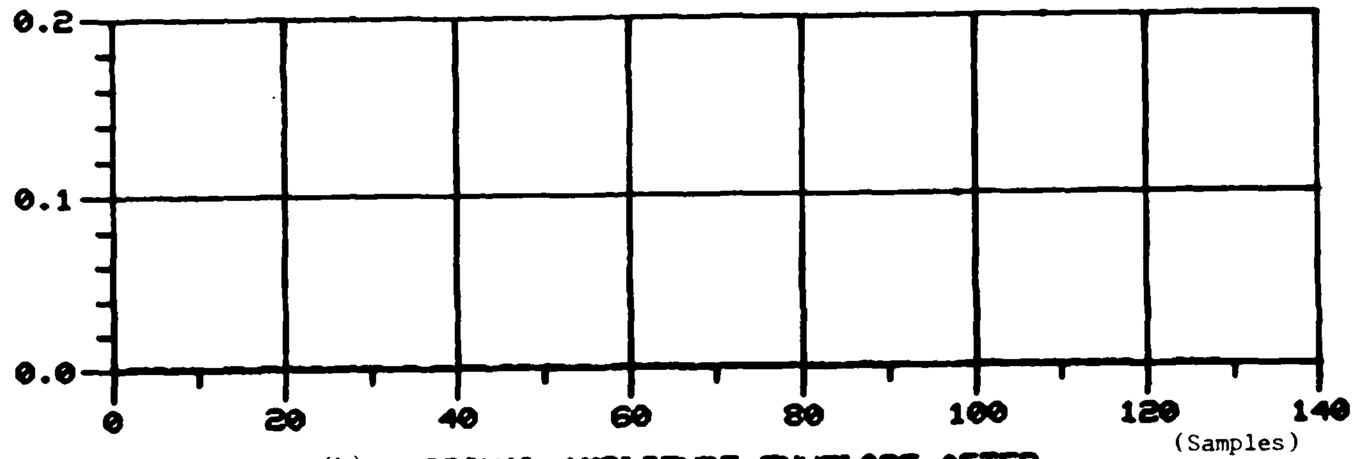
REVISION : JULY 2, 1980

FILE	AUTHOR(S)	ORIGINATED	REVISED
AMPH:S	S. M. DANIEL I. M. KERTESZ	4/19/80	7/01/80
ANTWT:S	S. M. DANIEL I. M. KERTESZ	4/19/80	6/11/80
BLINK:S	I. M. KERTESZ S. M. DANIEL	6/12/80	6/12/80
CHANNEL:S	S. M. DANIEL I. M. KERTESZ	5/04/80	6/26/80
FILTER:S	S. M. DANIEL I. M. KERTESZ	4/19/80	6/23/80
FFT2:S	PROF. P. RANSOM S. M. DANIEL	7/15/69	6/07/80
IMPULS:S	S. M. DANIEL I. M. KERTESZ	5/11/80	6/07/80
JCL:B	S. M. DANIEL G. C. WANG	6/20/80	7/01/80
JCL:EX	S. M. DANIEL G. C. WANG	6/22/80	6/23/80
JCLSIG:BL	S. M. DANIEL G. C. WANG	6/21/80	6/23/80

42 -	42.000	RW:S	S. M. DANIEL	6/15/78	4/19/80
43 -	43.000		I. M. KERTESZ		
44 -	44.000				
45 -	45.000	RWID:S	S. M. DANIEL	6/15/78	4/19/80
46 -	46.000		I. M. KERTESZ		
47 -	47.000				
48 -	48.000	PRTSGNLO:S	S. M. DANIEL	6/01/80	7/02/80
49 -	49.000		I. M. KERTESZ		
50 -	50.000				
51 -	51.000	SIGMAINO:S	S. M. DANIEL	4/19/80	7/01/80
52 -	52.000		I. M. KERTESZ		
53 -	53.000				
54 -	54.000	SIGNALO:S	S. M. DANIEL	6/08/80	6/10/80
55 -	55.000		I. M. KERTESZ		
56 -	56.000				
57 -	57.000	SIGSETO:S	S. M. DANIEL	6/01/80	7/01/80
58 -	58.000		I. M. KERTESZ		
59 -	59.000				
60 -	60.000	----------	----------	----------	----------



(a) SIGNAL AMPLITUDE ENVELOPE BEFORE



(b) SIGNAL AMPLITUDE ENVELOPE AFTER

Figure 3-5. Original Main-Port and Combined-Port Signal Amplitude Envelope Before and After Cancellation Using BCR Adaptive Processing

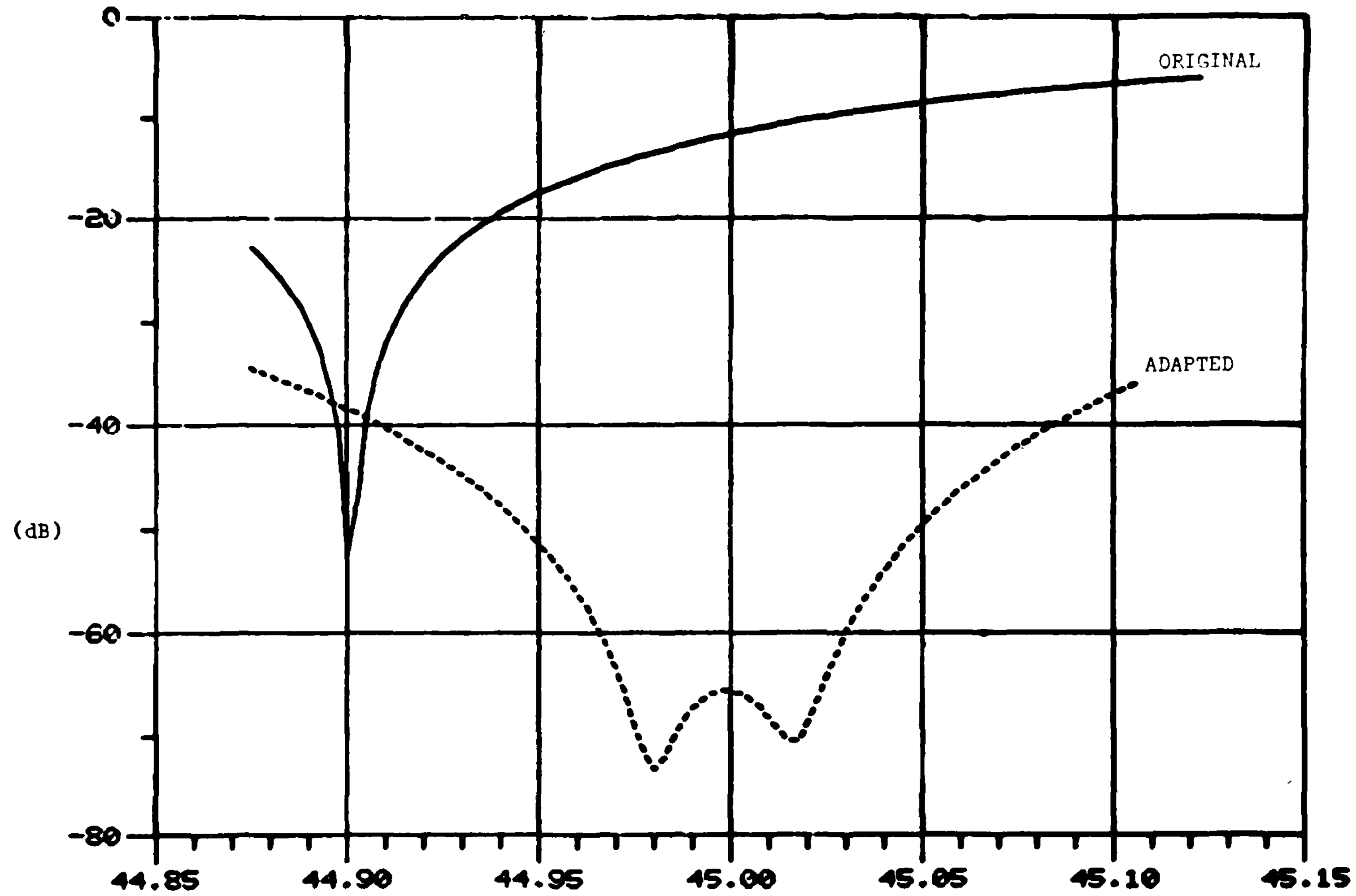


Figure 3-6. Original Main-Port and Adapted Combined-Port Center-Frequency Field Patterns

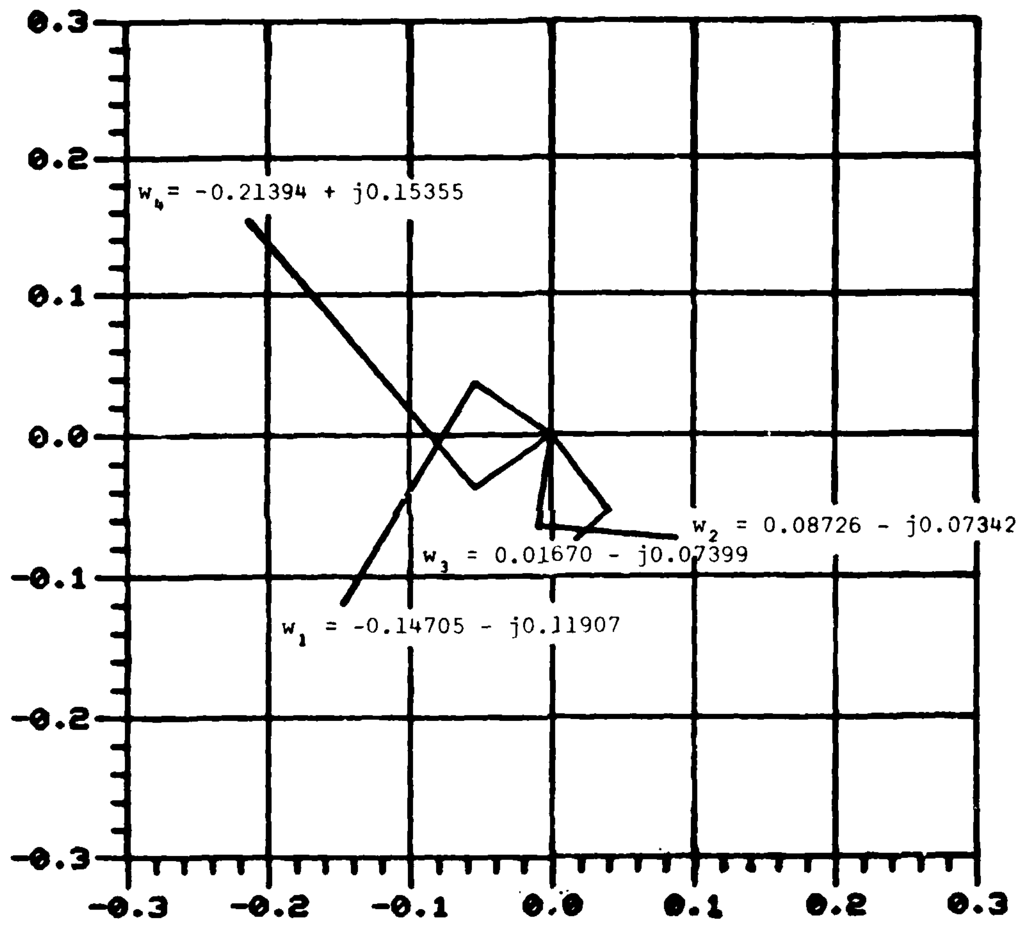


Figure 3-7. Evolution of Adaptive BCR Weights in Complex Plane

4.0 CONCLUSIONS AND RECOMMENDATIONS

The present project memorandum has been devoted to describing, in some detail, the development of a minimum-core signal generation program, a program for generating sampled baseband signals at each port of a multiport antenna system. Motivated by the underlying mathematical model defined in Project Memorandum 8512-01, Section 2.0 provides a specific parametric system description based on a normalized model amenable to easy generalization and suitable for computer simulation.

Section 3.0 discusses the actual development of the signal generation program, SIGGENO. First, the general methodology of a modularized program architecture is given in Section 3.1. Following this, a stage-by-stage specific program development in Section 3.2 begins with the description of the input data file consisting of a very readable table of pertinent system parameters. Presented subsequently, is the complete library of source subroutine modules comprising the virtual source signal generation program, SIGGENO:S, their functional description in connection with a minimum-core program architecture and a complete FORTRAN listing of each. The corresponding library of binary modules is introduced along with two special job control language programs, JCL:B and JCLSIGO:BL, the first for creating individual binary modules and the second for concatenating them into the composite binary module SIGGENO:B while simultaneously generating an executable load module SIGGENO:L. Section 3.2 ends by defining the procedure for executing SIGGENO:L, via job control program JCL:EX, and the formation of an output file, SIGGENO:O, for a specific example. Using an existing BCR adaptive processing program, the validation of SIGGENO is established beyond reasonable doubt.

The minimum-core architecture adopted for the signal generation program is crucial in generating data for a large-scale system. However, because of some inherently necessary redundancy, the present version is not time-optimal. Upon completing and qualifying the present version of the signal generation program, a limited effort was directed toward defining an alternative program structure that would possess both a minimum-core and a minimum time characteristics. A three-stage partitioning seemed to be the logical approach to take. The first stage was devoted to generating the sequence of sampled noise signal envelopes and storing them in a data file. Of course, this circumvented the need to regenerate this data for each port, which is unavoidably done in SIGGENO. The second stage was designed to simply generate the impulse responses of each and every channel associated with each and every port and output them into another data file. With the incident noise and impulse response data files at hand, these two stages of the alternate version of the signal generation program need not be run unless the maximum system dimensionality and/or system description needs to be changed. The third stage of the new program version, SIGGEN1, was constructed to generate a selected set of port signals from a selected set of incident noise sources. With this capability, several examples could be created by SIGGEN1 using the same two data files. However, because of the I/O burden required in the third stage, the speed advantage noted to date has been meager. Based on comparative speed performance for at least two isolated examples, there does not seem to be any substantial justification for adopting this alternate three-stage signal generation program architecture. For the time being, SIGGENO will be the preferred program version.

A possible future effort toward time-optimization of the signal generation program might be one involving a two-stage program formulation. The first stage could be designed to generate individual channel signals associated with each port. The second stage could simply involve the selected superposition of a subset of channel signals over a selected subset of port signals, generating thereby, a number of varying examples from one set of data generated in the first stage.

Other possible variations or additions to the existing signal generation program could be the generation of multipath and multitap signals. In both cases, the basic program will remain substantially intact. Subroutines SIGSETO and PRTSGNLO will require an appropriate number of new call statements. However, the functional aspects associated with multipath and multitap signals and thus the major burden thereof will be relegated to dedicated subroutine modules consistent with the overall program architecture employed.

BIBLIOGRAPHY

[1] Daniel, S. M., "BCR Adaptive Processing - Mathematical Model," Project Memorandum 8512-01, March 15, 1980.

[2] Van Valkenberg, M. E., "Introduction to Modern Network Synthesis," J. Wiley, New York, 1960.

BATCH ADAPTIVE PROCESSING AND THE BCR PROCESS

Prepared by:

S. M. Daniel
Project Leader
Advanced Technology & Systems Analysis

Reviewed by:

J. R. Roman
Advanced Technology & Systems Analysis

Approved by:

G. V. Morris
Section Manager
Advanced Technology & Systems Analysis

D. L. Howell
Chief Engineer
Radar Systems Office
Radar Operations

ACKNOWLEDGEMENT

My sincere appreciation is extended to Jaime R. Roman, who dedicated substantial time and effort in meticulously reviewing this memorandum and making a number of valuable suggestions that have enhanced the material presented.

TABLE OF CONTENTS

1.0 INTRODUCTION

BCR is an acronym for "Batch Covariance Relaxation" which was coined at Motorola's Government Electronics Division in connection with the development of a proprietary digital multisensor adaptive processor based on a minimum-mean-square (MMS) criterion [1]. The term "Batch" is intended to emphasize the batch mechanization embodying the suitable underlying process and distinguish it from a dynamic alternative. More specifically, this term refers to the fact that the underlying process is designed to operate on the multisensor sampled signal data in batches which are distinct but contiguous and of sufficient duration so as to allow for the necessary computational time of the process. In particular, the batch duration is chosen long enough so as to allow for the derivation of adaptive weights that will provide the optimal combination of multisensor data within the batch, in accordance with the MMS criterion. It should be understood that the adaptive weights derived from a multisensor signal data-batch must be applied to that very batch, if the full advantage of the batch approach is to be realized, in general.

The term "Covariance" refers to the computation of the covariance matrix computed over a batch of multisensor sampled signal data which are to be adaptively weighted. Such a matrix often arises in a variety of parameter estimation problems based on the MMS criterion. A large group of such problems gives rise to a system of linear equations

$$C\underline{w} + \underline{b} = \underline{0} \qquad (1\text{-}1)$$

where C is, in general, an N x N complex conjugate-symmetric covariance matrix, $\underline{b}$ is a complex N-vector contained in the space generated by C and $\underline{w}$ is the complex adaptive weight N-vector to be determined.

The term "Relaxation" refers to the nature of the computational method used to solve for a $\underline{w}$ that satisfies (1-1). The specific relaxation method used is the well-known Conjugate Gradients (CG) method, a numerically efficient, stable, finite-step, iterative descent procedure, a special case of the more general Conjugate Directions (CD) method [2].

Section 2.0 presents the Batch Adaptive Process in the context of an adaptive array sidelobe cancellation subsystem. Following a batch formulation of the underlying adaptive array problem, two alternative solution approaches are considered. The Batch Covariance Inversion (BCI) approach, more commonly referred to as Sample Matrix Inversion (SMI), is discussed briefly in order to provide a familiar background to the informed reader. Basically, BCI encompasses all techniques capable of producing a solution to (1-1) via an explicit inversi of the covariance matrix C. Since the existence of a <u>direct inverse</u>, C^{-1}, may not generally be guaranteed, one may be compelled to resort to covariance regularization and thus rely on the approximate <u>regularized inverse</u>, $\tilde{C}^{-1}$, at the expense of some error in the solution. Alternatively, whether or not C is full-rank, a <u>pseudoinverse</u>, C^{+}, may be constructed which will produce the unique minimum-norm solution to (1-1).

In contrast to BCI, the BCR approach, which encompasses all variations of the CD method, will produce the solution to (1-1) without inversion. In particular, the CG method, which is the main thrust of the present investigation, is designed to produce the minimum-norm solution to (1-1). Under certain conditions, it is possible to use the CG process to construct C^{-1}, if it exists. Considering that the algorithmic structure of the CG method lends itself to an architectural implementation that is considerably more efficient than that of any inversion procedure, (such as the standard or modified Cholesky Decomposition and the Singular Value Decomposition) it is highly recommended for batch adaptive processing. In large-scale applications when the dimensionality of C may exceed 100, a BCI approach will be vulnerable to roundoff error and, as a consequence, its operational effectiveness may deteriorate. It is here where the BCR approach clearly excels. Specifically, the CG method begins from an initial solution estimate, $\underline{w}^0$, and iteratively generates improved estimates $\underline{w}^1$, $\underline{w}^2$, ... until it terminates after, at most, N iterations, the dimensionality of system (1-1). If we wish to diminish the roundoff error in this final estimate, we simply consider it as an initial estimate and repeat the CG process, thereby producing a more refined estimate of the minimum-norm solution to (1-1).

Section 2.0 concludes with a block-diagram functional description of a batch adaptive processor.

Section 3.0 provides the mathematical basis of the BCR adaptive process. Included here is a detailed development of the general CD method followed by a similar development of its most efficient special case, the CG method. Subsequently, some of the important properties of the CG method are discussed. Finally, two numerical examples are given that demonstrate the validity of the methods discussed and provide a tangible insight into their properties.

Section 4.0 discusses the important extension of BCR to the linear-equality-constrained MMS problem in the context of adaptive array processing. Following a Lagrange-multiplier formulation of the underlying problem the mathematical development of the constrained BCR (CBCR) formulation is presented. Both the general constrained CD (CCD) and the special constrained CG (CCG) methods are derived. Numerical examples serve to illustrate the validity of the various formulations discussed. Finally, the application of CBCR to adaptive beamforming is noted to be particularly important when the covariance matrix inverse, C^{-1}, does not exist and the use of the pseudoinverse, C^{+}, is not valid.

2.0 BATCH ADAPTIVE PROCESSING

The concept of batch adaptive processing is presented in the context of an adaptive array sidelobe cancellation subsystem. A mathematical formulation of the underlying MMS parameter estimation problem leads to the special linear system of equations (1-1). Alternative approaches for solving (1-1) fall into two distinct categories; namely, inversion and relaxation. The inversion approach is examined briefly in order to add perspective to the relaxation approach which happens to be the main thrust of the present effort. In particular, the effort will be devoted to the investigation of the conjugate gradients (CG) method [2]. The primary motive for choosing this technique is that it happens to be a numerically robust and efficient procedure for solving (1-1) possessing, at the same time, a structure that lends itself to a number of architecturally-efficient, technically-reliable and cost effective implementations. A functional block diagram of the general batch adaptive processor is given.

2.1 Mathematical Formulation

Consider an adaptive array sidelobe cancellation subsystem consisting of a main narrow-beam low-sidelobe Taylor-weighted linear antenna array in combination with N omnidirectional weight-adjustable auxiliary antenna elements nonuniformly emplaced over the main-array aperture. The purpose of this arrangement is to suppress undesired sidelobe interference by appropriate auxiliary-weight adjustment. More specifically, the sidelobe interference assumed will consist of directionally-distinct continuous or pulsed wideband noise sources.

Project Memorandum 8512-01 [3] has already described an appropriate mathematical model for generating complex baseband signal samples received at the main and auxiliary ports of the system described above, due to a specified number of incident noise sources. The pertinent signals involved are defined to be

$s_0(m)$ = the m-th sample of the complex baseband scalar signal received at the main-antenna port

$\underline{s}(m)$ = the m-th sample of the complex baseband N-vector signal whose components correspond to received auxiliary-port signals

Letting

$\underline{w}(m)$ = the complex adaptive weight N-vector applied componentwise to the auxiliary-port N-vector signal, $\underline{s}(m)$, during the m-th sample time interval we may define the combined-port signal by

$$s_c(m) = \underline{s}^T(m)\underline{w}(m) + s_0(m) \tag{2-1}$$

= the m-th sample of the received complex baseband combined scalar signal resulting from the combination of the main-port signal and the weighted sum of the N auxiliary signal components during the m-th sample time interval

Based on physically meaningful arguments, the typical criterion of an adaptive array process is the familiar MMS criterion, or, more specifically, the minimum-combined-power criterion. In more precise terms, we wish to determine the weight vector $\underline{w}(m)$ which minimizes a convenient metric of combined power. Symbolically, this may be stated as

$$\min_{\underline{w}(m)} P_c(\underline{w}(m)) \tag{2-2}$$

where

$$P_c(\underline{w}(m)) = \| s_c(m) \|^2$$

= the combined signal power, a metric which must be defined in a manner suitable to the adaptive process employed

A continuous or dynamic adaptive process based on criterion (2-2) is designed to produce a new weight-vector value $\underline{w}(m)$ with each new set of signal samples $\{s_0(m), \underline{s}(m)\}$. Accordingly, a convenient combined-power metric is the exponential running average

$$P_c(\underline{w}(m)) = \rho P_c(\underline{w}(m-1)) + (1-\rho)|s_c(m)|^2 \tag{2-4}$$

and the corresponding samplewise weight-adjustment is of the general form

$$\underline{w}(m) = \underline{w}(m-1) + \Delta\underline{w}(m) \tag{2-5}$$

where the specific form of $\Delta\underline{w}(m)$ is peculiar to the actual dynamic adaptive process employed.

Given a temporally-stationary interference environment, a dynamic adaptive process will direct the evolution of the adaptive weight vector toward a steady-state value which will suffice to minimize the combined power henceforth. Depending on the bandwidth implied by ρ in (2-4), the nature of the actual adaptive process used and the interference environment, the convergence-time of the weight-vector, from an arbitrary initial value to its final steady-state value, will vary. Clearly, in the case of a stationary environment, the length of the convergence-time is of little consequence. However, convergence-time becomes important when the dynamic adaptive process has to deal with a nonstationary dynamically-changing environment. Depending on the nature of the nonstationarity, the performance of a given dynamic adaptive process could degrade considerably, to the point where it may become operationally ineffective.

Whereas a given dynamic adaptive process is vulnerable to a nonstationary environment [4], a batch process is designed to circumvent the central cause of this vulnerability: the dependence on <u>convergence-time</u>. In contrast with a dynamic process, a batch approach is designed to operate on a batch of signal data, of an appropriate number of samples, with a unique weight-vector value which minimizes the combined power within the batch. The tacit assumption here is that the underlying weight-estimation process terminate within the available batch-time. By its very nature, the batch approach is not affected by signal dynamics outside the bounds of the given batch it happens to be operating on. On the other hand, signal dynamics within the batch will appear "frozen," exhibiting, at worst, an increased dynamic range in signal values. As such, suppression of undesired interference signals within the batch will depend on their average power within the batch. In any case, transient effects that may be present in a dynamic approach will be absent in a batch approach. Consequently, <u>convergence-time</u> has no meaning in a batch process. Instead, what is significant here is <u>batch-time</u> or <u>response-time</u>, the time needed by the underlying weight-estimation process to produce or respond with an optimal weight vector for a given batch. Of course, the batch-time must be sufficiently long so that the statistical characteristics of the signal process may be adequately observed.

The batch formulation of the adaptive array problem described above can be made more precise, in mathematical terms, after defining some important pertinent quantities. Let

$\left\{s_0(m)\right\}_{m=1}^{M}$ = an M-sample main-port complex baseband scalar signal batch

$\left\{\underline{s}(m)\right\}_{m=1}^{M}$ = the corresponding M-sample complex baseband N-vector signal batch whose components represent the individual auxiliary-port signals

$\underline{w}$ = a constant complex adaptive weight N-vector applied componentwise to the auxiliary-port N-vector signal, $\underline{s}(m)$, for m=1, ..., M whence,

$$\left\{s_c(m)\right\}_{m=1}^{M} = \left\{\underline{s}^T(m)\underline{w} + s_0(m)\right\}_{m=1}^{M} \tag{2-6}$$

= the M-sample complex combined-port baseband scalar signal batch

Then, using compact Hilbert-space notation†, we may define the appropriate combined-power expression for a batch approach by

$$P_c(\underline{w}) \equiv \|s_c\|^2 \tag{2-7}$$

which, by (2-6), is the average power of the combined signal over the M-sample batch. Note that $P_c(\underline{w})$ is a positive-real scalar quadratic function of the complex N-vector $\underline{w}$. As such, $P_c(\underline{w})$ will attain a minimum value at some point $\underline{w}$, which need not necessarily be unique. From calculus we know that at such a minimum point the gradient of $P_c(w)$ with respect to $\underline{w}$ must necessarily vanish. More specifically, we wish to determine $\underline{w}$ for which

$$\begin{bmatrix} \nabla_{\underline{w}_r} \\ \\ \nabla_{\underline{w}_i} \end{bmatrix} P_c(\underline{w}) = \begin{bmatrix} \underline{0} \\ \\ \underline{0} \end{bmatrix} \tag{2-8}$$

or, more symbolically,

$$\nabla_{\underline{w}} P_c(\underline{w}) = \underline{0} \tag{2-9}$$

---

†Given complex-valued time-sequences $x = \left\{x(m)\right\}_{m=1}^{M}$ and $y = \left\{y(m)\right\}_{m=1}^{M}$, define their inner-product by

$$<x,y> = \frac{1}{M}\sum_{m=1}^{M} x^*(m)y(m)$$

and the metric of a given sequence as its self inner-product

$$\|x\|^2 = <x,x>$$

where we arbitrarily define the complex gradient operator [5] by

$$\nabla_{\underline{w}} = \nabla_{\underline{w}_r} + j\nabla_{\underline{w}_i} \tag{2-10}$$

In view of (2-6), and definitions

$$\begin{aligned} s_0 &= s_{0r} + js_{0i} \\ s_c &= s_{cr} + js_{ci} \\ \underline{s} &= \underline{s}_r + j\underline{s}_i \\ \underline{w} &= \underline{w}_r + j\underline{w}_i \end{aligned} \tag{2-11}$$

(2-7) may be rewritten as

$$\begin{aligned} P_c(\underline{w}) &= \langle s_c, s_c \rangle \\ &= \langle 1, s_c^* s_c \rangle \\ &= \langle 1, (s_{cr})^2 + (s_{ci})^2 \rangle \\ &= \langle 1, (\underline{s}_r^T \underline{w}_r - \underline{s}_i^T \underline{w}_i + s_{0r})^2 + (\underline{s}_r^T \underline{w}_i + \underline{s}_i^T \underline{w}_r + s_{0i})^2 \rangle \end{aligned} \tag{2-12}$$

whence, by (2-10), we get

$$\begin{aligned} \nabla_{\underline{w}} P_c(\underline{w}) &= \langle 1, (s_r^T s_{cr} + s_i^T s_{ci}) + j(s_r^T s_{ci} - s_i^T s_{cr}) \rangle \\ &= \langle 1, Re\left\{\underline{s}^{*T} s_c\right\} + jIm\left\{\underline{s}^{*T} s_c\right\} \rangle \\ &= \langle 1, \underline{s}^{*T} s_c \rangle \\ &= \langle \underline{s}, s_c \rangle \end{aligned} \tag{2-13}$$

which is simply a cross-correlation of the complex auxiliary signal N-vector $\underline{s}$ with the combined complex scalar signal s_c; clearly, a complex N-vector. Using (2-6), it is possible to simplify (2-13) into an expression that involves $\underline{w}$ explicity; that is,

$$\nabla_{\underline{w}} P_c(\underline{w}) = \langle \underline{s}, s_c \rangle$$

$$= \langle \underline{s}, \underline{s}^T \underline{w} + s_0 \rangle$$

$$= \langle \underline{s}, \underline{s}^T \rangle \underline{w} + \langle \underline{s}, s_0 \rangle$$

$$= C\underline{w} + \underline{b} \qquad (2\text{-}14)$$

where

$$C = \langle \underline{s}, \underline{s}^T \rangle \qquad (2\text{-}15)$$

= the complex N x N conjugate-symmetric covariance matrix of the complex auxiliary N-vector signal, $\underline{s}$, over the batch of M samples. Its general component is given by

$$C_{ij} = \frac{1}{M} \sum_{m=1}^{M} s_i^*(m) s_j(m) \qquad (2\text{-}16)$$

$$\underline{b} = \langle \underline{s}, s_0 \rangle \qquad (2\text{-}17)$$

= the complex N-vector representing the cross-correlation between the complex auxiliary N-vector signal, $\underline{s}$, and the complex main scalar signal, s_0. Its general component is given by

$$b_i = \frac{1}{M} \sum_{m=1}^{M} s_i^*(m) s_0(m) \qquad (2\text{-}18)$$

As a consequence of (2-14) the minimum-combined-power condition (2-9) leads to the complex linear system of equations

$$C\underline{w} + \underline{b} = \underline{0} \qquad (2\text{-}19)$$

involving the unknown complex weight N-vector $\underline{w}$. The section that follows discusses possible solutions to (2-19).

2.2 Solution Alternatives

Methods for solving (2-19) fall into a number of distinct categories including inversion and relaxation. To denote the association with batch processing and an accompanying covariance matrix, these two categories will be suggestively referred to by the names "Batch Covariance Inversion" (BCI) and "Batch Covariance Relaxation" (BCR). Although the emphasis of the present effort is exclusively on the conjugate gradients (CG) method, a relaxation technique, a brief examination of inversion will provide a useful perspective for understanding some essential aspects of the CG method.

2.2.1 Batch Covariance Inversion (BCI)

Formally, the solution to (2-19) is simply

$$\underline{\hat{w}} = -C^{-1}\underline{b} \tag{2-20}$$

assuming that the inverse of the covariance matrix, C^{-1}, exists. In general, C may be numerically ill-conditioned or even singular with rank less than N. Since C^{-1} may not exist under these conditions, one is compelled to seek either an approximate or a more generalized form of C^{-1}.

2.2.1.1 Regularized Inverse

Given a singular matrix C, it is possible to construct a full-rank approximate version, $\tilde{C}$, by augmenting C with a sufficient amount of diagonal variance; i.e.,

$$\tilde{C} = C + \varepsilon I \tag{2-21}$$

where $\varepsilon > 0$ is a small fraction of the largest diagonal element in C. $\tilde{C}$ is referred to as a regularization of C. The approximate solution of (2-18) is then

$$\underline{\tilde{w}} = -\tilde{C}^{-1}\underline{b} \tag{2-22}$$

where $\tilde{C}^{-1}$ is the regularized inverse. Clearly, the accuracy of $\underline{\tilde{w}}$ with, respect to satisfying (2-19) will depend directly on the magnitude of ε.

It is interesting to note here that ε may be viewed as the common variance of independent additive white noise processes associated with each auxiliary signal component of $\underline{s}$ and main signal s_0.

2.2.1.2 Pseudoinverse

Let $\mathcal{C}^N$ represent the N-dimensional complex space. In general, as $\underline{w}$ spans $\mathcal{C}^N$, $C\underline{w}$ will span a subspace $\mathcal{C}^R \subset \mathcal{C}^N$ where R = rank C. Then, (2-19) is satisfied by any $\underline{w}$ of the form

$$\underline{\tilde{w}} = \underline{w}^R + \underline{w}^{\bar{R}} \tag{2-23}$$

where $\underline{w}^R \in C^R$ is unique and $\underline{w}^{\bar{R}} \in \overline{C^R} = C^N - C^R$ is arbitrary. By construction,

$$
\begin{aligned}
\underline{0} &= C\underline{\tilde{w}} + \underline{b} \\
&= C(\underline{w}^R + \underline{w}^{\bar{R}}) + \underline{b} \\
&= C\underline{w}^R + \cancelto{0}{C\underline{w}^{\bar{R}}} + \underline{b} \\
&= C\underline{w}^R + \underline{b} \qquad (2\text{-}24)
\end{aligned}
$$

as expected. As a consequence, a desirable exact solution of (2-19) is

$$\underline{\hat{w}} = \underline{w}^R \qquad (2\text{-}25)$$

which happens to be the smallest-length solution of the form (2-23) since $\underline{w}^R$ and $\underline{w}^{\bar{R}}$ are mutually orthogonal. Solution (2-25) is referred to as the <u>minimum-norm solution</u> of (2-19).

Solution (2-25) may be obtained by using a unique special inverse operator, C^+, known as the <u>pseudoinverse</u> of C [6]. One construction of C^+ is given in the specialized theorem that follows.

<u>Theorem 2-1</u>. [Pseudoinverse of a Hermitian Matrix]

Let C be an N x N complex Hermitian (conjugate symmetric) matrix with rank $R \leq N$. Given its similarity transformation

$$C = E \, \Lambda \, E^{*T} \qquad (2\text{-}26)$$

where

E = an N x N complex unitary (orthonormal) matrix whose columns are the eigenvectors of C. This matrix is often referred to as the eigenvector matrix and has the unitary property that

$$E^{-1} = E^{*T} \qquad (2\text{-}27)$$

and, as such,

$$\|E\underline{z}\|^2 = \|\underline{z}\| \qquad (2\text{-}28)$$

indicating that E preserves the Euclidean norm[†]
of any N-vector it operates on.

Λ = an N x N real diagonal matrix whose diagonal elements are the eigenvalues of C. Note that when $R < N$, N-R eigenvalues of C are identically zero. An appropriate form of Λ is

$$\Lambda = \begin{bmatrix} \lambda_1 & & & & \\ & \ddots & & 0 & \\ & & \lambda_R & & \\ & 0 & & 0 & \\ & & & & \ddots \; 0 \end{bmatrix} = \begin{bmatrix} \Lambda_{11} & 0 \\ & \\ 0 & 0 \end{bmatrix} \tag{2-29}$$

Then, the pseudoinverse of C is given by

$$C^{+} = E\,\Lambda^{+}E^{*T} \tag{2-30}$$

where

$$\Lambda^{+} = \begin{bmatrix} \Lambda_{11}^{-1} & 0 \\ & \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \lambda_1^{-1} & & & \\ & \ddots & & \\ & & \lambda_R^{-1} & \\ & & & 0 \\ & & & & \ddots \; 0 \end{bmatrix} \tag{2-31}$$

= the pseudoinverse of Λ

[†]The <u>Euclidean norm</u> of any vector $\underline{z} \in C^N$ is defined by

$$\|z\| = \sqrt{\sum_{n=1}^{N} z_n^* z_n}$$

which represents its length.

and the minimum-norm solution of (2-19) is given by

$$\underline{\hat{w}} = -C^{+}\underline{b} \tag{2-32}$$

Proof: Consider (2-19) and define the residual vector

$$\underline{r} = C\underline{w} + \underline{b} \tag{2-33}$$

We wish to determine $\underline{w}$ which will minimize the Euclidean norm of this residual. Then, writing

$$\begin{aligned} \|r\|^2 &= \|C\underline{w} + \underline{b}\|^2 \\ &= \|E\Lambda E^{*T}\underline{w} + \underline{b}\|^2 \\ &= \|E(\Lambda E^{*T}\underline{w} + E^{*T}\underline{b})\|^2 \\ &= \|\Lambda E^{*T}\underline{w} + E^{*T}\underline{b}\|^2 \end{aligned} \tag{2-34}$$

which follows by property (2-28). Now, define the following two vectors

$$E^{*T}\underline{w} = \underline{\omega} = \begin{bmatrix} \underline{u} \\ \underline{v} \end{bmatrix} \tag{2-35}$$

$$E^{*T}\underline{b} = \underline{\beta} = \begin{bmatrix} \underline{d} \\ \underline{e} \end{bmatrix} \tag{2-36}$$

where $\underline{\omega}$ is shown to be a concatenation of $\underline{u}$ and $\underline{v}$ representing the first R components and the remaining N-R components of $\underline{\omega}$, respectively. A similar partitioning of $\underline{\beta}$ is implied in (2-36). Then, upon substituting (2-35) and (2-36) in (2-34), we get

$$\begin{aligned} \|\underline{r}\|^2 &= \|\Lambda\underline{\omega} + \underline{\beta}\|^2 \\ &= \|\Lambda_{11}\underline{u} + \underline{d}\|^2 + \|\underline{e}\|^2 \end{aligned} \tag{2-37}$$

which follows from (2-29). Note that $\|r\|^2$ is minimized by any $\underline{\omega}$ of the form

$$\begin{aligned}
\underline{\tilde{\omega}} &= \begin{bmatrix} \underline{u} \\ \underline{v} \end{bmatrix} \\
&= \begin{bmatrix} -\Lambda_{11}^{-1}\underline{d} \\ \underline{v} \end{bmatrix} \\
&= \begin{bmatrix} -\Lambda_{11}^{-1}\underline{d} \\ \underline{0} \end{bmatrix} + \begin{bmatrix} \underline{0} \\ \underline{v} \end{bmatrix} \qquad (2\text{-}38)
\end{aligned}$$

where $\underline{0}$ is the R or (N-R)-dimensional zero depending on its location, and $\underline{v}$ is any (N-R) complex vector. Clearly, a unique minimum-norm $\underline{\tilde{\omega}}$ that minimizes $\|\underline{r}\|^2$ results by choosing $\underline{v} = \underline{0}$. By (2-35), (2-36) and property (2-27), it follows from (2-38) that the general exact solution to (2-19) becomes

$$\begin{aligned}
\underline{\tilde{w}} &= E\underline{\tilde{\omega}} \\
&= E\begin{bmatrix} -\Lambda_{11}^{-1}\underline{d} \\ \underline{0} \end{bmatrix} + E\begin{bmatrix} \underline{0} \\ \underline{v} \end{bmatrix} \\
&= -E\begin{bmatrix} \Lambda_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \underline{d} \\ \underline{e} \end{bmatrix} + E\begin{bmatrix} \underline{0} \\ \underline{v} \end{bmatrix} \\
&= -E\,\Lambda^{+}E^{*T}\underline{b} + E\begin{bmatrix} \underline{0} \\ \underline{v} \end{bmatrix} \\
&= \underline{w}^{R} + \underline{w}^{\overline{R}} \qquad (2\text{-}39)
\end{aligned}$$

as stated in (2-23). Clearly, by orthogonality of $\underline{w}^R$ and $\underline{w}^{\overline{R}}$, the minimum-norm solution to (2-19) is given by

$$\underline{\hat{w}} = -E\,\Lambda^{+}E^{*T}\underline{b} \qquad (2\text{-}40)$$

which is identical to $\underline{w}^R$ and is mathematically equivalent to $E\underline{\hat{\omega}}$, where $\underline{\hat{\omega}}$ is the unique minimum-norm value of (2-38). We have thus proved that there exists an operator

$$C^+ = E\,\Lambda^+ E^{*T} \tag{2-41}$$

the pseudoinverse of C, such that

$$\underline{\hat{w}} = -C^+\underline{b} \tag{2-42}$$

is the unique minimum-norm solution to (2-19), where, Λ^+ itself denotes the pseudoinverse of Λ in direct accordance to definition (2-41). Also, it should be clear that when C is full-rank (R=N), $C^+ = C^{-1}$.

2.2.2 Batch Covariance Relaxation (BCR)

Although the BCI approach is intuitively appealing and there exist a number of numerically-efficient algorithms for computing an "inverse" of the conjugate-symmetric matrix C, the major benefit in deriving an "inverse" is an intended efficient evaluation of a multiplicity of solutions to (2-18) involving a number of distinct forcing vectors, $\underline{b}$. However, in the interest of minimizing implementation complexity and insuring reliable numerical performance, especially with increasing dimensionality N, it is imperative that indirect approaches for solving (2-18) be considered. The CG method, a particularly efficient special case of the general class of CD methods is inherently a numerically-robust, finite-step, iterative relaxation technique for producing the minimum-norm solution to (2-18) without the need for explicity computing an "inverse." However, if necessary, it is possible to compute an inverse operator, C^x, as a by-product of the CG process†, which, under certain conditions, happens to be the desired pseudoinverse, C^+.

2.2.2.1 Method of Conjugate Directions (CD)

The origin of the CG method may be traced back to the independent investigations by Maguus Hestenes and Eduard Steifel which culminated in their combined paper [2] in 1952. Both investigators, aware of previous work [7] dealing with the solution of (2-18), recognized that it could be generalized into a category of relaxation techniques that constitute the class of Conjugate Directions (CD) methods. They succeeded in developing the CG method and showing that it is a special computationally-efficient CD method. A detailed mathematical development of the CD and CG methods is deferred to Section [illegible]. For the present, a brief outline of the CD method suffices to subsequently introduce the CG method.

†An inverse operator, C^x, may be defined for any CD method. Under certain conditions, C^x is a generalized inverse while under more restrictive additional conditions, it is the unique pseudoinverse, C^+.

The general CD method is a finite-step iterative procedure for producing an "exact solution" to the complex linear system (2-19), namely,

$$\underline{Cw} + \underline{b} = \underline{0} \tag{2-43}$$

Starting with an arbitrary initial estimate, $\underline{w}^0$, the desired solution is obtained by successive optimal relaxation along a special set of search vectors $\underline{p}^0$, p^1, ... which satisfy the C-conjugacy (C = orthogonality) property

$$(\underline{p}^i, Cp^j) = 0 \; ; \; (i \neq j) \tag{2-44}$$

More specifically, upon defining the residual vector

$$\underline{r}^0 = C\underline{w}^0 + \underline{b}$$

at the initial estimate $\underline{w}^0$, the CD algorithm consists of the following set of recursive steps

$$\alpha_i = \frac{(\underline{p}^i, \underline{r}^i)}{(p^i, C\underline{p}^i)} \tag{2-45.1}$$

$$\underline{w}^{i+1} = \underline{w}^i - \alpha_i \underline{p}^i \tag{2-45.2}$$

$$\underline{r}^{i+1} = \underline{r}^i - \alpha_i C\underline{p}^i \tag{2-45.3}$$

repeated a number of iterations $I \leq N$, where N is the dimensionality of system (2-43). Note that the CD procedure terminates in $I \leq N$ iterations when $\|\underline{r}\| < \varepsilon \|\underline{b}\|$, a logical stopping criterion. The quantity $\varepsilon \ll 1$ is chosen commensurate to the computing precision available.

It should be mentioned here that the sequence of C-conjugate vectors, $\underline{p}^0$, $\underline{p}^1$, ..., may be derived by means of a Gram-Schmidt process. Specifically, employing a complete set of orthonormal N-vectors $\{\underline{e}^i\}_{i=0}^{N-1}$, the Gram-Schmidt (GS) process may be used to generate the set of C-conjugate directions satisfying the conjugacy condition (2-44). As a consequence, the k-th conjugate-direction vector will be given by

---

†The inner-product notation $(\cdot, \cdot)$ is reserved for the vectors in the complex weight N-space, C^N, and must be distinguished from $\langle \cdot, \cdot \rangle$, the counterpart in time-space.

$$\underline{p}^k = \underline{e}^k + \sum_{j=0}^{k-1} \beta_{k,j}\underline{p}^j \qquad (2\text{-}46)$$

the familiar telescoping form.

2.2.2.2 Method of Conjugate Gradients (CG)

Recall from Section 2.1 that the residual vector, $\underline{r}^{i+1}$, in (2-45.3) is, in fact, the complex gradient $\nabla_w P_c(\underline{w})$ evaluated at $\underline{w} = \underline{w}^{i+1}$. The CG method is a special CD case where the C-conjugate vectors $\underline{p}^0$, $\underline{p}^1$, ... are generated via a built-in GS process using successive gradients at each iteration. In more specific terms, upon defining the residual and conjugate-direction vectors by

$$\underline{r}^0 = C\underline{w}^0 + \underline{b} \qquad (2\text{-}47\text{-}1)$$

$$\underline{p}^0 = \underline{r}^0 \qquad (2\text{-}47.2)$$

the CG algorithm consists of the following recursive steps

$$\alpha_i = \frac{(\underline{p}^i, \underline{r}^i)}{(\underline{p}^i, C\underline{p}^i)} \qquad (2\text{-}48.1)$$

$$\underline{w}^{i+1} = \underline{w}^i - \alpha_i\underline{p}^i \qquad (2\text{-}48.2)$$

$$\underline{r}^{i+1} = \underline{r}^i - \alpha_i C\underline{p}^i \qquad (2\text{-}48.3)$$

$$\beta_i = \frac{\|\underline{r}^{i+1}\|^2}{\|\underline{r}^i\|^2} \qquad (2\text{-}48.4)$$

$$\underline{p}^{i+1} = \underline{r}^i + \beta_i\underline{p}^i \qquad (2\text{-}48.5)$$

repeated a number of iterations $I \leq N$.

It is important to note here that the successive gradients $\underline{r}^0$, $\underline{r}^1$, ... may be shown to be mutually orthogonal (or, conjugate) and hence qualify as basis vectors for a GS expansion. The CG method bears its name for this obvious reason. Above all, the greatest significance of the CG method is in the fact that, as a direct consequence of the particular choice of the basis vectors, the built-in GS process (see (2-47.2), (2-48.4) and (2-48.5)) does not telescope as expected in general. In contrast to the increasing number of terms involved in (2-46), the GS process in the CG method generates the new C-conjugate vector $\underline{p}^{i+1}$ from current values of gradient and C-conjugate vectors $\underline{r}^i$ and $\underline{p}^i$, respectively. The computational simplicity of the CG method over that of the general CD method is clear.

2.2.2.3 Intrinsic Inverse

It will be shown in the next section that if $\underline{w}^0 = \underline{0}$, the solution to (2-18) obtained via the general CD method may be written in the form

$$\hat{\underline{w}} = -C^x \underline{b} \tag{2-49}$$

where

$$C^x = \sum_{i=1}^{I-1} \frac{\underline{p}^i \underline{p}^{i*T}}{(\underline{p}^i, C\underline{p}^i)} \tag{2-50}$$

is a *special inverse operator constructed from* successive C-conjugate vectors. In general, $C^x \neq C^+$. This may be demonstrated via two simple examples given below.

Example 2-1. In system (2-43), let $\underline{b} = \underline{e}^1$, a nontrivial eigenvector of C with associated eigenvalue λ_1. Starting with an initial solution estimate, $\underline{w}^0 = \underline{0}$, the CG method, (2-48), will yield the exact solution

$$\hat{\underline{w}} = -\frac{1}{\lambda_1}\underline{e}^1 \tag{2-51}$$

in a single iteration even if rank $C > 1$. In this case,

$$C^x = \frac{\underline{e}^1 \underline{e}^{1*T}}{\lambda_1}$$

$$= E\Lambda^x E^{*T} \tag{2-52}$$

which is the pseudoinverse, C^+, only when rank $C = 1$. What is more significant here is that the CG solution, (2-51), is the minimum-norm solution, (2-42), even though $C^x \neq C^+$. In the sense that C^x is tailored to $\underline{b}$, it may be called the <u>intrinsic inverse</u> of C with respect to $\underline{b}$. Analogously, Λ^x is the intrinsic inverse of the eigenvalue matrix Λ.

<u>Example 2-2</u>. Suppose that, in (2-43), $C = I$ and $\underline{b} = \underline{1}$, the uniform vector. Then, $C^+ = C^{-1} = I$. Starting with an initial solution estimate, $\underline{w}^0 = \underline{0}$, the CG method, (2-48), will yield the unique solution

$$\hat{\underline{w}} = - \underline{1} \tag{2-53}$$

in a single iteration even though C is full-rank. However,

$$\begin{aligned} C^x &= \frac{\underline{b}\ \underline{b}^{*T}}{(\underline{b},\ C\underline{b})} \\ &= \frac{\underline{b}\ \underline{b}^{*T}}{\|\underline{b}\|^2} \\ &= \frac{1}{N} U \end{aligned} \tag{2-54}$$

where U is the uniform N x N matrix of all 1's and N is the system dimensionality. Clearly, $C^x \neq C^+ = C^{-1}$. We have thus shown that C^x is different from C^{-1}, even though the latter exists.

Motivated by these two examples and recalling a number of others where $C^x = C^{-1}$ when the eigenvalues of C are nonzero and distinct, it is intuitively appealing to state the following conjecture conerning a condition under which $C^x = C^{-1}$ might be true.

<u>Conjecture 2-2</u>. [Condition under which the CG Intrinsic Inverse is the Actual Inverse]

In (2-43), let C have distinct nonzero eigenvalues and $\underline{b}$ equal to an exhaustive linear combination of the eigenvectors of C; i.e.,

$$\underline{b} = \sum_{i=1}^{N} a_i \underline{e}^i \tag{2-55}$$

where N is the system dimensionality, $a_i \neq 0 \ \forall i \in [1,N]$ and $\{\underline{e}^i\}_{i=1}^N$ is the complete set of eigenvectors of C. Then, the CG intrinsic inverse, C^x, as given by (2-50) is identically equal to the actual inverse, C^{-1}. (2-41).

Comment: By Example 2-1, $C^x \neq C^{-1}$ if $\underline{b}$ is not exhaustive linear combination of eigenvectors as indicated in (2-55). By Example 2-2, $C^x \neq C^{-1}$ if the eigenvalues of C are identical, independent of the choice of $\underline{b}$. This seems to suggest that any multiplicity of eigenvalues of C would lead to a $C^x \neq C^{-1}$. A number of specific numerical examples have substantiated this observation. In fact, the conjecture has held true in a number of examples where the stated hypotheses were satisfied. Of course, the latter observation does not constitute a proof and, as such, the stated condition is a conjecture and not a theorem. The obvious next step is to actually state this conjecture as a theorem and provide a rigorous proof, something beyond the objective of the present work.

An immediate consequence of the above conjecture is the following proposition which intends to guarantee that $C^x = C^{-1}$, assuming C^{-1} exists.

Proposition 2-3. [Construction of C^{-1} Via CG]

Given system (2-19), assume that C has distinct nonzero eigenvalues and let

$$\underline{b} = C\underline{d} \tag{2-56}$$

where $\underline{d} \in C^N \ni E^{*T}\underline{d}$ has no zero elements. Then, the CG intrinsic inverse with respect to $\underline{b}$ is identical to the ordinary inverse.†

†The reader may be motivated to generalize this proposition by removing this last restriction. It is intuitively appealing to state that, given (2-56), $C^x = C^{-1}$ with probability one. The proof of the resulting corollary, if valid, would be much more complex. Note that an event is said to occur with probability one if the occurrence frequency of the complement event is zero.

Comment: Note that, via (2-56), $\underline{b}$ is an exhaustive sum of all the columns of C. Using the similarity transformation for C, (2-56) may be rewritten as

$$\underline{b} = E \Lambda E^{*T} \underline{d} \tag{2-57}$$

Then, assuming that $E^{*T}\underline{d}$ has no zero elements and that Λ is full-rank, $\exists | \{a_i\}_{i=1}^{N} \in C^1 \ni$

$$a_i = \lambda_i \sum_{j=1}^{N} d_j e_j^{*i} \neq 0 \tag{2-58}$$

and

$$b = \sum_{i=1}^{N} a_i \underline{e}^i \tag{2-59}$$

Then, by Conjecture 2-2, $C^x = C^{-1}$.

It should be mentioned, in passing, that Conjecture 2-2 and Proposition 2-3 apply to the general CD method when the C-conjugate vectors involved are contained in the eigenspace of C. This may be guaranteed by construction. However, since the present investigation is primarily focused on the CG method, we will examine the general CD method only to the extent necessary for enhancing the understanding of the CG method.

2.3 Functional Description

Figure 2-1 shows the essential functional blocks that comprise a general batch process as it applies to the specific adaptive array cancellation subsystem discussed in Section 2-1.

As indicated, main and auxiliary-port baseband signal samples, $s_0(m)$ and $\underline{s}(m)$, are presented to a "C and $\underline{b}$ Computation Block" and respective "Delay Memory Buffers." After an M-sample batch-time, C and $\underline{b}$ become available to the "Batch Algorithm Block," which subsequently produces the adaptive weight vector, $\underline{w}$, in an L-sample period not exceeding the available batch-time. Then, for an M-sample period of time, $\underline{w}$ is applied onto the auxiliary signal samples comprising the original batch used to compute C and $\underline{b}$. The resulting weighted auxiliary signal sample sum, $\underline{s}^T(m-M-L)\underline{w}$, is finally added synchronously to corresponding main signal samples, $s_\ell(m-M-L-K)$, appropriately delayed by K samples to account for the elapsed time through the combiner. The final combined output signal is designated by $s_c(m-M-L-K-1)$, clearly indicating a processor pipeline delay of M+L+K+1.

It should be mentioned that the minimum value of L is determined by the computation-time of the batch algorithm. A value of L larger than this minimum may be chosen depending on practical considerations. Although C and $\underline{b}$ must be computed over no less than an L-sample signal batch, larger batches may be used for practical reasons including a potential need to time-share the batch algorithm.

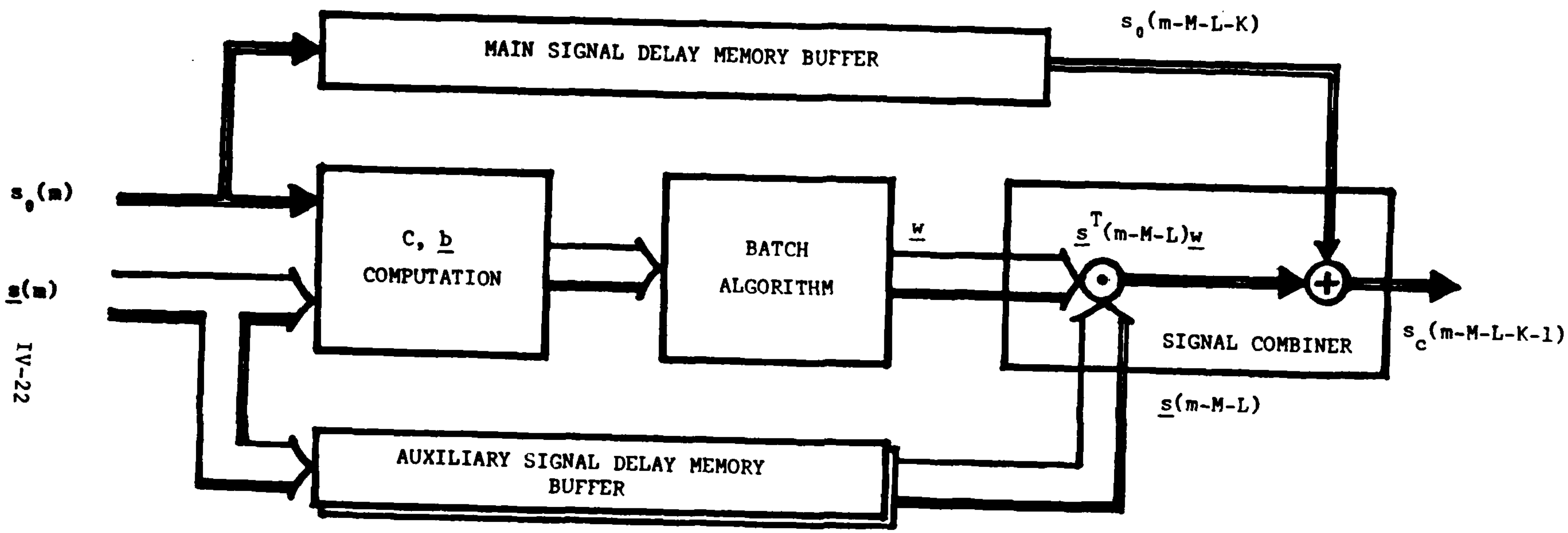


Figure 2-1. Functional Block Mechanization of a General Batch Adaptive Processor Applicable to an Adaptive Array Sidelobe Cancellation System

3.0 MATHEMATICAL BASIS OF THE BCR PROCESS

An appreciation of the BCR process may be derived from a better understanding of its mathematical basis. In this section, a detailed development of the general CD method leads logically to the derivation of the special CG method. Specific numerical examples serve to illustrate the validity and some unique features of the CG method, the relaxation algorithm actually adopted for the BCR process.

3.1 Detailed Mathematical Development

The derivation of the CG method is most clearly motivated from the mathematical development of the general CD method. Accordingly, the CD method is derived first. The CG method is developed subsequently as a special case of the general CD method. Before proceeding, however, some preliminary remarks are in order.

Recall that the batch formulation of the adaptive array problem considered in the previous section has led to the complex linear system

$$C\underline{w} + \underline{b} = \underline{0} \tag{3-1}$$

where C is an N x N conjugate-symmetric matrix, $\underline{b}$ is a complex forcing N-vector and $\underline{w}$ is the complex adaptive weight N-vector to be determined. Recall that (3-1) simply states the necessary condition that $P_c(\underline{w})$ attain a minimum at some value of $\underline{w}$. More specifically, (3-1) requires that the complex gradient of $P_c(\underline{w})$ with respect to $\underline{w}$ be identically equal to zero at a minimum point, $\underline{w}$.

The desired complex adaptive weight N-vector, $\underline{w}$, may be determined by means of the CD and CG methods to be derived. Central to this development are the algebraic properties of $\underline{b}$ and C in (3-1). To start with, it is important to note a fundamental property of the complex N-vector $\underline{b}$ based on the geometrical significance of (3-1). Letting $\mathcal{C}^N$ denote the complete <u>complex N-space</u>, define the <u>eigenspace of matrix C</u> by

$$\mathcal{C}^R \equiv \{\underline{v} \in \mathcal{C}^N : C\underline{w} = \underline{v},\ \forall \underline{w} \in \mathcal{C}^N\} \tag{3-2}$$

Note that $\mathcal{C}^R$ is a <u>subspace of $\mathcal{C}^N$</u> generated by C upon operating on all members of $\mathcal{C}^N$. The superscript R in $\mathcal{C}^R$ refers to the fact that it may be spanned by a linear combination of no less than R independent complex N-vectors. In fact, R = rank C $\leq$ N. In view of (3-2), the <u>complement space</u>, $\overline{\mathcal{C}^R} = \mathcal{C}^N - \mathcal{C}^R$, is the <u>null-space of C</u>. Clearly, $C\underline{w} = \underline{0}$ if $\underline{w} \in \overline{\mathcal{C}^R}$. Since (3-1) is satisfied exactly, it follows that $\underline{b} \cap \overline{\mathcal{C}^R} = \emptyset$; that is $\underline{b}$ is entirely contained in the eigenspace of C. This property of $\underline{b}$ is pertinent to the mathematical development that follows.

The special properties of matrix C are also crucial in the subsequent discussion. Note that C is a Hermitian matrix with the conjugate-symmetric property

$$C = C^{*T} \tag{3-3}$$

An important consequence of this property is embodied in the following theorem.

> <u>Theorem 3-1.</u> [Eigenvalue Property of a Hermitian Matrix C]
>
> The eigenvalues of an N x N complex Hermitian matrix C are non-negative real.

<u>Proof:</u> Let $\underline{e} \in C^R$ be a nontrivial eigenvalue of C with a nonzero eigenvalue λ. Then, by definition,

$$C\underline{e} = \lambda\underline{e} \tag{3-4}$$

where $\underline{e}$ is understood to be normalized to a unit Euclidean length; i.e.,

$$\|\underline{e}\|^2 = (\underline{e}, \underline{e}) = \underline{e}^{*T}\underline{e} = 1 \tag{3-5}$$

Then, it follows from (3-4) and (3-5) that

$$\begin{aligned}(\underline{e}, C\underline{e}) &= \underline{e}^{*T}C\underline{e} \\ &= \underline{e}^{*T}\lambda\underline{e} \\ &= \lambda\underline{e}^{*T}\underline{e} \\ &= \lambda \end{aligned} \tag{3-6}$$

whence, (3-3) implies that

$$
\begin{aligned}
\lambda^* &= (\underline{e}, C\underline{e})^* \\
&= (\underline{e}^{*T} C\underline{e})^* \\
&= e^T C^* \underline{e}^* \\
&= (C^{*T}\underline{e})^T \underline{e}^* \\
&= (C\underline{e})^T \underline{e}^* \\
&= \lambda \underline{e}^T \underline{e}^* \\
&= \lambda \underline{e}^{*T} \underline{e} \\
&= \lambda \qquad\qquad (3\text{-}7)
\end{aligned}
$$

Hence, the nontrivial eigenvalues of C are positive real. If $\underline{e} \in \overline{C^R}$, it follows by (3-2) and (3-4) that $\lambda = 0$. Therefore, the eigenvalues of C are non-negative real.

∎

An important implication of the above result is the non-negative definite property of C in the following corollary.

Corollary 3-2. [Non-negative Definite Property of a Hermitian Matrix C]

Let C be any N x N Hermitian matrix. Then for any $\underline{w} \in C^N$,

$$(\underline{w}, C\underline{w}) = a \geq 0 \qquad (3\text{-}8)$$

where a is real.

Proof: Let

$$\underline{w} = \sum_{i=1}^{N} d_i \underline{e}^i$$

$$= \sum_{i=1}^{R} d_i \underline{e}^i + \sum_{i=R+1}^{N} d_i \underline{e}^i$$

$$= \underline{w}^R + \underline{w}^{\overline{R}} \qquad (3\text{-}9)$$

where $\underline{w}^R \in C^R$ and $\underline{w}^{\overline{R}} \in \overline{C^R}$. Also, $\{\underline{e}^i\}_{i=1}^{N}$ is a complete orthonormal basis of complex N-vectors in C^N, the first R of which are the nontrivial eigenvectors of C contained in C^R while the remaining N-R are contained in $\overline{C^R}$. Note that (3-19) may be written as

$$\underline{w} = ED \qquad (3\text{-}10)$$

where E is an N x N complex matrix whose columns are the eigenvectors $\{\underline{e}^i\}_{i=1}^{N}$ and D is a diagonal matrix of the corresponding coefficients $\{d_i\}_{i=1}^{N}$. Recalling that C may be written as the similarity transformation

$$C = E \Lambda E^{*T} \qquad (3\text{-}11)$$

where Λ is the diagonal matrix of eigenvalues of C, we see that by the unitary property† of E,

$$(\underline{w}, C\underline{w}) = \underline{w}^{*T} C \underline{w}$$

$$= D^* E^{*T} E \Lambda E^* E D$$

$$= D^* \Lambda D$$

$$= \sum_{i=1}^{R} \lambda_i |d_i|^2$$

$$= (\underline{w}^R, C\underline{w}^R)$$

$$\geq 0 \qquad (3\text{-}12)$$

clearly, a non-negative real quantity. In particular, note that when $\underline{w} = \underline{w}^R$, (3-12) is identically equal to zero.

---

†An N x N matrix E is unitary if $E^{-1} = E^{*T}$.

Another property of C that pertains to the discussion to follow is included in the following lemma.

Lemma 3-3. [Mutual Conjugacy of Two Vectors with Respect to a Hermitian Matrix C]

Let C be an N x N complex Hermitian matrix. Then, for some $\underline{u}, \underline{v} \in C^N$,

$$(\underline{u}, C\underline{v}) = 0 \tag{3-13}$$

if an only if

$$(\underline{v}, C\underline{u}) = 0 \tag{3-14}$$

By (3-13), $\underline{u}$ is orthogonal to $C\underline{v}$ or C-conjugate to $\underline{v}$. Similarly (3-14) states that $\underline{v}$ is C-conjugate to $\underline{u}$. When both (3-13) and (3-14) hold, $\underline{u}$ and $\underline{v}$ are said to be mutally conjugate with respect to C, or mutually C-conjugate.

Proof: Assume that (3-13) holds for some $\underline{u}, \underline{v} \in C^N$. We wish to show that (3-14) holds by implication. Accordingly,

$$\begin{aligned} 0 &= (\underline{u}, C\underline{v}) \\ &= (\underline{u}, C\underline{v})^{*T} \\ &= (\underline{u}^{*T} C\underline{v})^{*T} \\ &= \underline{v}^{*T} C^{*T} \underline{u} \\ &= (\underline{v}, C\underline{u}) \end{aligned} \tag{3-15}$$

as desired. The reverse implication is the identical argument in reverse. ∎

3.1.1 Derivation of the CD Method

Consider the complex linear system (3-1). Let $\underline{w}^0$ be an arbitrarily chosen initial estimate of its general solution, $\underline{\tilde{w}}$, defined by (2-23). Corresponding to this estimate, define the initial residual vector

$$\underline{r}^0 = C\underline{w}^0 + \underline{b} \tag{3-16}$$

which, as described previously, represents the complex gradient of the combined power function, $P_c(\underline{w})$, with respect to $\underline{w}$, evaluated at $\underline{w}^0$. Then, assuming that rank $C = R \leq N$, there exists a set of Q linearly independent N-vectors

$$\left\{\underline{p}^k\right\}_{k=0}^{Q-1} \in C^N \tag{3-17}$$

where $Q \leq R$, such that, a general solution to (3-1) may be expressed as the linear combination

$$\tilde{\underline{w}} = \underline{w}^0 - \sum_{k=0}^{Q-1} \alpha_k \underline{p}^k \tag{3-18}$$

where $\left\{\alpha_k\right\}_{k=0}^{Q-1}$ are expansion coefficients to be determined. Since the residual vector, $\tilde{\underline{r}}$, corresponding to solution $\tilde{w}$ happens to be identically equal to zero, it follows from (3-16) and (3-18) that

$$\underline{r}^0 = \sum_{k=0}^{Q-1} \alpha_k C\underline{p}^k \tag{3-19}$$

If a linearly independent set of vectors $\left\{p^k\right\}_{k=0}^{Q-1}$ could be constructed such that any two different vectors could be mutually C-conjugate, then the computation of the expansion coefficients would follow readily. In fact, if

$$\left.\begin{aligned} &(\underline{p}^i, C\underline{p}^j) = 0 \\ &i, j \in [0, Q-1] \; ; \; i \neq j \end{aligned}\right\} \tag{3-20}$$

then, in view of (3-19)

$$\left.\begin{aligned} &\alpha_k = \frac{(\underline{p}^k, \underline{r}^k)}{(\underline{p}^k, C\underline{p}^k)} \\ &\forall k \in [0, Q-1] \end{aligned}\right\} \tag{3-21}$$

provided that $(\underline{p}^k, C\underline{p}^k) \neq 0$, or, more precisely, $C\underline{p}^k \neq \underline{0}$. This clearly indicates that the vectors $\{\underline{p}^k\}_{k=0}^{Q-1}$ must intersect the eigenspace of C to be admissible. The appropriateness of the C-conjugacy condition (3-20) is proved in the lemma that follows.

<u>Lemma 3-4</u>. [Consistency of Linear Independence and Mutual C-Conjugacy of Vectors $\{\underline{p}^k\}_{k=0}^{Q-1}$]

Assume that vectors $\{p^k\}_{k=0}^{Q-1}$ are mutually C-conjugate in accordance to (3-20). This choice is consistent with the linear independence assumption of these vectors.

<u>Proof</u>: Let $\{\underline{p}^k\}_{k=0}^{Q-1}$ be a set of linearly independent vectors. Then, by definition,

$$\sum_{k=0}^{Q-1} \alpha_k \underline{p}^k = \underline{0} \tag{3-22}$$

if and only if $\alpha_k = 0, \ \forall\ k \in [0, Q-1]$. Assuming that

$$\left.\begin{array}{l} C\underline{p}^j \neq 0 \\ \forall\ j \in [0, Q-1] \end{array}\right\} \tag{3-23}$$

it follows by (3-19) and (3-21) that

$$\begin{aligned} 0 &= \left(\sum_{k=0}^{Q-1} \alpha_k \underline{p}^k,\ C\underline{p}^j\right) \\ &= \alpha_j(\underline{p}^j,\ C\underline{p}^j) \end{aligned} \tag{3-24}$$

which implies that $\alpha_j = 0$, since, by Corollary 3-2, $(\underline{p}^j, C\underline{p}^j) \neq 0$ $\forall\ j \in [0, Q-1]$. Hence, linear independence of the set of Q vectors $\{p^k\}_{k=1}^{Q-1}$ is consistent with their mutual C-conjugacy. ∎

The general solution (3-18) may be viewed as a finite sequence of estimates $\{\underline{w}^i\}_{i=0}^{Q-1}$ where

$$\underline{w}^i = \underline{w}^0 - \sum_{k=0}^{i-1} \alpha_k \underline{p}^k \qquad (3\text{-}25)$$

written, alternatively, as the recursion relation

$$\underline{w}^{i+1} = \underline{w}^i - \alpha_i \underline{p}^i \qquad (3\text{-}26)$$

where i = 0, 1, ..., Q-1. Clearly, solution (3-18) is simply

$$\tilde{\underline{w}} = \underline{w}^{Q-1} \qquad (3\text{-}27)$$

the last iterate of (3-26). Upon substituting (3-26) into (3-1), we get a similar recursion relation for the set of corresponding residual vectors $\{r^i\}_{i=1}^{Q-1}$; i.e.,

$$\underline{r}^{i+1} = \underline{r}^i - \alpha_i C\underline{p}^i \qquad (3\text{-}28)$$

where i = 0, 1, ..., Q-1. The residual of the solution (3-27), clearly the last iterate of (3-28), is identically equal to zero as mentioned previously; that is,

$$\tilde{\underline{r}} = \underline{r}^{Q-1} = \underline{0} \qquad (3\text{-}29)$$

which is consistent with expression (3-19). In view of (3-21), (3-26) and (3-28), we have proved one form of the general CD method included in the following theorem.

Theorem 3-5. [Standard Form of the CD Method]

Consider the linear system

$$C\underline{w} + \underline{b} = \underline{0} \qquad (3\text{-}30)$$

where C is an N x N complex Hermitian matrix of rank R, $\underline{b}$ is a complex N-vector contained in C^R, the eigenspace of C, and $\underline{w}$ is the unknown complex N-vector to be determined. Let

$$\left\{\underline{p}^i\right\}_{i=0}^{Q-1} \qquad (3\text{-}31)$$

be a set of Q, $Q \leq R \leq N$, linearly independent complex N-vectors having the properties of

(1) Nonzero intersection with eigenspace C^R; i.e.,

$$\left. \begin{array}{l} \underline{p}^i \cap C^R \neq \emptyset \\ \\ \forall\, i \in [0,\ Q-1] \end{array} \right\} \quad (3\text{-}32)$$

(2) Spanning the eigenspace C^R via a linear combination; i.e.,

$$C^R \in \left\{ \underline{v} \in C^N : \underline{v} = \sum_{i=0}^{Q-1} a_i \underline{p}^i \ ;\ \forall\, a_i \in C^1,\ i \in [0,\ Q-1] \right\} \quad (3\text{-}33)$$

where C^1 is the space of complex scalars.

(3) Mutual C-conjugacy property; i.e.,

$$\left. \begin{array}{l} (\underline{p}^i,\ C\underline{p}^j) = 0 \\ \\ \forall\, i,\ j \in [0,\ Q-1];\ i \neq j \end{array} \right\} \quad (3\text{-}34)$$

Then, a general solution to (3-30) is the last iterate, $\underline{w}^{Q-1}$, of the Q-step iterative procedure that begins with an initial solution estimate $\underline{w}^0$ and corresponding residual vector $\underline{r}^0 = C\underline{w}^0 + \underline{b}$ and repeats relations

$$\alpha_i = \frac{(\underline{p}^i,\ \underline{r}^0)}{(\underline{p}^i,\ C\underline{p}^i)} \quad (3\text{-}35.1)$$

$$\underline{w}^{i+1} = \underline{w}^i - \alpha_i \underline{p}^i \quad (3\text{-}35.2)$$

$$\underline{r}^{i+1} = \underline{r}^i - \alpha_i C\underline{p}^i \quad (3\text{-}35.3)$$

for $i = 0, 1, \ldots, Q-1$. This procedure will be referred to as the standard form of the conjugate directions (CD) method, where conjugate directions refers to the C-conjugate set of vectors $\{\underline{p}^i\}_{i=0}^{Q-1}$

Proof: The hypotheses and the standard CD method stated by this theorem follows from preceding discussion starting with Section 3.1.

Although the standard CD method is strictly correct as stated in (3-35), attempts to minimize roundoff error in actual computation has led to a slight reformulation of the algorithm [2]. The basis for the alternate CD algorithm is the following lemma.

Lemma 3-6. [Special Properties of $(\underline{p}^i, \underline{r}^j)$ in CD Method]

In the standard CD method defined by Theorem 3-5, the inner product $(\underline{p}^i, \underline{r}^j)$ has the following properties:

$$\left.\begin{aligned} &(\underline{p}^i, \underline{r}^j) = (\underline{p}^i, \underline{r}^0) \\ &\forall\ i \geq j\ ;\ i,\ j \in [0,\ Q-1] \end{aligned}\right\} \tag{3-36}$$

and

$$\left.\begin{aligned} &(\underline{p}^i, \underline{r}^j) = 0 \\ &\forall\ i < j\ ;\ i \in [0,\ Q-1],\ j \in [1,\ Q-1] \end{aligned}\right\} \tag{3-37}$$

Proof: By the C-conjugacy property (3-34) and recursion (3-35.3), we see that

$$\begin{aligned} (\underline{p}^i, \underline{r}^i) &= (\underline{p}^i, \underline{r}^{i-1}) - \cancelto{0}{\alpha_i(\underline{p}^i, C\underline{p}^{i-1})} \\ &= (\underline{p}^i, \underline{r}^{i-2}) - \cancelto{0}{\alpha_{i-1}(\underline{p}^i, C\underline{p}^{i-2})} \\ &\ \vdots \\ &= (\underline{p}^i, \underline{r}^0) \end{aligned} \tag{3-38}$$

$\forall\ i \geq j\ ;\ i,\ j \in [0,\ Q-1]$. This is precisely (3-36). On the other hand, using the additional expression for expansion coefficient (3-35.1) and incorporating (3-38), we see that, for $i < j$,

$$
\begin{aligned}
(\underline{p}^i, \underline{r}^j) &= (\underline{p}^i, \underline{r}^{j-1}) - \alpha_{j-1}(\underline{p}^i - C\underline{p}^{j-1}) \\
&\vdots \\
&= (\underline{p}^i, \underline{r}^i) - \alpha_i(\underline{p}^i, C\underline{p}^i) \\
&= 0
\end{aligned}
\tag{3-39}
$$

$\forall i < j$; $i \in [0, Q-1]$, $j \in [1, Q-1]$. This completes the proof. ∎

The alternate form of the CD method may now be stated in the following theorem.

Theorem 3-7. [Alternate Form of the CD Method]

Given all the hypotheses of Theorem 3-5, a general solution to (3-30) is the last iterate, $\underline{w}^{Q-1}$, of the Q-step iterative procedure that begins with an aribitrary initial solution estimate, $\underline{w}^0$, and the corresponding residual vector, $\underline{r}^0 = C\underline{w}^0 + \underline{b}$, and repeats relations

$$\alpha_i = \frac{(\underline{p}^i, \underline{r}^i)}{(\underline{p}^i, C\underline{p}^i)} \tag{3-40.1}$$

$$\underline{w}^{i+1} = \underline{w}^i - \alpha_i\underline{p}^i \tag{3-40.2}$$

$$\underline{r}^{i+1} = \underline{r}^i - \alpha_i C\underline{p}^i \tag{3-40.3}$$

for $i = 0, 1, \ldots, Q-1$. This procedure will be referred to as the alternate form of the conjugate directions (CD) method.

Proof: Result (3-40) follows from Theorem 3-6 and Lemma 3-7. ∎

The description of the CD method is not complete without a means for constructing the set of linearly independent C-conjugate vectors, $\{\underline{p}^i\}_{i=0}^{Q-1}$. One possible construction involves the use of a complete orthogonal basis of vectors, $\{\underline{q}^i\}_{i=0}^{N-1}$, which can span the complex N-space, C^N, via a linear combination in the sense of (3-33). Subsequently, using a procedure similar to the Gram-Schmidt process, it is possible to generate a set of mutually C-conjugate vectors $\{\underline{p}^i\}_{i=0}^{Q-1}$, as desired. This construction scheme is presented in the following lemma.

Lemma 3-8. [Construction of C-Conjugate set of N-vectors $\{\underline{p}^i\}_{i=0}^{Q-1}$ from a Complete Orthogonal Set $\{\underline{q}^i\}_{i=0}^{N-1}$]

Let $\{\underline{q}^i\}_{i=0}^{N-1}$ be a complete orthogonal basis of C^N; i.e.,

$$C^N = \left\{ \underline{v} : \underline{v} = \sum_{i=0}^{N-1} a_i \underline{q}_i \; ; \; \forall\, a_i \in C^1,\ i \in [0,N-1] \right\} \tag{3-41}$$

Then, a set of Q linearly independent C-conjugate vectors $\{\underline{p}^i\}_{i=0}^{Q-1}$ may be defined as follows. The first vector is arbitrarily defined by

$$\underline{p}^0 = \underline{q}^{k_0} \tag{3-42}$$

where $k_0 = \min[0, N-1] \ni C\underline{p}^0 \neq \underline{0}$. The general such vector is given by

$$\underline{p}^i = \underline{q}^{k_i} + \sum_{j=0}^{i-1} \beta_{i,j} \underline{p}^j \tag{3-43}$$

where

$$\beta_{i,j} = \frac{(\underline{q}^{i_1}, C\underline{p}^j)}{(\underline{p}^j, C\underline{p}^j)} \tag{3-44}$$

and $\{k_i\}$ is a subsequence of indices in the range [0, N-1] while i and j range over [0, Q-1] and [0, Q-2], respectively, with $i > j$. Furthermore, $Q \leq R = \text{rank } C$.

Proof: Following the Gram-Schmidt (GS)† procedure, the first of the C-conjugate vectors, $\underline{p}^0$, is defined by

$$\underline{p}^0 = \underline{q}^{k_0} \tag{3-45}$$

†In the standard GS procedure $\underline{p}^1$ is constructed to be orthogonal to $\underline{p}^0$ by removing from q^{k_1} its orthogonal projection, $\beta_{1,0}\underline{p}^0$, onto $\underline{p}^0$. In contrast, the GS procedure appropriate for the CD method requires that $\underline{p}^1$ and $\underline{p}^0$ be mutually C-conjugate.

where $k_0 = \min[0, N-1] \ni \underline{g}^0 \cap C^R \neq \emptyset$, or $C\underline{g}^{k_0} = C\underline{p}^0 \neq \underline{0}$. The next C-orthogonal vector is given by

$$\underline{p}^1 = \underline{g}^{k_1} + \beta_{1,0}\underline{p}^0 \tag{3-46}$$

where $k_1 = \min[k_0+1, N-1] \ni \underline{g}^{k_1} \cap C^R \neq \emptyset$, or $C\underline{p}^1 \neq \underline{0}$. The GS coefficient $\beta_{1,0}$ is determined by invoking the desired C-conjugacy between $\underline{p}^1$ and $\underline{p}^0$

$$\begin{aligned} 0 &= (\underline{p}^1, C\underline{p}^0) \\ &= (\underline{g}^{k_1} + \beta_{1,0}(\underline{p}^0, C\underline{p}^0) \\ &= (\underline{g}^{k_1}, C\underline{p}^0) + \beta_{1,0}(\underline{p}^0, C\underline{p}^0) \end{aligned} \tag{3-47}$$

yielding

$$\beta_{1,0} = -\frac{(\underline{g}^{k_1}, C\underline{p}^0)}{(\underline{p}^0, C\underline{p}^0)} \tag{3-48}$$

which is well-defined, since $(\underline{p}^0, C\underline{p}^0) > 0$ by Corollary 3-2. Proceeding accordingly, the general C-conjugate vector is given by the familiar telescoping series

$$\underline{p}^i = \underline{g}^{k_i} + \sum_{j=0}^{i-1} \beta_{i,j}\underline{p}^j \tag{3-49}$$

where $k_i = \min[k_{i-1}+1, N-1] \ni \underline{g}^{k_i} \cap C^R \neq \emptyset$, or $C\underline{p}^i \neq \underline{0}$. By mutual C-conjugacy of $\underline{p}^i$ and the set of previously defined vectors $\{\underline{p}^j\}_{j=0}^{i-1}$, we can determine the GS coefficients $\{\beta_{i,j}\}_{j=0}^{i-1}$. In fact, for $j \in [0, i-1]$, (3-48) implies that

$$\begin{aligned} 0 &= (\underline{p}^i, C\underline{p}^j) \\ &= (\underline{g}^{k_i} + \beta_{i,j}\underline{p}^j, C\underline{p}^j) \\ &= (\underline{g}^{k_i}, C\underline{p}^j) + \beta_{i,j}(\underline{p}^j, C\underline{p}^j) \end{aligned} \tag{3-50}$$

yielding

$$\beta_{i,j} = -\frac{(\underline{g}^{k_i}, C\underline{p}^j)}{(\underline{p}^j, C\underline{p}^j)} \tag{3-51}$$

where $k_i \in [0, N-1] \ni \underline{g}^{k_i} \cap C^R \neq \emptyset$, $i \in [0, Q-1]$, $j \in [0, Q-2]$ and $i > j$. Note that $Q \leq R = \text{rank } C$, since it takes no more than R linearly independent vectors to span C^R.

Variations on the basic construction approach described above are possible. However, the most interesting such construction is the built-in GS procedure of the CG method developed next.

3.1.2 Derivation of the CG Method

The CG method is a special case of the CD method in which the basis vectors are the iteratively generated gradients $\{\underline{r}^i\}_{i=1}^{Q-1}$ which are used to construct corresponding C-conjugate search vectors $\{\underline{p}^i\}_{i=1}^{Q-1}$ via a procedure similar to that of Lemma 3-8. It turns out that this particular choice of basis vectors leads to a relatively simple expression for the general C-conjugate vector. In contrast to the telescoping complexity (3-43) in the CD method, the general C-conjugate vector in the CG method is given by a first-order recursion involving the current gradient vector and the previous C-conjugate vector as indicated in (2-48.3). This constitutes the major advantage of the CG method over the general CD method which becomes especially *significant with increasing system dimensionality*, N.

The rigorous derivation of the CG method follows logically from the general CD theory already developed, as presented in the following theorem.

<u>Theorem 3-9</u>. [Conventional Form of the CG Method]

Consider the linear system

$$C\underline{w} + \underline{b} = \underline{0} \tag{3-52}$$

where C is an N x N complex Hermitian matrix of rank R, $\underline{b}$ is a complex N-vector contained in C^R, the eigenspace of C, and $\underline{w}$ is the unknown complex N-vector to be determined. Then, a <u>general solution</u> to (3-52) is the last iterate, $\underline{w}^{Q-1}$, of a Q-step iterative procedure that begins with an initial solution estimate, $\underline{w}^0$, and a corresponding residual, $\underline{r}^0 = C\underline{w}^0 + \underline{b}$, and repeats relations

$$\alpha_i = \frac{\|\underline{r}^i\|^2}{(\underline{p}^i, C\underline{p}^i)} \tag{3-53.1}$$

$$\underline{w}^{i+1} = \underline{w}^i - \alpha_i \underline{p}^i \tag{3-53.2}$$

$$\underline{r}^{i+1} = \underline{r}^i - \alpha_i C\underline{p}^i \tag{3-53.2}$$

$$\beta_i = \frac{\|\underline{r}^{i+1}\|^2}{\|\underline{r}^i\|^2} \tag{3-53.4}$$

$$\underline{p}^{i+1} = \underline{r}^{i+1} + \beta_i \underline{p}^i \tag{3-53.5}$$

for $i = 0, 1, \ldots, Q-1$. This procedure will be referred to as the <u>conventional form of the conjugate gradients (CG) method</u>, where conjugate gradients refers to the orthogonal set of gradient basis vectors, $\{\underline{r}^i\}_{i=0}^{Q-1}$, generated.

Proof: Let $\underline{w}^0$ be an arbitrarily chosen initial estimate. If the corresponding residual or gradient vector $\underline{r} = \underline{0}$, $Q = 1$ and the theorem holds trivially. Assume that $\underline{r}^0 \neq \underline{0}$. Then, let the initial C-conjugate vector be defined by

$$\underline{p}^0 = \underline{r}^0 \tag{3-54}$$

By Theorem 3-7, relations (3-40.1) and (3-40.3), it follows that

$$\alpha_0 = \frac{\|\underline{r}^0\|^2}{(\underline{p}^0, C\underline{p}^0)} \tag{3-55.1}$$

$$\underline{r}^1 = \underline{r}^0 - \alpha_0 C\underline{p}^0 \tag{3-55.2}$$

With $\underline{r}^1$ at hand, define the second C-conjugate vector

$$\underline{p}^1 = \underline{r}^1 + \beta_{1,0}\underline{p}^0 \tag{3-56}$$

Then, by (3-54) and Lemma 3-6,

$$0 = (\underline{p}^0, \underline{r}^1)$$

$$= (\underline{r}^0, \underline{r}^1) \qquad (3\text{-}57)$$

implying that the first two gradient vectors, $\underline{r}^0$ and $\underline{r}^1$, are mutually orthogonal. This qualifies then as basic vectors, as desired.

For $\underline{p}^1$ to be an admissible C-conjugate vector, it must be C-orthogonal to $\underline{p}^0$. In fact, if $\underline{r}^1 \neq \underline{0}$, there exists a coefficient $\beta_{1,0}$ such that the C-conjugacy holds nontrivially. Invoking the C-conjugacy condition, and using (3-54) - (3-57), we have

$$0 = (\underline{p}^1, C\underline{p}^0)$$

$$= \frac{1}{\alpha_0} (\underline{r}^1 + \beta_{1,0}\, \underline{p}^0, \underline{r}^0 - \underline{r}^1)$$

$$= \cancelto{0}{(\underline{r}^1, \underline{r}^0)} - \|\underline{r}^1\|^2 + \beta_{1,0}\, \|\underline{r}^0\|^2 - \cancelto{0}{(\underline{p}^0, \underline{r}^1)} \qquad (3\text{-}58)$$

whence,

$$\beta_{1,0} = \frac{\|\underline{r}^1\|^2}{\|\underline{r}^0\|^2} \qquad (3\text{-}59)$$

which is well-defined, since $\underline{r}^0 \neq \underline{0}$.

In view of (3-56) and (3-57),

$$(\underline{p}^1, \underline{r}^1) = (\underline{r}^1 + \beta_{1,0}\, \underline{p}^0, \underline{r}^1)$$

$$= \|\underline{r}^1\|^2 \qquad (3\text{-}60)$$

so that, by Theorem 3-7,

$$\alpha_1 = \frac{\|\underline{r}^1\|^2}{(\underline{p}^1, C\underline{p}^1)} \qquad (3\text{-}61.1)$$

$$\underline{r}^2 = \underline{r}^1 - \alpha_1 C\underline{p}^1 \tag{3-61.2}$$

Now, define the third C-conjugate vector by

$$\underline{p}^2 = \underline{r}^2 + \beta_{2,0}\underline{p}^0 + \beta_{2,1}\underline{p}^1 \tag{3-62}$$

Before proceeding with the actual determination of constants $\beta_{2,0}$ and $\beta_{2,1}$, it is important to establish the orthogonality of $\underline{r}^2$ to $\underline{r}^0$ and $\underline{r}^1$. By (3-54), (3-56) and Lemma 3-6,

$$\begin{aligned} 0 &= (\underline{p}^0,\underline{r}^2) \\ &= (\underline{r}^0,\underline{r}^2) \end{aligned} \tag{3-63.1}$$

$$\begin{aligned} 0 &= (\underline{p}^1,\underline{r}^2) \\ &= (\underline{r}^1 + \beta_{1,0}\underline{p}^0,\underline{r}^2) \\ &= (\underline{r}^1,\underline{r}^2) + \cancel{\beta_{1,0}(\underline{p}^0,\underline{r}^2)} \rightarrow 0 \end{aligned} \tag{3-63.2}$$

which, combined with (3-57), may be summarized as

$$\left.\begin{aligned} &(\underline{r}^i,\underline{r}^j) = 0 \\ &\forall\, i,j \quad [0,2] \;;\; i \neq j \end{aligned}\right\} \tag{3-64}$$

the desired orthogonality property of the first three gradient vectors.

It is now possible to specify (3-61). Invoking the C-conjugacy condition, and using (3-54), (3-55.2), (3-58) and (3-64) we get

$$\begin{aligned} 0 &= (\underline{p}^2,C\underline{p}^0) \\ &= (\underline{r}^2 + \beta_{2,0}\underline{p}^0 + \beta_{2,1}\underline{p}^1,C\underline{p}^0) \\ &= (\underline{r}^2 + \beta_{2,0}\underline{p}^0,\underline{r}^0 - \underline{r}^1) \\ &= \beta_{2,0}\|\underline{r}^0\|^2 \end{aligned} \tag{3-65}$$

implying that

$$\beta_{2,0} = 0 \tag{3-66}$$

Similarly, using (3-60), (3-66) and (3-61.2),

$$\begin{aligned} 0 &= (\underline{p}^2, C\underline{p}^1) \\ &= (\underline{r}^2 + \beta_{2,1}\underline{p}^2, C\underline{p}^2) \\ &= (\underline{r}^2 + \beta_{2,1}\underline{p}^1, \underline{r}^2 - \underline{r}^1) \\ &= \|\underline{r}^2\|^2 + \beta_{2,1}(\underline{p}^1, \underline{r}^1) \\ &= \|\underline{r}^2\|^2 + \beta_{2,1}\|\underline{r}^1\|^2 \end{aligned} \tag{3-67}$$

which leads to

$$\beta_{2,1} = \frac{\|\underline{r}^2\|^2}{\|\underline{r}^1\|^2} \tag{3-68}$$

From (3-66) and (3-68) we can conclude that (3-62) reduces to an expression for $\underline{p}^2$ dependent on $\underline{r}^2$ and $\underline{p}^1$ but not on $\underline{p}^0$. We are thus tempted to conclude, that (3-53.5) holds in general. To prove this assumption, we use induction. Assume that, in the k-th iteration, we have

$$\underline{p}^k = \underline{r}^k + \beta_k \underline{p}^k \tag{3-69.1}$$

$$(\underline{r}^i, \underline{r}^j) = 0 \;,\; \forall\; i,j \in [0,k] \;;\; i \neq j \tag{3-69.2}$$

We wish to show that (3-69) holds for the (k+1)st iteration. Noting that

$$\left.\begin{aligned} &\underline{r}^{\ell+1} = \underline{r}^\ell - \alpha_\ell C\underline{p}^\ell \\ &\forall\; \ell \in [0,k] \end{aligned}\right\} \tag{3-70}$$

by Theorem 3-7, assume that the corresponding C-conjugate vector is given by

$$\underline{p}^{k+1} = \underline{r}^{k+1} + \sum_{\ell=0}^{k} \beta_{k+1,\ell}\underline{p}^{\ell} \tag{3-71}$$

We wish to show that (3-69) implies that the only nonzero coefficient in (3-71) is $\beta_{k+1,k}$ and is defined consistently with (3-53.4).

Letting $\ell \leq k$, Lemma 3-6 implies that

$$\begin{aligned} 0 &= (\underline{p}^{\ell},\underline{r}^{k+1}) \\ &= (\underline{r}^{\ell} + \beta_{\ell,\ell-1}\underline{p}^{\ell-1},\underline{r}^{k+1}) \\ &= (\underline{r}^{\ell},\underline{r}^{k+1}) \end{aligned} \tag{3-72}$$

which extends the orthogonality condition (3-69.2) to the (k+1)st iteration. In addition, using (3-70) and invoking C-conjugacy, we get

$$\begin{aligned} 0 &= (\underline{p}^{k+1},C\underline{p}^{\ell}) \\ &= (\underline{r}^{k+1} + \sum_{\ell=0}^{k} \beta_{k+1}\underline{p}^{\ell},C\underline{p}^{\ell}) \\ &= (\underline{r}^{k+1} + \beta_{k+1,\ell}\underline{p}^{\ell},\underline{r}^{\ell} - \underline{r}^{\ell+1}) \\ &= -(\underline{r}^{k+1},\underline{r}^{\ell+1}) + \beta_{k+1,\ell}(\underline{p}^{\ell},\underline{r}^{\ell}) \\ &= -(\underline{r}^{k+1},\underline{r}^{\ell+1}) + \beta_{k+1,\ell}\|\underline{r}^{\ell}\|^2 \end{aligned} \tag{3-73}$$

which leads to

$$\beta_{k+1,\ell} = \left\{ \begin{array}{ll} \dfrac{\|\underline{r}^{k+1}\|^2}{\|\underline{r}^{k}\|^2} & ; \quad \ell = k \\ \\ 0 & ; \quad \ell \leq k \end{array} \right\} \equiv \beta_k \tag{3-74}$$

In view of (3-69) and (3-74), (3-69) has been extended to the (k+1)st iteration and thus holds in general. As a consequence, the CG procedure (3-53) is valid for i = 0, 1, ..., Q-1 where Q is the iteration number for which $\underline{r}^Q = \underline{0}$. This completes the proof.

∎

As in the case of the general CD method, the CG method has standard and alternate forms as stated in the following two theorems.

Theorem 3-10. [Standard Form of the CG Method]

Given all the hypotheses of Theorem 3-9, a general solution to (3-52) is the last iterate, $\underline{w}^{Q-1}$, of the Q-step iterative procedure that begins with an arbitrary initial solution estimate, $\underline{w}^0$, and the corresponding C-conjugate and residual vectors $\underline{p}^0 = \underline{r}^0 = C\underline{w}^0 + \underline{b}$ and repeats relations

$$\alpha_i = \frac{(\underline{p}^i, \underline{r}^0)}{(\underline{p}^i, C\underline{p}^i)} \tag{3-75.1}$$

$$\underline{w}^{i+1} = \underline{w}^i - \alpha_i \underline{p}^i \tag{3-75.2}$$

$$\underline{r}^{i+1} = \underline{r}^i - \alpha_i C\underline{p}^i \tag{3-75.3}$$

$$\beta_i = \frac{\|\underline{r}^{i+1}\|^2}{\|\underline{r}^i\|^2} \tag{3-75.4}$$

$$\underline{p}^{i+1} = \underline{r}^{i+1} + \beta_i \underline{p}^i \tag{3-75.5}$$

for i = 0, 1, ..., Q-1. This procedure is referred to as the standard form of the conjugate gradients (CG) method.

Proof: Result (3-75) follows from Theorems 3-5 and 3-9.

∎

Theorem 3-11. [Alternate Form of the CG Method]

Given all the hypotheses of Theorem 3-9, a general solution to (3-52) is the last iterate, $\underline{w}^{Q-1}$, of the Q-step iterative procedure that begins with an arbitrary initial solution estimate, $\underline{w}^0$, and the corresponding C-conjugate and residual vectors $\underline{p}^0 = \underline{r}^0 = C\underline{w}^0 + \underline{b}$ and repeats relations

$$\alpha_i = \frac{(\underline{p}^i, \underline{r}^i)}{(\underline{p}^i, C\underline{p}^i)} \tag{3-76.1}$$

$$\underline{w}^{i+1} = \underline{w}^i - \alpha_i \underline{p}^i \tag{3-76.2}$$

$$\underline{r}^{i+1} = \underline{r}^i - \alpha_i C\underline{p}^i \tag{3-76.3}$$

$$\beta_i = \frac{\|\underline{r}^{i+1}\|^2}{\|\underline{r}^i\|^2} \tag{3-76.4}$$

$$\underline{p}^{i+1} = \underline{r}^{i+1} + \beta_i \underline{p}^i \tag{3-76.5}$$

for i = 0, 1, ..., Q-1. This procedure is referred to as the alternate form of the conjugate gradients (CG) method.

Proof: Result (3-76) follows from Theorems 3-7 and 3-9.

3.2 Properties of the CG Method

The CG method has a number of special properties, some of which are discussed below. As in the case of the general CD method, the CG method may be designed to yield a minimum-norm solution as well as an intinsic inverse. Its numerically-efficient, flexible and stable structure characterizes its overall numerical robustness.

3.2.1 Minimum-Norm Solution

The condition under which the CG method yields the unique minimum-norm solution to (3-52) is stated in the following theorem.

Theorem 3-12. [Condition for Minimum-Norm CG Solution to (3-52)]

The CG method as presented in Theorems 3-9, 3-10 and 3-11 yield the unique minimum-norm solution (2-42), of system (3-52), when $\underline{w}^0 \in C^R$; that is, when the initial estimate is contained within the eigenspace of the N x N Hermitian matrix C in (3-52).

Proof: Note that, by (2-18), $\underline{b} \in C^R$. Then, by definition (3-33), it follows that $\underline{p}^0$, $\underline{r}^0 \in C^R$ for any initial estimate, $\underline{w}^0$. In fact, by definition (3-33), all vectors in the CG procedure are contained in C^R, with the exception of $\underline{w}^{i+1}$. A choice $\underline{w}^0 \in C^R$ guarantees that $\underline{w}^{i+1} \in C^R$. Since $\underline{r}^{Q-1} = \underline{0}$ for some $Q \leq \text{rank } C \leq N$, $\underline{w}^{Q-1} = \hat{\underline{w}}$, the unique minimum-norm solution (2-42).

3.2.2 Intrinsic Inverse

The CG method yields an intrinsic inverse as stated in the following theorem.

Theorem 3-13. [The CG Intrinsic Inverse of Matrix C in (3-52)]

The intrinsic inverse, C^x, of matrix C in (3-52) is given by

$$C^x = \sum_{i=0}^{Q-1} \frac{\underline{p}^i \underline{p}^{i*T}}{(\underline{p}^i, C\underline{p}^i)} \tag{3-77}$$

Proof: Let $\underline{w}^0 = \underline{0}$ in Theorem 3-10, whence $\underline{r}^0 = \underline{b}$. By Theorem 3-12, $\underline{w}^{Q-1} = \hat{\underline{w}}$, the minimum-norm solution. Then, using (3.75.1) and (3-75.2), we see that

$$\hat{\underline{w}} = -\sum_{i=0}^{Q-1} \alpha_i \underline{p}^i$$

$$= -\sum_{i=0}^{Q-1} \frac{(\underline{p}^i,\underline{b})}{(\underline{p}^i,C\underline{p}^i)}\,\underline{p}^i$$

$$= -\sum_{i=0}^{Q-1} \frac{\underline{p}^i\underline{p}^{i*T}}{(\underline{p}^i,C\underline{p}^i)}\,\underline{b}$$

$$= -\,C^x\underline{b} \tag{3-78}$$

where C^x is given by (3-77).

It should be noted that the properties of the CD intrinsic inverse discussed in Section 2.2.2.3 apply equally well to the CG intrinsic inverse.

3.2.3 Numerical Robustness

It has been shown that the CG method is a computationally efficient form of the general CD method. As a consequence, a finite-word machine computation using the CG method will yield a smaller roundoff error in the final solution. However, the most important feature of the general CD method and, hence, the CG method, is its inherent numerical stability. That is, each new weight-vector iterate, $\underline{w}^{i+1}$, reduces the value of the combined power metric

$$P_c(\underline{w}) = \|\underline{s}^T\underline{w} + s_0\|^2$$

$$= (\underline{w},\langle\underline{s},\underline{s}^T\rangle\underline{w}) + 2\mathcal{Re}(\underline{w},\langle\underline{s},s_0\rangle) + \|s_0\|^2$$

$$= (\underline{w},C\underline{w}) + 2\mathcal{Re}(\underline{w},\underline{b}) + P_0 \tag{3-79}$$

where P_0 is the main-port power over an M-sample batch of signal data.

Noting that $P_C(\underline{w})$ is a nonnegative-real quadratic function of $\underline{w}$ that represents a hyperparaboloidal surface in (2N+1)-space, its iterative reduction with each new weight estimate may be viewed geometrically as a descent process on the hypersurface along C-conjugate search directions. This descent-property which guarantees the numerical stability of the CG method is demonstrated in the following theorem.

Theorem 3-14. [Descent Property of the CG Method]

Let $\underline{w}$ be the current estimate of some iteration in the CG procedure and consider the variation of $P_c(\underline{w})$ along the cord

$$\underline{w}' = \underline{w} - \alpha\underline{p} \tag{3-80}$$

where α is a scalar factor. Then, $P_c(\underline{w}')$ attains a unique minimum when

$$\alpha = \frac{(\underline{p},\underline{r})}{(\underline{p},C\underline{p})} \tag{3-81.1}$$

$$= \frac{\|\underline{r}\|^2}{(\underline{p},C\underline{p})} \tag{3-81.2}$$

It is then said that $P_c(\underline{w}')$ has been relaxed optimally along $-\underline{p}$.

Proof: By (3-79) and (3-80)

$$P_c(\underline{w}') = (w',C\underline{w}') + 2Re(\underline{w}',\underline{b}) + P_0$$

$$= (\underline{w} - \alpha\underline{p},C(\underline{w} - \alpha\underline{p})) + 2Re(\underline{w} - \alpha\underline{p},\underline{b}) + P_0$$

$$= |\alpha|^2(\underline{p},C\underline{p}) - 2Re\{\alpha^*(\underline{p},C\underline{w} + \underline{b})\} + P_c(\underline{w})$$

$$= |\alpha|^2(p,C\underline{p}) - 2Re\{\alpha^*(\underline{p},\underline{r})\} + P_c(\underline{w}) \tag{3-82}$$

where α is assumed to be complex, in general. The value of α that minimizes (3-82) may be found by setting its "complex derivative" with respect to α equal to zero; i.e.,

$$0 = \frac{d}{d\alpha} P_c(\underline{w}')$$

$$= \left(\frac{d}{d\alpha_r} + j\frac{d}{d\alpha_i}\right) P_c(\underline{w}')$$

$$= \alpha(\underline{p},C\underline{p}) - (\underline{p},\underline{r}) \tag{3-83}$$

which, for the CG case, leads to the desired result (3-81), where α is nonnegative real by Corollary 3-2. In the general CD case where (3-83) also applies, α may be complex.

Upon substituting (3-81.1) in (3-82), we get

$$P_c(\underline{w}') = \left|\frac{(\underline{p},\underline{r})}{(\underline{p},C\underline{p})}\right|^2 (p,Cp) - 2\mathcal{Re}\ \frac{(\underline{p},\underline{r})^*(\underline{p},\underline{r})}{(\underline{p},C\underline{p})} + P_c(\underline{w})$$

$$= P_c(\underline{w}) - \frac{|(\underline{p},\underline{r})|^2}{(\underline{p},C\underline{p})}$$

$$\leq P_c(w)$$

the desired descent property.

As mentioned previously, the CG procedure will terminate at an iteration when $\|\underline{r}\| < \varepsilon \|\underline{b}\|$, where ε is a positive real scalar on the order of the precision of the sampled signal data and the machine employed. For example, if the data used is accurate to 8 bits, it is reasonable to chose $\varepsilon = 2^{-8}$, assuming the machine computation is sufficiently precise to offset any significant roundoff error buildup.

Of course, with increasing dimensionality N, it will be less and less likely that the roundoff error could be contained well enough in order that the above stopping criterion make sense. It has been shown [2] that the roundoff error buildup is worst in the standard CG method defined by (3-75). The conventional form (3-53) will reduce this error.

A further improvement may be realized in the alternate form (3-76). Finally, additional accuracy could be achieved if the GS coefficient is redefined by

$$\beta_i = -\frac{(\underline{r}^{i+1},C\underline{p}^i)}{(\underline{p}^i,C\underline{p}^i)} \tag{3-84}$$

where (3-53.1) and (3-76.3) have been used to modify (3.76.4).

As N becomes larger and larger, even these measures become less effective in containing the roundoff error to a sufficiently low level. If, in fact, the roundoff error is so large that the stopping criterion is far from being satisfied, it is always possible to use the final weight estimate as an initial estimate in a second CG execution. Then, the roundoff error associated with the final estimate during the first execution could be significantly reduced during the second.

Clearly, the CG method possesses an efficient iterative structure with an inherent numerical stability and the capability of minimizing roundoff error. For these reasons, it is a numerically-robust computational procedure.

3.2.4 Cancellation Ratio Computation

In connection with the sidelobe cancellation system considered, the combined-port signal power, $P_C(\underline{w})$, over an M-sample batch has been given in (3-79). This expression may be rewritten as

$$\begin{aligned} P_C(\underline{w}) &= (\underline{w},C\underline{w}) + (\underline{w},\underline{b}) + (\underline{w},\underline{b})^* + P_0 \\ &= (\underline{w},\underline{r}) + (\underline{w},\underline{b})^* + P_0 \end{aligned} \tag{3-85}$$

since $\underline{r} = C\underline{w} + \underline{b}$. Note that (3-85) is completely defined by the main-port power, P_0, the cross-correlation vector, $\underline{b}$, and CG variables $\underline{w}$ and $\underline{r}$. As such, (3-85) is in a convenient form to compute the cancellation ratio as part of the BCR process. Specifically, this ratio is given by

$$CR(\underline{w}) = 10 \log_{10} \left[\frac{P_0}{P_0 + (\underline{w},\underline{b})^* + (\underline{w},\underline{r})} \right] \tag{3-86}$$

If (3-86) were appended to the BCR process as a performance monitor, it would provide a numerically-reliable stopping criterion. In fact, by the descent property of Theorem 3-14, we should observe an increase in $CR(\underline{w})$ with each new iterate, $\underline{w}$. When $CR(\underline{w})$ stops decreasing appreciably or increases because of excessive roundoff error, the process may be terminated using the present or previous value of $\underline{w}$, as appropriate, and reinitiated as necessary.

3.3 Illustrative Examples

The overall understanding of the CG procedure may be enhanced by actually working out specific numerical examples. Below, we consider two simple examples of a two-dimensional system (3-1). The first example involves a full-rank complex Hermitian matrix C. It is solved by three different methods: inversion, CD and CG. The second example involves a real-valued singular matrix C. It is solved by regularized inversion, pseudoinversion, CD and CG. In as much as it is possible, the features of the CG method are illustrated quantitatively.

3.3.1 Full-Rank Covariance Case

Consider the complex system

$$A\underline{w} + \underline{a} = \underline{0} \tag{3-87}$$

where

$$\left.\begin{aligned} A &= \begin{pmatrix} 1 & j \\ 1 & 1 \end{pmatrix} \\ \underline{a} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{aligned}\right\} \tag{3-88}$$

We wish to determine $\underline{w}$ which minimizes the performance index

$$J(\underline{w}) = \|A\underline{w} + \underline{a}\|^2 \tag{3-89}$$

that is, the value of $\underline{w}$ at which the "complex gradient" $\nabla_{\underline{w}} J(\underline{w})$ vanishes. This leads to the linear system

$$C\underline{w} + \underline{b} = \underline{0} \tag{3-90}$$

where

$$C = A^{*T}A$$

$$= \begin{pmatrix} 1 & 1 \\ -j & 1 \end{pmatrix} \begin{pmatrix} 1 & j \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1+j \\ 1-j & 2 \end{pmatrix} \qquad (3\text{-}91)$$

$$\underline{b} = A^{*T}\underline{a}$$

$$= \begin{pmatrix} 1 & 1 \\ -j & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -j \end{pmatrix} \qquad (3\text{-}92)$$

We will solve (3-90) for $\underline{w}$ by the following three methods.

<u>Inversion Solution</u>:

By inspection,

$$C^{-1} = \frac{1}{2} \begin{pmatrix} 2 & -(1+j) \\ -(1-j) & 2 \end{pmatrix} \qquad (3\text{-}93)$$

so that, the desired unique solution to (3-90) is given by

$$\underline{w} = -C^{-1}\underline{b}$$

$$= -\frac{1}{2} \begin{pmatrix} 2 & -(1+j) \\ -(1-j) & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

$$= -\frac{1}{2}\begin{pmatrix} 1+j \\ -(1+j) \end{pmatrix}$$

$$= -\frac{1+j}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad (3\text{-}94)$$

which happens to satisfy (3-87) and certainly (3-90).

CD Solution:

Below we will use Theorem 3-7 and Lemma 3-8. First, choose basis vectors

$$\underline{q}^0 = [1 \quad 0]^T \qquad (3\text{-}95.1)$$

$$\underline{q}^1 = [0 \quad 1]^T \qquad (3\text{-}95.2)$$

Then, construct C-conjugate vectors $\underline{p}^0$ and $\underline{p}^1$ according to Theorem 3-9, as follows:

$$\underline{p}^0 = \underline{q}^0 \qquad (3\text{-}96.1)$$

$$\underline{p}^1 = \underline{q}^1 + \beta_0 \underline{p}^0 \qquad (3\text{-}96.2)$$

where, by (3-44)

$$\beta_0 = -\frac{(\underline{q}^1, C\underline{p}^0)^*}{(\underline{p}^0, C\underline{p}^0)}$$

$$= -\frac{(1 + j)}{2} \qquad (3\text{-}97)$$

since, by (3-95) and (3-96)

$$(\underline{q}^1, C\underline{p}^0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}^T \begin{pmatrix} 2 \\ 1-j \end{pmatrix} = 1-j$$

$$(\underline{p}^0, C\underline{p}^0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} 2 \\ 1-j \end{pmatrix} = 2$$

Hence, by (3-96),

$$\underline{p}^0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3-98.1}$$

$$\underline{p}^1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \frac{1+j}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} -(1+j) \\ 2 \end{pmatrix} \tag{3-98.2}$$

Letting $\underline{w}^0 = \underline{0}$ and thus $\underline{r}^0 = \underline{b}$, we follow the CD process as defined by (3-40). In fact,

$$\alpha_0 = \frac{(\underline{p}^0, \underline{r}^0)}{(\underline{p}^0, C\underline{p}^0)}$$

$$= \frac{1}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

$$= \frac{1}{2} \tag{3-99}$$

and

$$\underline{r}^1 = \underline{r}^0 - \alpha_0 C\underline{p}^0$$

$$= \begin{pmatrix} 1 \\ -j \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 2 \\ 1-j \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 0 \\ -(1+j) \end{pmatrix} \qquad (3\text{-}100)$$

Since

$$C\underline{p}^1 = \frac{1}{2} \begin{pmatrix} 2 & 1+j \\ 1-j & 2 \end{pmatrix} \begin{pmatrix} -(1+j) \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

then

$$\alpha_1 = \frac{(\underline{p}^1, \underline{r}^1)}{(p^1, C\underline{p}^1)}$$

$$= \frac{1}{4} \begin{pmatrix} -(1-j) \\ 2 \end{pmatrix}^T \begin{pmatrix} 0 \\ -(1+j) \end{pmatrix}$$

$$= \frac{-1}{2} (1+j) \qquad (3\text{-}101)$$

and

$$\underline{r}^2 = \underline{r}^1 - \alpha_1 C\underline{p}^1$$

$$= \frac{1}{2} \begin{pmatrix} 0 \\ -(1+j) \end{pmatrix} + \frac{1}{2} (1+j) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (3\text{-}102)$$

Finally, the desired unique solution is given by

$$\underline{\hat{w}} = \underline{w}^0 - \alpha_0 \underline{p}^0 - \alpha_1 \underline{p}^1$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{4}(1+j)\begin{pmatrix} -(1+j) \\ 2 \end{pmatrix}$$

$$= -\frac{1+j}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{3-103}$$

which is identical to the inversion solution, (3-94).

In this simple example it is possible to cite, specifically, a number of features that characterize the CD procedure. By (3-101), we have substantiated the fact that, in the CD method, the relaxation coefficient, α, is generally complex - an observation made in the proof of Theorem 3-14.

Another important property of the general CD method is the diagonalization of matrix C via the transformation.

$$D = P^{*T}CP$$

$$= (\underline{p}^0 \ p^1)^{*T} C(\underline{p}^0 \ \underline{p}^1)$$

$$= \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \tag{3-104}$$

which is simply a manifestation of the C-conjugacy property. It should be emphasized here that $\underline{p}^0$ and $\underline{p}^1$ are not eigenvectors of C, and D is not its eigenvalue matrix. Actually, the eigenvalues of C are $2 + \sqrt{2}$ and $2 - \sqrt{2}$.

The descent-property may be quantified by direct substitution of $\underline{w}^0$, $\underline{w}^2 = \underline{w}^0 - \alpha_0\underline{p}^0$ and $\underline{w}^2 = \hat{\underline{w}}$ into the quadratic performance index (3-89). The three corresponding successive values of (3-89) are found to be

$$J_0 = J(\underline{w}^0)$$

$$= ||\underline{a}||^2$$

$$= 1 \tag{3-105.1}$$

$$J_1 = J(\underline{w}^1)$$

$$= \| A\underline{w}^1 + \underline{a} \|^2$$

$$= \left\| -\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix} \right\|^2$$

$$= \left\| \frac{1}{2} \begin{pmatrix}1\\-1\end{pmatrix} \right\|^2$$

$$= \frac{1}{2} \tag{3-105.2}$$

which, by (3-89), could have been computed by the equivalent relation

$$J_1 = (\underline{w}^1,\underline{r}^1) + (\underline{w}^1,\underline{b})^* + J_0 \tag{3-105.3}$$

Since we have already shown that

$$J_2 = J(\underline{w}^2) = 0 \tag{3-105.4}$$

we have illustrated the descent property for the general CD method; namely, $J_0 > J_1 > J_2$.

A final property of the CD method which may be demonstrated in the present example is the construction of the intrinsic inverse, C^x. By (3-77),

$$C^x = \frac{\underline{p}^0\underline{p}^{0*T}}{(\underline{p}^0,C\underline{p}^0)} - \frac{\underline{p}^1\underline{p}^{1*T}}{(\underline{p}^1,C\underline{p}^1)}$$

$$= \frac{1}{2}\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} - \frac{1}{4}\begin{pmatrix}2 & -2(1+j)\\-2(1-j) & 4\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}2 & -(1+j)\\-(1-j) & 2\end{pmatrix} \tag{3-106}$$

Note that, in this case, $C^x = C^{-1}$, the inverse given in (3-93).

CG Solution:

Here, we will use the conventional form of the CG method as given in (3-53) of Theorem (3-9). Starting with initial conditions,

$$\left.\begin{aligned} \underline{w}^0 &= \underline{0} \\ \underline{p}^0 &= \underline{r}^0 = \underline{b} = \begin{pmatrix} 1 \\ -j \end{pmatrix} \end{aligned}\right\}. \qquad (3\text{-}107)$$

and $\|\underline{r}^0\|^2 = 2$, we proceed with the first iteration. Since

$$C\underline{p}^0 = \begin{pmatrix} 2 & 1+j \\ 1-j & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -j \end{pmatrix} = \begin{pmatrix} 3-j \\ 1-3j \end{pmatrix}$$

$$(\underline{p}^0, C\underline{p}^0) = \begin{pmatrix} 1 \\ j \end{pmatrix}^T \begin{pmatrix} 3-j \\ 1-3j \end{pmatrix} = 6$$

then,

$$\alpha_0 = \frac{\|r^0\|^2}{(\underline{p}^0, C\underline{p}^0)}$$

$$= \frac{1}{3} \qquad (3\text{-}108)$$

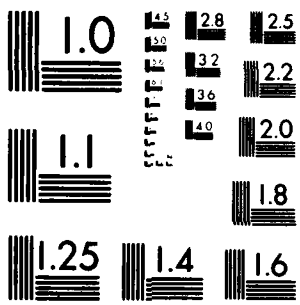
$$\underline{w}^1 = \underline{w}^0 - \alpha_0 \underline{p}^0$$

$$= \underline{0} - \frac{1}{3}\begin{pmatrix} 1 \\ -j \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} -1 \\ j \end{pmatrix} \qquad (3\text{-}109)$$

AD-A109 927 MOTOROLA INC TEMPE AZ GOVERNMENT ELECTRONICS DIV F/G 20/14
BATCH COVARIANCE RELAXATION (BCR) ADAPTIVE PROCESSING.(U)
AUG 81 S M DANIEL, I KERTESZ F30602-80-C-0031
UNCLASSIFIED 8512-F RADC-TR-81-212 NL

3 of 7
AD A109927



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

$$\underline{r}^1 = \underline{r}^0 - \alpha_0 C\underline{p}^0$$

$$= \begin{pmatrix} 1 \\ -j \end{pmatrix} - \frac{1}{3} \begin{pmatrix} 3-j \\ 1-3j \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 1 \\ -j \end{pmatrix} \tag{3-110}$$

$$\beta_0 = \frac{\|\underline{r}^1\|^2}{\|\underline{r}^0\|^2}$$

$$= \left(\frac{2}{9}\right) \left(\frac{1}{2}\right)$$

$$= \frac{1}{9} \tag{3-111}$$

and

$$\underline{p}^1 = \underline{r}^1 + \beta_0 \underline{p}^0$$

$$= \frac{1}{3}\begin{pmatrix} j \\ -1 \end{pmatrix} + \frac{1}{9} \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

$$= \frac{1}{9} \begin{pmatrix} 1+3j \\ -(3+j) \end{pmatrix} \tag{3-112}$$

In the second iteration, since

$$C\underline{p}^1 = \frac{1}{9} \begin{pmatrix} 2 & 1+j \\ 1-j & 2 \end{pmatrix} \begin{pmatrix} 3j+1 \\ -(3+j) \end{pmatrix}$$

$$= \frac{1}{9} \begin{pmatrix} 2(3j+1) - (1+j)(3+j) \\ (1-j)(3j+1) - 2(3+j) \end{pmatrix}$$

$$= \frac{2}{9} \begin{pmatrix} j \\ -1 \end{pmatrix}$$

$$(\underline{p}^1, C\underline{p}^1) = \frac{2}{81}\begin{pmatrix}1-3j\\-3+j\end{pmatrix}^T\begin{pmatrix}j\\-1\end{pmatrix}$$

$$= \left(\frac{2}{81}\right)6$$

$$= \frac{4}{27}$$

then,

$$\alpha_1 = \frac{\|\underline{r}^1\|^2}{(\underline{p}^1, C\underline{p}^1)}$$

$$= \left(\frac{2}{9}\right)\left(\frac{27}{4}\right)$$

$$= \frac{3}{2} \qquad (3\text{-}113)$$

$$\underline{w}^2 = \underline{w}^1 - \alpha_1\underline{p}^1$$

$$= \frac{1}{3}\begin{pmatrix}-1\\j\end{pmatrix} - \left(\frac{3}{2}\right)\left(\frac{1}{9}\right)\begin{pmatrix}1+3j\\-(j+3)\end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix}-1\\j\end{pmatrix} - \frac{1}{6}\begin{pmatrix}1+3j\\-3-j\end{pmatrix}$$

$$= \frac{1}{6}\begin{pmatrix}-3-3j\\3+3j\end{pmatrix}$$

$$= \frac{1+j}{2}\begin{pmatrix}1\\-1\end{pmatrix} \qquad (3\text{-}114)$$

$$\underline{r}^2 = \underline{r}^1 - \alpha_1 C\underline{p}^1$$

$$= \frac{1}{3}\begin{pmatrix} j \\ -1 \end{pmatrix} - \left(\frac{3}{2}\right)\left(\frac{2}{9}\right)\begin{pmatrix} j \\ -1 \end{pmatrix}$$

$$= \underline{0} \qquad (3\text{-}115)$$

The CG process is terminated at this point of the second iteration, since $\|\underline{r}^2\| = \underline{0}$. Obviously, $\hat{\underline{w}} = \underline{w}^2$, which agrees with previous results, (3-94) and (3-103).

As a consequence of this example, it is possible to cite some specific properties of the CG method. In contrast to the CD method, the relaxation coefficient, α, in the CG method is always real.

As expected, the C-conjugacy property in the CG method gives rise to the diagonalization of matrix C via the transformation

$$D = P^{*T}CP$$

$$= (\underline{p}^0\ \underline{p}^1)^{*T}C(\underline{p}^0\ \underline{p}^1)$$

$$= \begin{pmatrix} 6 & 0 \\ 0 & 4/27 \end{pmatrix} \qquad (3\text{-}116)$$

Note that this value of D is different from that of (3-104) in the CD solution. Similarly, $\underline{p}^0$ and $\underline{p}^1$ are not eigenvectors of C, and D is not its eigenvalue matrix.

The descent property of the CG method may also be demonstrated. At the initial estimate, $\underline{w}^0 = \underline{0}$, the performance index value is

$$J_0 = 1 \qquad (3\text{-}117.1)$$

as in the CD case. At the first iterate, $\underline{w}^1$,

$$J_1 = J(\underline{w}^1)$$

$$= \| A\underline{w}^1 + \underline{a} \|^2$$

$$= \left\| \frac{1}{3}\begin{pmatrix} 1 & j \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ j \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\|^2$$

$$= \left\| \frac{1}{3}\begin{pmatrix} -2 \\ -1+j \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\|^2$$

$$= \left\| \frac{1}{9}\begin{pmatrix} 1 \\ -1+j \end{pmatrix} \right\|^2$$

$$= \frac{1}{3} \tag{3-117.2}$$

or, equivalently, by (3-89)

$$J^1 = (\underline{w}^1,\underline{r}^1) + (\underline{w}^1,\underline{b})^* + J_0$$

$$= \frac{1}{9}\begin{pmatrix} -1 \\ -j \end{pmatrix}^T \begin{pmatrix} j \\ -1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -1 \\ j \end{pmatrix}^T \begin{pmatrix} 1 \\ j \end{pmatrix} + 1$$

$$= 0 - \frac{2}{3} + 1$$

$$= \frac{1}{3} \tag{3-117.2'}$$

and, as before,

$$J_2 = 0 \tag{3-117.3}$$

Clearly, $J_0 > J_1 > J_2$, as in the CD case. In comparison, J_1 is smaller in the CG case because the relaxation is along $-\underline{r}^0$, the direction of steepest descent.

The CG intrinsic inverse may now be computed by using (3-77). In fact,

$$C^x = \frac{\underline{p}^0\underline{p}^{0*T}}{(\underline{p}^0, C\underline{p}^0)} + \frac{\underline{p}^1\underline{p}^{1*T}}{(\underline{p}^1, C\underline{p}^1)}$$

$$= \frac{1}{6}\begin{pmatrix} 1 \\ -j \end{pmatrix}\begin{pmatrix} 1 & j \end{pmatrix} + \left(\frac{27}{4}\right)\left(\frac{1}{81}\right)\begin{pmatrix} 1+3j \\ -(3+j) \end{pmatrix}\begin{pmatrix} 1-3j & -(3-j) \end{pmatrix}$$

$$= \frac{1}{6}\begin{pmatrix} 1 & j \\ -j & 1 \end{pmatrix} + \frac{1}{12}\begin{pmatrix} 10 & -6-8j \\ -6+8j & 10 \end{pmatrix}$$

$$= \frac{1}{12}\begin{pmatrix} 12 & -6(1+j) \\ -6(1-j) & 12 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 & -(1+j) \\ -(1-j) & 2 \end{pmatrix} \tag{3-118}$$

which happens to be the inverse, C^{-1}, in (3-93).

It should be noted that the CG procedure took 2 iterations to produce the solution, which corresponds to the rank matrix C. The probability that it could have terminated in one iteration is zero since the eigenvalues of C are distinct and $\underline{b}$ is a linear combination of the two associated eigenvectors. The latter claim follows directly from the initial discussion in Section 2.2.2.3.

3.3.2 Singular Covariance Case

Consider next a simple example of a singular system (3-87), where

$$\left.\begin{aligned} A &= \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \\ \underline{a} &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned}\right\} \tag{3-119}$$

We wish to solve (3-90), where

$$C = 2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{3-120}$$

$$\underline{b} = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{3-121}$$

Since C is clearly singular, an inversion solution is not possible. Below, we will solve (3-90) with the C and $\underline{b}$ values given above via the approximate <u>regularized inverse</u>, the <u>pseudoinverse</u>, <u>CD</u> and <u>CG</u>.

<u>Regularized Inversion Schedule</u>:

With reference to Section 2.2.1.1, define the <u>regularized matrix C</u> by

$$\tilde{C} = 2\begin{pmatrix} 1+\varepsilon & 1 \\ 1 & 1+\varepsilon \end{pmatrix} \tag{3-122}$$

Then, the <u>regularized inverse</u> is simply

$$\tilde{C}^{-1} = \frac{1}{2\varepsilon(2+\varepsilon)}\begin{pmatrix} 1+\varepsilon & -1 \\ -1 & 1+\varepsilon \end{pmatrix} \tag{3-123}$$

whence,

$$\begin{aligned} \underline{\tilde{w}} &= - \tilde{C}^{-1}\underline{b} \\ &= - \frac{1}{(2+\varepsilon)}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned} \tag{3-124}$$

At this regularized inverse solution, the performance index (3-89) becomes

$$\begin{aligned} J(\underline{\tilde{w}}) &= \| A\underline{\tilde{w}} + \underline{a} \|^2 \\ &= \left\| - \frac{1}{2+\varepsilon}\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\|^2 \end{aligned}$$

$$= \left(\frac{1}{\varepsilon+2}\right)^2 \quad 2\varepsilon^2$$

$$= 2\left(\frac{\varepsilon}{\varepsilon+2}\right)^2$$

$$\approx \frac{\varepsilon^2}{2} \tag{3-125}$$

when $\varepsilon \ll 1$. *Note that*

$$\lim_{\varepsilon \to 0} \underline{\tilde{w}} = -\frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{3-126}$$

at which value $J(\underline{\tilde{w}}) = 0$. Clearly then, *(3-126) is the* <u>*minimum-norm*</u> <u>solution</u> which may be obtained by pseudoinversion according to Theorem 2-1.

<u>Pseudoinversion Solution</u>:

The eigenvalues of C, given by (3-120), satisfy the characteristic equation

$$\begin{aligned} 0 &= \phi(\lambda) \\ &= \det |C-\lambda I| \\ &= (\lambda-2)^2 - 4 \\ &= \lambda^2 - 4\lambda \end{aligned} \tag{3-127}$$

The actual eigenvalues are $\lambda_1 = 4$ and $\lambda_2 = 0$, the latter indicative of the singularity of C. The corresponding eigenvectors are $\underline{e}^1 = (1/\sqrt{2})[1 \;\; 1]^T$ $\underline{e}^2 = (1/\sqrt{2})[1 \;\; -1]^T$. Then, by (2-26) *in Theorem 2-1*

$$\begin{aligned} C &= E\Lambda^{+}E^{*T} \\ &= 2\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \end{aligned} \tag{3-128}$$

and by (2-30), the <u>pseudoinverse of C</u> is given by

$$C^{+} = E\Lambda^{+}E^{*T}$$

$$= \frac{1}{8}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

$$= \frac{1}{8}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}$$

$$= \frac{1}{8}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} \qquad (3\text{-}129)$$

so that, by (2-32), the minimum-norm solution is

$$\underline{\hat{Q}} = -\,C^{+}\underline{b}$$

$$= -\,\frac{1}{4}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$= -\,\frac{1}{2}\begin{pmatrix}1\\ 1\end{pmatrix} \qquad (3\text{-}130)$$

which agrees with the limiting regularized solution (3-126).

CD Solution:

To solve (3-90) with $C,\underline{b}$ as defined in (3-120) and (3-121), we will use the basis vectors (3-95) and C-conjugate vector expressions (3-96). As in the previous example, since

$$(\underline{q}^{1}, C\underline{p}^{0}) = 2\begin{pmatrix}0\\ 1\end{pmatrix}^{T}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = 2$$

$$(\underline{p}^{0}, C\underline{p}^{0}) = 2\begin{pmatrix}1\\ 0\end{pmatrix}^{T}\begin{pmatrix}1\\ 1\end{pmatrix} = 2$$

then, by (3-44)

$$\beta_0 = -\frac{(\underline{q}^1, C\underline{p}^0)^*}{(\underline{p}^0, C\underline{p}^0)}$$

$$= -1 \qquad (3\text{-}131)$$

so that, by (3-96)

$$\underline{p}^0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (3\text{-}132.1)$$

$$\underline{p}^1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad (3\text{-}132.2)$$

With $\underline{w}^0 = \underline{0}$ and thus $\underline{r}^0 = \underline{b}$, we follow CD process (3-40):

$$\alpha_0 = \frac{(\underline{p}^0, \underline{r}^0)}{(\underline{p}^0, C\underline{p}^0)}$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T 2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= 1 \qquad (3\text{-}133)$$

and

$$\underline{r}^1 = \underline{r}^0 - \alpha_0 C\underline{p}^0$$

$$= 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \underline{0} \qquad (3\text{-}134)$$

at which point, the CD process terminates. The one-iteration solution is then

$$\hat{\underline{w}} = \underline{w}^0 - \alpha_0 \underline{p}^0$$
$$= -\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (3\text{-}135)$$

Note that this solution is a generalized solution and not the minimum-norm solution (3-130). In fact, the norm of solution (3-135) is 1, which is greater than the minimum-norm of $\sqrt{2}/2$. Of course, either solution satisfies (3-90) with C and $\underline{b}$ values given in (3-120) and (3-121).

As expected, even in this singular example, the C-conjugacy property leads to the diagonalization of matrix C via the transformation

$$D = P^{*T} C P$$
$$= (\underline{p}^0 \ \underline{p}^1)^{*T} C (\underline{p}^0 \ \underline{p}^1)$$
$$= 2\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$
$$= 2\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \qquad (3\text{-}136)$$

The fact that 2 is the nontrivial eigenvalue of C in the present example is coincidental with the choice of basis vectors in the CD process. Certainly, $\underline{p}^0$ and $\underline{p}^1$ are not eigenvectors of C.

The descent property is obvious in this case. Since $J_0 = 2$ and $J_1 = 0$, $J_0 > J_1$ holds.

The CD intrinsic inverse, C^x, as defined by (3-77) is computed to be

$$C^x = \frac{\underline{p}^0 \, \underline{p}^{0*T}}{(\underline{p}^0, C\underline{p}^0)}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{3-137}$$

which, interestingly, is twice as large as the pseudoinverse (3-129). Note that since C^x is not equal to the pseudoinverse, it is a generalized inverse. If $\underline{p}^0$ were chosen equal to the nontrivial eigenvector $\underline{e}^1 = [1 \quad 1]^T$, C^x would be identical to the pseudoinverse.

CG Solution:

Starting with initial conditions

$$\left.\begin{aligned} \underline{w}^0 &= \underline{0} \\ \underline{p}^0 &= \underline{r}^0 = \underline{b} \end{aligned}\right\} \tag{3-138}$$

and $\|\underline{r}^0\|^2 = 8$, we solve (3-90) for the C and $\underline{b}$ values defined in (3-120) and (3-121). Since

$$C\underline{p}^0 = 4\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 8\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$(\underline{p}^0, C\underline{p}^0) = 16\begin{pmatrix} 1 \\ 1 \end{pmatrix}^T \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 32 \tag{3-139}$$

then,

$$\alpha_0 = \frac{\|\underline{r}^0\|^2}{(\underline{p}^0, C\underline{p}^0)}$$

$$= \frac{1}{4} \tag{3-140}$$

$$\underline{w}^1 = \underline{w}^0 - \alpha_0 \underline{p}^0$$

$$= -\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} \tag{3-141}$$

$$\underline{r}^1 = \underline{r}^0 - \alpha_0 C\underline{p}^0$$

$$= 2\begin{pmatrix}1\\1\end{pmatrix} - \frac{1}{4}(8)\begin{pmatrix}1\\1\end{pmatrix}$$

$$= 0 \tag{3-142}$$

at which point the CG process terminates. Clearly, $\hat{\underline{w}} = \underline{w}^1$, which happens to be the minimum-norm solution, (3-130). This is as expected, since $\underline{w}^0 = \underline{0} \in \overline{C^R}$.

As expected, the C-conjugacy property leads to the diagonalization of matrix C; i.e.,

$$D = P^{*T}CP$$

$$= (\underline{p}^0 \ \underline{p}^1)^{*T}C(\underline{p}^0 \ \underline{p}^1)$$

$$= \begin{pmatrix}32 & 0\\0 & 0\end{pmatrix} \tag{3-143}$$

The descent property is obvious since $J_0 = 2$ and $J_1 = 0$ and $J_0 > J_1$ holds.

The CG intrinsic inverse is given by

$$C^x = \frac{\underline{p}^0 \ \underline{p}^{0*T}}{(\underline{p}^0, C\underline{p}^0)}$$

$$= \frac{1}{8}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} \tag{3-144}$$

which happens to be identical to the pseudoinverse, (3-129). Again, this is as expected, since $\underline{w}^0 = \underline{0}$ and $\underline{p}^0 = \underline{b} \in \overline{C^R}$.

4.0 CONSTRAINED BATCH ADAPTIVE PROCESSING

Recall that, in the sidelobe cancellation system under consideration, a number of omnidirectional auxiliary antenna elements are adaptively weighted and combined with an established narrow-beam main antenna array. In so doing, it is inevitable that the original mainbeam gain will not remain intact. While this effect is expected when undesired sidelobe interference is present, it could be more pronounced if a desired signal of significant relative power is incident along the mainbeam direction. As the number of adjustable auxiliaries increases, this problem becomes correspondingly more important.

What is needed to alleviate an undesired variation in mainbeam signal gain is an appropriate constraint on the adaptive weights applied over the auxiliaries. Conceptually, one such approach would consist of guaranteeing that the gain contribution by the auxiliary subarray in the mainbeam direction remain fixed during system operation. The appropriate criterion would then be minimum-combined-power subject to a fixed auxiliary subarray gain constraint. In mathematical terms, this may be stated as the constrained minimization problem

$$\min_{\underline{w}} \; P_c(\underline{w}) \tag{4-1.1}$$

$$\text{subject to:} \quad (\underline{e},\underline{w}) = d \tag{4-1.2}$$

where

$$P_c(\underline{w}) = \| \underline{s}^T \underline{w} + s_0 \|^2$$

= the combined signal power over an M-sample batch

d = a fixed real gain of the auxiliary subarray along the mainbeam direction

$\underline{e}$ = a nonzero complex constraining N-vector on the auxiliary weight vector $\underline{w}$ which guarantees a fixed auxiliary subarray gain d in the mainbeam direction

Note that (4-1) is a general enough statement to encompass the fully-adaptive problem. In fact, in the absence of a main antenna, which corresponds to $s_0 = 0$, (4-1) states the adaptive beamforming problem [8].

The discussion that follows presents the conventional approach for solving (4-1) based on the Lagrange-multiplier (LM) method [9] and involving the inversion of the covariance matrix $C = \langle \underline{s}, \underline{s}^T \rangle$. However, when C^{-1} does not exist, this approach is not valid. The constrained BCR formulation is introduced as a means of circumventing the need to invert C. More specifically, a detailed mathematical development of the constrained CD method (CCD) precedes the specific constrained CG method (CCG). Numerical examples are used to contrast the inversion and relaxation approaches.

4.1 Lagrange-Multiplier Method

The solution to (4-1) may be obtained by the minimization of the Lagrangian function

$$L(\underline{w},\lambda_1,\lambda_2) = \frac{1}{2} P_c(\underline{w}) + \lambda_1(Re(\underline{e},\underline{w})-d) + \lambda_2(Im(\underline{e},\underline{w})) \tag{4-2}$$

with respect to $\underline{w}$ and the real-valued Lagrange multipliers, λ_1 and λ_2. This involves solving the complex linear system that results from setting the complex gradient, $\nabla_w L(\underline{w},\lambda_1,\lambda_2)$, to zero simultaneously with the equality constraint. The resulting constrained solution, $\underline{\hat{w}}^c$, is defined in the following theorem.

Theorem 4-1. [Lagrange-Multiplier Solution to the Linearly Constrained Adaptive Nulling Problem, (4-1)]

The Lagrange-multiplier solution to (4-1), based on the Lagrangian function (4-2) is given by

$$\underline{\hat{w}}^c = \underline{\hat{w}} - \lambda C^{-1}\underline{e} \tag{4-3}$$

where

$$\underline{\hat{w}} = - C^{-1}\underline{b} \tag{4-4}$$

= the familiar unconstrained inversion solution

$$\lambda = \lambda_1 + j\lambda_2 \tag{4-5.1}$$

$$= - \frac{d - (\underline{e},\underline{\hat{w}})}{(\underline{e},C^{-1}\underline{e})} \tag{4-5.2}$$

for any $\underline{e} \in C^N$, provided that C^{-1} exists.

Proof: Setting the "complex gradient" $\nabla_{\underline{w}} L(\underline{w}, \lambda_1, \lambda_2)$ to zero, we get

$$\underline{0} = C\underline{w} + \underline{b} + \lambda_1 \underline{e} + j\lambda_2 \underline{e}$$

$$= C\underline{w} + \underline{b} + \lambda \underline{e} \tag{4-6}$$

where λ is given by (4-5.1). Then, given that C^{-1} exists, the desired constrained solution (4-3) follows immediately, in view of (4-4). Upon substituting (4-3) into constraint equation (4-1.2) we get the composite complex Lagrange multiplier solution. In fact,

$$d = (\underline{e}, \underline{\hat{w}}^c)$$

$$= (\underline{e}, \underline{\hat{w}}) - \lambda(\underline{e}, C^{-1}\underline{e}) \tag{4-7}$$

which yields the desired result, (4-5.2). ∎

Based on Theorem 4-1 we can derive the solution for the special fully-adaptive or adaptive beamforming problem [10]. This is stated in the corollary that follows.

Corollary 4-2. [Lagrange-Multiplier Solution to the Fully-Adaptive Problem, (4-1)]

Consider the fully-adaptive special case of (4-1). In the absence of a fixed main antenna, $\underline{b} = \underline{0}$. Then, assuming C^{-1} exists, the Lagrange-multiplier solution to (4-1) is given by

$$\underline{\hat{w}}^c = \frac{d}{(\underline{e}, C^{-1}\underline{e})} C^{-1}\underline{e} \tag{4-8}$$

for any $\underline{e} \in C^N$, provided that C^{-1} exists.

Proof: When $\underline{b} = 0$, $\underline{w} = -C^{-1}\underline{b} = \underline{0}$. Then, the familiar "steering vector" solution (4-8) follows immediately from Theorem 4-1.

Of course, in the above development, there is no particular restriction that d be real. In fact, it may be complex, in general. Of greater concern to the problem at hand is the reliance that C^{-1} exist. If C^{-1} does not exist, one must resort to alternate approaches†, including regularization of C in which case the inevitable error in solution accuracy must be tolerated.

In line with the main thrust of the work presented here, the constrained problem (4-1) will be addressed via a BCR approach. As such, C^{-1} need not exist.

4.2 Derivation of Constrained BCR (CBCR)

Consider the constrained minimization problem (4-1). Note that the equality constraint (4-1.2) simply states that the desired solution, $\hat{\underline{w}}^c$, lies on a constraining hyperplane whose orientation is defined by a normal vector $\underline{e}$ and its normal displacement from the origin is determined by d.

More specifically, the desired solution $\hat{\underline{w}}^c$ is a vector which originates from the origin and terminates at a point in the constraining hyperplane at which the combined power $P_c(\underline{w})$ is a minimum. At such a point, the complex gradient $\nabla_{\underline{w}} P_c(\underline{w})$ projected on the hyperplane must vanish.

In general, any vector $\underline{v} \in C^N$ may be expressed as an additive composition of two N-vectors, $\underline{v}^{\parallel}$ and $\underline{v}^{\perp}$, its parallel and perpendicular parts with respect to the constraining hyperplane. That is,

$$\underline{v} = \underline{v}^{\parallel} + \underline{v}^{\perp} \tag{4-9}$$

where, by definition,

$$(\underline{e}, \underline{v}^{\parallel}) = 0 \tag{4-10.1}$$

$$(\underline{e}, \underline{v}) = (\underline{e}, \underline{v}^{\perp}) \tag{4-10.2}$$

†Private discussions with J. R. Roman, Advanced Technology and Systems Analysis Section, Systems Office, Radar Operations, Motorola Government Electronics Division, Tempe, Arizona.

Then,

$$\underline{v}^{\parallel} = \underline{v} - \underline{v}^{\perp}$$

$$= \underline{v} - \frac{\underline{e}}{\|\underline{e}\|^2} (\underline{e}, \underline{v})$$

$$= \left(I - \frac{\underline{e}\,\underline{e}^{*T}}{\|\underline{e}\|^2} \right) \underline{v}$$

$$= P\underline{v} \tag{4-11}$$

where P is the desired <u>projection operator</u>, a unitary N x N matrix.

In view of the above, the constrained minimization of (4-1) leads to the simultaneous system

$$P(C\underline{w} + \underline{b}) = \underline{0} \tag{4-12.1}$$

$$(\underline{e}, \underline{w}) = d \tag{4-12.2}$$

where, (4-12.1) requires that the projected gradient $\nabla_{\underline{w}} P_c(\underline{w})$ vanish at a point $\underline{w}$, and (4-12.2) forces this $\underline{w}$ to be a point on the constraining hyperplane. Figure 4-1 serves to demonstrate these results.

With the constrained minimization formulation at hand and the geometrical interpretation provided in Figure 4-1, we wish to appropriately redefine the CD and CG methods for arriving at the desired solution $\hat{\underline{w}}^c$. Logically, we wish to choose an initial solution estimate $\underline{w}^0$ lying on the constraining hyperplane and define CD and CG iterative relaxation procedures that are totally confined on the hyperplane. These two constrained methods are developed in the next two sections.

4.2.1 Constrained CD (CCD) Method

Consider the specialization of the CD method, already developed in Section 2.2.2.1, for solving the linearly constrained problem (4-1), or, more specifically, the equivalent statement (4-12). To begin with, choose an initial solution estimate, $\underline{w}^0$, that lies entirely on the constraining hyperplane. That is, choose $\underline{w}^0$ such that

$$(\underline{e}, \underline{w}^0) = d \tag{4-13}$$

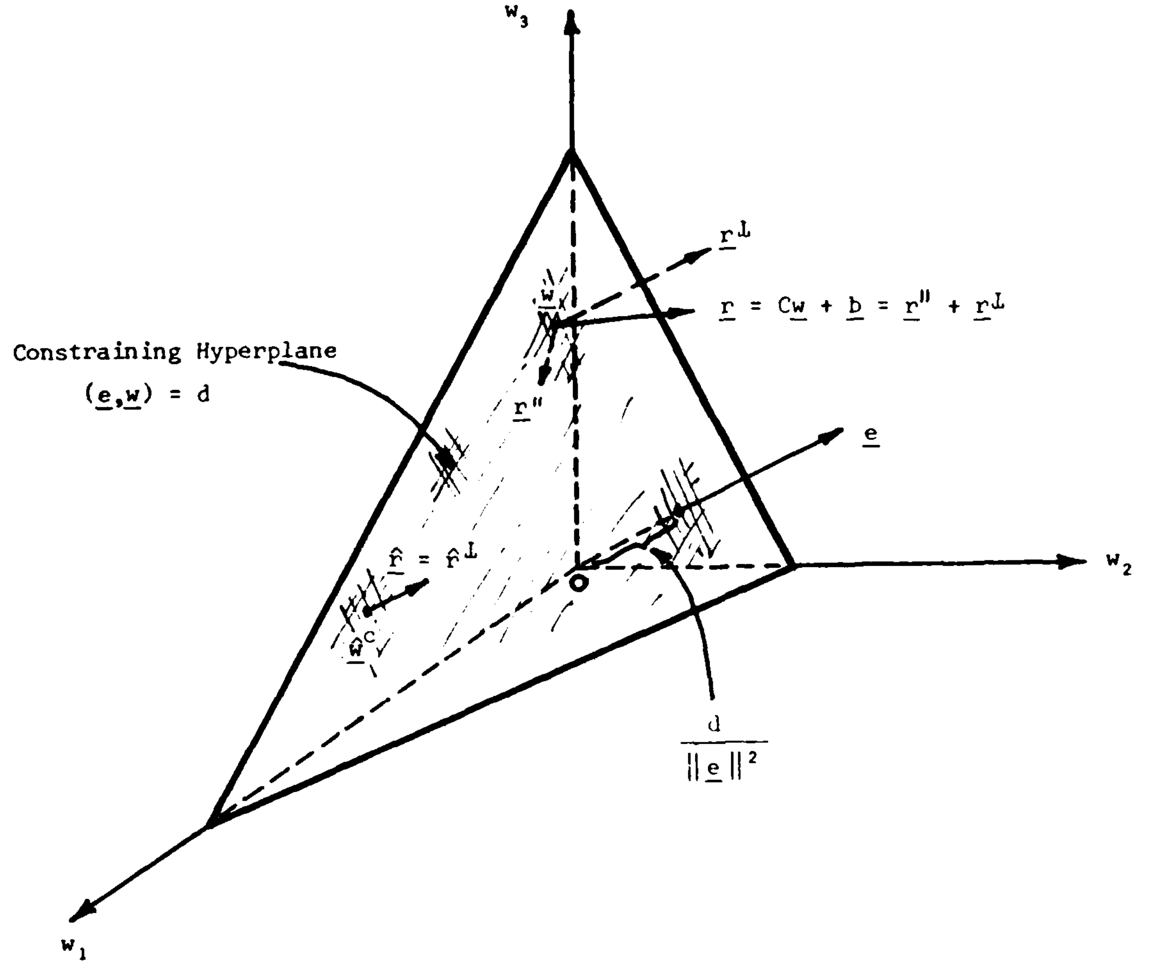


Figure 4-1. Geometrical Interpretation of the Constrained Minimization Formulation (4-12), of Problem (4-1).

Then, the desired solution, $\underline{\hat{w}}^c$, may be expressed as

$$\underline{\hat{w}}^c = \underline{w}^0 - \sum_{k=0}^{Q-1} \alpha_k \underline{p}^k \qquad (4\text{-}14)$$

where $Q \leq N$, the dimensionality of $\underline{w}$, $\{\alpha_k\}_{k=0}^{Q-1}$ is a set of expansion coefficients corresponding to a set of linearly independent set of vectors $\{p^k\}_{k=0}^{Q-1}$. With the restrictions that these vectors be parallel to the constraining hyperplane; that is,

$$(\underline{e},\underline{p}^k) = \underline{0}$$

$$\longrightarrow P\underline{p}^k = \underline{p}^k \qquad (4\text{-}15)$$

$$\forall\, k \in [0,Q-1]$$

(4-14) is meaningful, since by (4-13) and (4-15)

$$(\underline{e},\underline{\hat{w}}^c) = d \qquad (4\text{-}16)$$

as desired. Then, by (4-12.1), the projected gradient will vanish at $\underline{\hat{w}}^c$. Letting

$$\underline{r}^0 = P(C\underline{w}^0 + \underline{b}) \qquad (4\text{-}17)$$

be the projected gradient, or residual, at the initial estimate $\underline{w}^0$, (4-14) implies that

$$\underline{0} = \underline{\hat{r}}^c$$

$$= \underline{r} - \sum_{k=0}^{Q-1} \alpha_k PC\underline{p}^k \qquad (4\text{-}18)$$

We wish to solve for the set of expansion coefficients $\{\alpha_k\}_{k=0}^{Q-1}$. One efficient such solution is specified in the following lemma.

Lemma 4-3. [Solution of $\{\alpha_k\}_{k=0}^{Q-1}$ Via PC-Conjugacy of $\{\underline{p}^k\}_{k=0}^{Q-1}$]

There exists a linearly independent set of PC-conjugate vectors $\{\underline{p}^k\}_{k=0}^{Q-1}$ such that

$$\alpha_i = \frac{(\underline{p}^i, \underline{r}^i)}{(\underline{p}\ , PC\underline{p}^i)} \tag{4-19}$$

$$\forall i \in [0, Q-1]$$

Proof: By (4-18) and (4-15)

$$(\underline{p}^i, \underline{r}^0) = \sum_{k=0}^{Q-1} \alpha_k (\underline{p}^i, PC\underline{p}^k) \tag{4-20}$$

and

$$(\underline{p}^i, PC\underline{p}^k) = (\underline{p}^i, PCP\underline{p}^k) \tag{4-21}$$

respectively. Since P and C are conjugate symmetric, PCP is also conjugated symmetric. In fact,

$$\begin{aligned}(PCP)^{*T} &= P^{*T} C^{*T} P^{*T} \\ &= PCP\end{aligned} \tag{4-22}$$

Then, by Lemmas 3-3 and 3-4, it is possible to assume PCP-conjugacy on the set of linearly independent vectors $\{\underline{p}^k\}_{k=0}^{Q-1}$. But, (4-21) implies that a PC-conjugacy may be assumed, equivalently. Applying PC-conjugacy on (4-20), we get the desired result.

∎

It is now possible to state one version of the constrained CD method for solving (4-12), the equivalent statement to (4-1).

Theorem 4-4. [Standard Form of the CCD Method]

Consider the system

$$P(C\underline{w} + \underline{b}) = \underline{0} \tag{4-23.1}$$

$$(\underline{e},\underline{w}) = d \tag{3-23.2}$$

where C is an N x N complex Hermitian matrix of rank R, $\underline{b}$ is a complex N-vector in C^R, the eigenspace of C, e is a complex constraining N-vector, d is a real-valued scalar, P is a complex N x N unitary matrix

$$P = I - \frac{\underline{e}\,\underline{e}^{*T}}{\|\underline{e}\|^2} \tag{4-24}$$

the projection operator which produces the projection of any vector $\underline{v} \in C^N$ onto the hyperplane (4-23.2), and $\underline{w}$ is the complex N-vector to be determined. Let

$$\left\{\underline{p}^k\right\}_{k=0}^{Q-1} \tag{4-25}$$

be a set of Q, $Q < R \leq N$, linearly independent complex N-vectors having the properties of

(1) Nonzero intersection with PC^R, the projected eigenspace of C; i.e.,

$$\left.\begin{array}{l} \underline{p}^i \cap PC^R \neq \emptyset \\ \forall\, i \in [0,Q-1] \end{array}\right\} \tag{4-26}$$

(2) Spanning the constrained eigenspace, PC^R, via a linear combination; i.e.,

$$PC^R = \left\{ P\underline{v} \in C^N : Pv = \sum_{i=0}^{Q-1} a_i \underline{p}^i \;;\; \forall\, \underline{v} \in C^N,\ a_i \in C^1,\ i \in [0,Q-1] \right\} \tag{4-27}$$

(3) Mutual PC-conjugacy property, i.e.,

$$\left.\begin{aligned} &(\underline{p}^i, PC\underline{p}^j) = 0 \\ &\forall\, i, j \in [0, Q-1];\ i \neq j \end{aligned}\right\} \qquad (4\text{-}28)$$

Then, a general solution to (4-31) is the last iterate, $\underline{w}^{Q-1}$, of the Q-step iterative procedure that begins with an initial solution estimate, $\underline{w}^0$ that satisfies (4-23.2) and a corresponding residual vector $\underline{r}^0 = P(C\underline{w}^0 + \underline{b})$ and repeats relations

$$\alpha_i = \frac{(\underline{p}^i, \underline{r}^0)}{(\underline{p}^i, PC\underline{p}^i)} \qquad (4\text{-}29.1)$$

$$\underline{w}^{i+1} = \underline{w}^i - \alpha_i \underline{p}^i \qquad (4\text{-}29.2)$$

$$\underline{r}^{i+1} = \underline{r}^i - \alpha_i PC\underline{p}^i \qquad (4\text{-}29.3)$$

for $i = 0, 1, \ldots, Q-1$. This procedure will be referred to as the standard form of the constrained conjugate direction (CCD) method.

Proof: Relation (4-29.1) is that already proved in the discussion preceeding this theorem. Referring to (4-14) it is possible to identify a sequence of solution estimates $\{\underline{w}^k\}_{k=0}^{Q-1}$ such that $\underline{w}^{i+1}$ is given by (4-29.2). Upon substituting (4-29.2) into (4-23.1), we can define a corresponding sequence of residuals, $\{\underline{r}^k\}_{k=0}^{Q-1}$, such that, $\underline{r}^{i+1}$ is given by (4-29.3). The hypotheses stated are similar to those of Theorem 3-5 except for the modifications needed to incorporate the projection operator. Finally, note that since the constraining vector $\underline{e}$ is eliminated from the set $\{\underline{p}^i\}_{i=0}^{Q-1}$, $Q \leq R-1$.

∎

An alternate form of the CCD method that is numerically more robust may be derived by using the following lemma.

Lemma 4-5. [Special Properties of $(\underline{p}^i, \underline{r}^j)$ in the CCD Method]

In the CCD method as defined by Theorem 4-4, the ineer product $(\underline{p}^i, \underline{r}^j)$ has the following properties

$$(\underline{p}^i,\underline{r}^j) = (\underline{p}^i,\underline{r}^0) \quad \forall\, i \geq j \;;\; i,j \in [0,Q-1] \tag{4-30}$$

and

$$(\underline{p}^i,\underline{r}^i) = 0 \quad \forall\, I < j \;;\; i \in [0,\, Q-1],\; j \in [1,\, Q-1] \tag{4-31}$$

Proof: Given the PC-conjugacy condition (4-28) and recursion (4-29.3), results (4-30) and (4-31) follow in the same way as those of Lemma 3-6.

The alternate form of the CCD algorithm may now be stated in the following theorem.

Theorem 4-6. [Alternate Form of the CCD Method]

Given all the hypothesis of Theorem 4-4, a general solution to (4-23) is the iterate, $\underline{w}^{Q-1}$, of the Q-step iterative procedure that begins with an arbitrary initial estimate, $\underline{w}^0$, in the constraining hyperplane (4-23.2) and the corresponding residual vector, $\underline{r}^0 = P(C\underline{w}^0 + \underline{b})$, and, upon setting $\underline{p}^0 = \underline{r}^0$, repeats relations

$$\alpha_i = \frac{(\underline{p}^i,\underline{r}^i)}{(\underline{p}^i,PC\underline{p}^i)} \tag{4-32.1}$$

$$\underline{w}^{i+1} = \underline{w}^i - \alpha_i\underline{p}^i \tag{4-32.2}$$

$$\underline{r}^{i+1} = \underline{r}^i - \alpha_i PC\underline{p}^i \tag{4-32.3}$$

for i = 0, 1, ..., Q-1. This procedure will be referred to as the alternate form of the constrained conjugate directions (CCD) method.

Proof: Result (4-32) follows from Theorem 4-4 and Lemma 4-5. ∎

Of course, a means for constructing the set of PC-conjugate set of vectors $\{\underline{p}^i\}_{i=0}^{Q-1}$ is crucial to the complete description of the CCD method. A construction similar to that given in Lemma 3-8 for the normal CD method may also be used for the CCD method. This is described in the following lemma.

Lemma 4-7. [Construction of PC-conjugate set of N-vectors $\{p^i\}_{i=0}^{Q-1}$ from a Complete Orthogonal Set $\{q^i\}_{i=0}^{N-1}$]

Let $\{q^i\}_{i=0}^{N-1}$ be a complete orthogonal basis of the projected space PC^N, i.e.,

$$PC^N = \left\{ \underline{v} \;:\; \underline{v} = \sum_{i=0}^{N-1} a_i \underline{p}^i \;,\; \underline{q}^i = P\underline{q}^i \;, \forall\, a^i \in C^1 \;,\; i \in [0,N-1] \right\} \tag{4-33}$$

Then, a set of Q linearly independent PC-conjugate vectors $\{\underline{p}^i\}_{i=0}^{Q-1}$ may be defined as follows. The first vector is arbitrarily defined by

$$\underline{p}^0 = \underline{q}^{k_0} \tag{4-34}$$

where $k_0 = \min[0,N-1] \ni PC\underline{p}^0 \neq \underline{0}$. The general such vector is given by

$$\underline{p}^i = \underline{q}^{k_i} + \sum_{j=0}^{i-1} \beta_{i,j} \underline{p}^j \tag{4-35}$$

where

$$\beta_{i,j} = \frac{(\underline{q}^{i_1}, PC\underline{p}^j)^*}{(\underline{p}^j, C\underline{p}^j)} \tag{4-36}$$

and $\{k_i\}$ is a subsequence of indices in the range [0,N-1] while i and j range over [0,Q-1] and [0,Q-2], respectively, with $i > j$. Futhermore, $Q < R = \text{rank}\, C$.

Proof: Except for the PC-conjugacy condition, the proof is identical to that of Theorem 4-6.

Variations of the construction given above are possible. For example, the orthogonality restriction on $\{q^i\}_{i=0}^{N-1}$ could be removed. In fact, it would suffice if this set of vectors span the projected space PC^N. Furthermore, it is possible to derive the unique minimum-norm solution using the CCD method if $\underline{w}^0 \in PC^R$, the projected eigenspace of C. This, of course, follows directly from (4-14).

4.2.2 Constrained CG (CCG) Method

The derivation of the CCG method follows as a special case of the CCD method, where the orthogonal basis vectors are the successive projected gradients $\{\underline{r}^i\}_{i=0}^{Q-1}$. The conventional version of the CCG method is as stated in the following theorem.

Theorem 4-8. [Conventional Form of the CCG Method]

Consider the system

$$P(C\underline{w} + \underline{b}) = \underline{0} \tag{4-37.1}$$

$$(\underline{e},\underline{w}) = d \tag{4-37.2}$$

where C is an N x N complex Hermitian matrix of rank R, $\underline{b}$ is a complex N-vector contained in C^R, the eigenspace of C, $\underline{e}$ is a complex constraining N-vector, d is a real-valued scalar, P is a complex N x N unitary matrix

$$P = I - \frac{\underline{e}\,\underline{e}^{*T}}{\|\underline{e}\|^2} \tag{4-38}$$

the projection generator that produces the projection of any vector $\underline{v} \in C^N$ onto the hyperplane (4-37.2) and $\underline{w}$ is the complex N-vector to be determined.

The general solution to (4-37) is the last iterate, $\underline{w}^{Q-1}$, of a Q-step iterative procedure that begins with an initial solution estimate, $\underline{w} \in PC^N$, the corresponding projected residual, $\underline{r}^0 = P(C\underline{w}^0 + \underline{b})$, and, upon setting $\underline{p}^0 = \underline{r}^0$, repeats relations

$$\alpha_i = \frac{\|\underline{r}^i\|^2}{(\underline{p}^i, PC\underline{p}^i)} \qquad (4\text{-}39.1)$$

$$\underline{w}^{i+1} = \underline{w}^i - \alpha_i \underline{p}^i \qquad (4\text{-}39.2)$$

$$\underline{r}^{i+1} = \underline{r}^i - \alpha_i PC\underline{p}^i \qquad (4\text{-}39.3)$$

$$\beta_i = \frac{\|\underline{r}^{i+1}\|^2}{\|\underline{r}^i\|^2} \qquad (4\text{-}39.4)$$

$$\underline{p}^{i+1} = \underline{r}^{i+1} + \beta_i \underline{p}^i \qquad (4\text{-}39.5)$$

for $i = 0, 1, \ldots, Q-1$. This procedure is referred to as the conventional form of the constrained conjugate gradients (CCG) method.

Proof: Except for using Lemma 4-5, Theorem 4-6 and the PC-conjugacy condition, result (4-39) follows the same arguments in the proof of Theorem 3-9.

The standard and alternate forms of the CCG method differ from the conventional form only in the definition of expansion coefficients; that is,

$$\alpha_i = \frac{(\underline{p}^i, \underline{r}^0)}{(\underline{p}^i, PC\underline{p}^i)} \qquad (4\text{-}40)$$

for the standard CCG method, and

$$\alpha_i = \frac{(\underline{p}^i, \underline{r}^i)}{(\underline{p}^i, PC\underline{p}^i)} \qquad (4\text{-}41)$$

for the alternate CCG method.

As in the case of the unconstrained CG method, the CCG method is a descent, or relaxation method. Figure 4-2 attempts to heuristically demonstrate the geometrical interpretation of the CCG method. Noted there is the unconstrained solution, $\hat{\underline{w}}$, location in the geometrical center of a continuum of concentric ellipsoidal contours of equal combined powers. For this 3-dimensional system, it is shown that the CCG process begins at an initial estimate on the constraining hyperplane and undergoes a relaxation there in two steps to the final constrained solution, $\hat{w}^c$, along appropriate PC-conjugate vectors. Significantly, the CCG process is shown to converge in two and not three iterations. This is as it should be, since the constrained minimum sought lies in the geometrical center of the elliptical contours resulting from the intersection of the hyperplane with the ellipsoidal contours. Clearly, a degree of freedom has been lost by imposing the equality constraint (4-44.2).

It should be noted here that in computing the projection of any vector $v \in C^N$ onto the hyperplane via $P\underline{v}$, it is advisable to perform this efficiently as follows.

$$P\underline{v} = \left(I - \frac{\underline{e}\,\underline{e}^{*T}}{\|\underline{e}\|^2} \right) \underline{v}$$

$$= \underline{v} - \frac{\underline{e}}{\|\underline{e}\|^2} (\underline{e}, \underline{v}) \tag{4-42}$$

thus avoiding the need to store or perform a matrix operation with P.

This will directly effect the efficiency of computing $PC\underline{p}$ (in (4-39.1). Alternatively, one may precompute PC initially if C need not be preserved for any other reason.

Finally, noting that in (4-39.2) $p^i \in PC^R$, a choice of initial solution estimate $\underline{w}^0 \in PC^R$ will guarantee that $\hat{\underline{w}}c = \underline{w}Q^{-1} \in PC^R$, and thus be the unique minimum-norm solution. As an example, an appropriate initial solution estimate is $\underline{w}^0 = d\underline{e}/\|\underline{e}\|^2$.

4.3 Illustrative Examples

Two examples will be presented to demonstrate the validity of the closed-form Lagrange-multiplier method and the iterative CCG alternative. The examples are identical to those of Section 3.3, except for the use of an appropriate equality constraint in each case. Of particular significance is the second example which involves a singular covariance matrix. Although an approximate regularized constrained solution is possible in this case, a pseudoinverse approach does not make sense. In contrast, the CCG solution is the exact constrained solution.

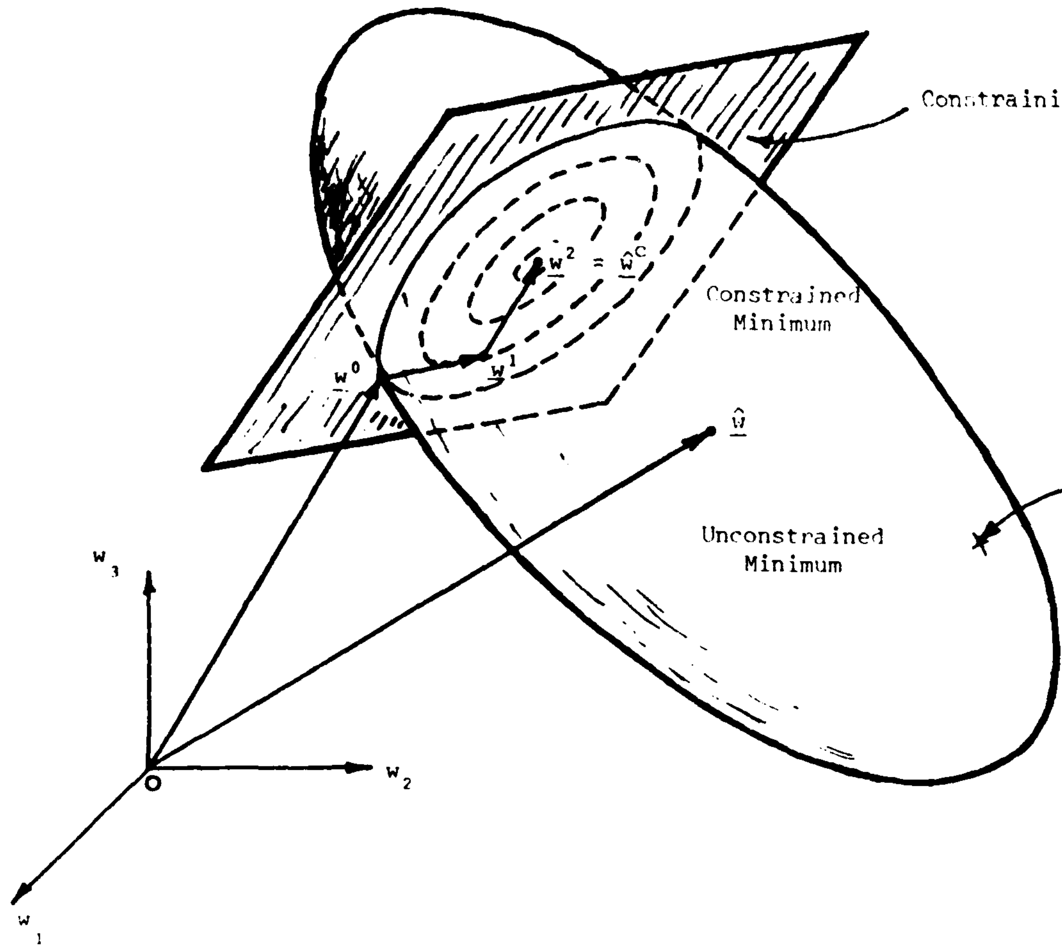


Figure 4-[illegible]. Geometrical Iterpretation of CGM Method.

4.3.1 Full-Rank Covariance Case

Consider solving the linear equality-constraint minimization problem

$$\min_{\underline{w}} J(\underline{w}) \tag{4-43.1}$$

$$(\underline{e},\underline{w}) = d \tag{4-43.2}$$

where,

$$J(\underline{w}) = \|A\underline{w} + \underline{a}\|^2 \tag{4-44}$$

$$\left.\begin{aligned} A &= \begin{pmatrix} 1 & j \\ 1 & 1 \end{pmatrix} \\ \underline{a} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \underline{e} &= \begin{pmatrix} j \\ 1 \end{pmatrix} \\ d &= 1 \end{aligned}\right\} \tag{4-45}$$

Note that (4-44) is the performance index used in the example presented in Section 3.3.1. Already computed there are

$$C = A^{*T}A$$

$$= \begin{pmatrix} 2 & 1+j \\ 1-j & 2 \end{pmatrix} \tag{4-46.1}$$

$$b = A^{*T}\underline{a}$$

$$= \begin{pmatrix} 1 \\ -j \end{pmatrix} \tag{4-46.2}$$

We will solve for $\underline{\hat{w}}^C$ by the Lagrange-multiplier method of Theorem 4-1 and by the CCG method of Theorem 4-4.

Lagrange-Multiplier Solution:

By Theorem 4-1,

$$\underline{\hat{w}}^C = \underline{\hat{w}} - \lambda C^{-1}\underline{e} \tag{4-47}$$

where

$$\begin{aligned}\underline{\hat{w}} &= - C^{-1}\underline{b} \\ &= - \frac{1+j}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix}\end{aligned} \tag{4-48}$$

is the unconstrained solution already computed in (3-94). Also,

$$\begin{aligned}(\underline{e},\underline{\hat{w}}) &= - \frac{1+j}{2}\begin{pmatrix} -j \\ 1 \end{pmatrix}^T\begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ &= \frac{(1+j)^2}{2} \\ &= j\end{aligned} \tag{4-49}$$

and, using, C^{-1} from (3-93),

$$\begin{aligned}C^{-1}\underline{e} &= \frac{1}{2}\begin{pmatrix} 2 & -(1+j) \\ -(1-j) & 2 \end{pmatrix}\begin{pmatrix} j \\ 1 \end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix} -1+j \\ 1-j \end{pmatrix} \\ &= \frac{1-j}{2}\begin{pmatrix} -1 \\ 1 \end{pmatrix}\end{aligned} \tag{4-50}$$

$$(\underline{e}, C^{-1}\underline{e}) = \frac{1-j}{2}\begin{pmatrix}-j \\ 1\end{pmatrix}^T\begin{pmatrix}-1 \\ 1\end{pmatrix}$$

$$= \frac{(1-j)(1+j)}{2}$$

$$= 1 \qquad (4\text{-}51)$$

By (4-5.2), the complex composite Lagrange multiplier is

$$\lambda = -\frac{d - (\underline{e}, \underline{\hat{w}})}{(\underline{e}, C^{-1}\underline{e})}$$

$$= -(1-j) \qquad (4\text{-}52)$$

Then, using (4-47), the desired constrained solution is

$$\underline{\hat{w}}^c = \underline{\hat{w}} - \lambda C^{-1}\underline{e}$$

$$= -\frac{1+j}{2}\begin{pmatrix}1 \\ -1\end{pmatrix} + \frac{(1-j)^2}{2}\begin{pmatrix}-1 \\ 1\end{pmatrix}$$

$$= -\frac{1}{2}\begin{pmatrix}1+j \\ -1-j\end{pmatrix} - \frac{1}{2}\begin{pmatrix}-2j \\ 2j\end{pmatrix}$$

$$= -\frac{1}{2}\begin{pmatrix}1-j \\ -1+j\end{pmatrix}$$

$$= -\frac{1-j}{2}\begin{pmatrix}1 \\ -1\end{pmatrix} \qquad (4\text{-}53)$$

The validity of (4-53) may now be checked. Given that $d = 1$, we note that $\underline{\hat{w}}^c$ satisfies constraint (4-43.2); i.e.,

$$(\underline{e}, \underline{\hat{w}}^c) = -\frac{1-j}{2}\begin{pmatrix}-j \\ 1\end{pmatrix}^T\begin{pmatrix}1 \\ -1\end{pmatrix}$$

$$= \frac{(1-j)(1+j)}{2}$$

$$= 1 \tag{4-54}$$

We need to check whether the projected gradient

$$P\nabla_{\underline{w}} J(\underline{w}) = P(C\underline{w} + \underline{b}) \tag{4-55}$$

vanishes at $\hat{\underline{w}}^c$. By (4-46) and (4-53),

$$C\hat{\underline{w}}^c + \underline{b} = -\frac{1-j}{2}\begin{pmatrix} 2 & 1+j \\ 1-j & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

$$= -\frac{(1-j)}{2}\begin{pmatrix} 1-j \\ -1-j \end{pmatrix} + \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

$$= -\frac{1}{2}\begin{pmatrix} -2j \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

$$= \begin{pmatrix} j \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

$$= \begin{pmatrix} 1+j \\ 1-j \end{pmatrix} \tag{4-56}$$

Then, by (4-42) and (4-45)

$$P(C\hat{\underline{w}}^c + \underline{b}) = P\begin{pmatrix} 1+j \\ 1-j \end{pmatrix}$$

$$= \begin{pmatrix} 1+j \\ 1-j \end{pmatrix} - \frac{1}{2}\begin{pmatrix} j \\ 1 \end{pmatrix}\left[\begin{pmatrix} -j \\ 1 \end{pmatrix}^T \begin{pmatrix} 1+j \\ 1-j \end{pmatrix}\right]$$

$$= \begin{pmatrix} 1+j \\ 1-j \end{pmatrix} - (1-j) \begin{pmatrix} j \\ 1 \end{pmatrix}$$

$$= \underline{0} \tag{4-57}$$

As a consequence of (4-57) and (4-54), we have verified that (4-53) is, in fact, the correct constrained solution.

CCG Solution:

We will use Theorem 4-8 to solve (4-43), or, equivalently,

$$P(C\underline{w} + \underline{b}) = \underline{0} \tag{4-58.1}$$

$$(\underline{e},\underline{w}) = d \tag{4-58-2}$$

We choose an initial solution estimate that satisfies constraint (4-43.2); i.e.,

$$\underline{w}^0 = \frac{d}{\|e\|^2} \underline{e}$$

$$= \frac{1}{2} \begin{pmatrix} j \\ 1 \end{pmatrix} \tag{4-59}$$

with corresponding projected gradient

$$r^0 = P(C\underline{w}^0 + \underline{b})$$

$$= P \left[\frac{1}{2} \begin{pmatrix} 2 & 1+j \\ 1-j & 2 \end{pmatrix} \begin{pmatrix} j \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -j \end{pmatrix} \right]$$

$$= P \left[\frac{1}{2} \begin{pmatrix} 1+3j \\ 3+j \end{pmatrix} + \begin{pmatrix} 1 \\ -j \end{pmatrix} \right]$$

$$= P\left[\frac{1}{2}\begin{pmatrix}3+3j\\3-j\end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix}3+3j\\3-j\end{pmatrix} - \frac{1}{4}\begin{pmatrix}j\\1\end{pmatrix}\left[\begin{pmatrix}-j\\1\end{pmatrix}^T\begin{pmatrix}3+3j\\3-j\end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix}3+3j\\3-j\end{pmatrix} - \frac{3-2j}{2}\begin{pmatrix}j\\1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1\\j\end{pmatrix} \tag{4-60}$$

where we have used (4-42). Note that $\underline{r}^0$ is parallel to the hyperplane defined by (4-58.2). In fact,

$$(\underline{e},\underline{r}^0) = \frac{1}{2}\begin{pmatrix}-j\\1\end{pmatrix}^T\begin{pmatrix}1\\j\end{pmatrix} = 0 \tag{4-61}$$

Setting $\underline{p}^0 = \underline{r}^0$, we follow procedure (4-39).

$$\|\underline{r}^0\|^2 = \frac{1}{2} \tag{4-62}$$

$$C\underline{p}^0 = \frac{1}{2}\begin{pmatrix}2 & 1+j\\1-j & 2\end{pmatrix}\begin{pmatrix}1\\j\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1+j\\1+j\end{pmatrix}$$

$$= \frac{1+j}{2}\begin{pmatrix}1\\1\end{pmatrix} \tag{4-63}$$

$$PC\underline{p}^0 = \frac{1+j}{2}\begin{pmatrix}1\\1\end{pmatrix} - \frac{1}{2}\left[\frac{1+j}{2}\begin{pmatrix}-j\\1\end{pmatrix}^T\begin{pmatrix}1\\1\end{pmatrix}\right]\begin{pmatrix}j\\1\end{pmatrix}$$

$$= \frac{1+j}{2}\begin{pmatrix}1\\1\end{pmatrix} - \frac{1}{2}\begin{pmatrix}j\\1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1\\j\end{pmatrix} \tag{4-64}$$

where we have used (4-42). Note again that $PC\underline{p}^0$ is in the hyperplane (4-58.2). Continuing,

$$(\underline{p}^0, PC\underline{p}^0) = \frac{1}{4}\begin{pmatrix}1\\-j\end{pmatrix}^T\begin{pmatrix}1\\j\end{pmatrix}$$

$$= \frac{1}{2} \tag{4-65}$$

$$\alpha_0 = \frac{||\underline{r}^0||^2}{(\underline{p}^0, PC\underline{p}^0)}$$

$$= 1 \tag{4-66}$$

$$\underline{w}^1 = \underline{w}^0 - \alpha_0\underline{p}^0$$

$$= \frac{1}{2}\begin{pmatrix}j\\1\end{pmatrix} - \frac{1}{2}\begin{pmatrix}1\\j\end{pmatrix}$$

$$= \frac{-1}{2}\begin{pmatrix}1-j\\-1+j\end{pmatrix}$$

$$= -\frac{1-j}{2}\begin{pmatrix}1\\-1\end{pmatrix} \tag{4-67}$$

$$\underline{r}^1 = \underline{r}^0 - \alpha_0 PC\underline{p}^0$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ j \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 1 \\ j \end{pmatrix}$$

$$= \underline{0} \tag{4-68}$$

Since $\|\underline{r}^1\| = 0$, the process terminates here. Then,

$$\underline{w}^1 = \underline{\hat{w}}^c \tag{4-69}$$

which agrees with the Lagrange-multiplier solution, (4-53).

It should be noted here that, whereas the unconstrained solution, $\underline{\hat{w}}$, takes 2 CG iterations, the CCG process required only 1 to produce the constrained solution $\underline{\hat{w}}^c$. This is as expected and demonstrated in Figure 4-1.

4.3.2 Singular Covariance Case

Consider solving the singular linear equality-constraint minimization problem

$$\min_{\underline{w}} J(\underline{w}) \tag{4-70.1}$$

$$(\underline{e},\underline{w}) = d \tag{4-70.2}$$

where,

$$J(\underline{w}) = \|A\underline{w} + \underline{a}\|^2 \tag{4-71}$$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\underline{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\underline{e} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$d = 1 \tag{4-72}$$

Note that $J(\underline{w})$ is the performance index used in the example presented in Section 3.3.2. Alr ly computed there are

$$C = A^{*T} A$$

$$= 2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{4-73.1}$$

$$\underline{b} = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{4-73.2}$$

where C is, clearly, singular.

We will solve for $\underline{\hat{w}}^c$ by the Lagrange-multiplier method of Theorem 4-1 using the approximate regularized inverse, $\tilde{C}^{-1}$, of C. Subsequently, we will examine the Lagrange multiplier solution using the pseudoinverse, C^+, of C. Finally, we will solve for $\underline{\hat{w}}^c$ via the CCG method of Theorem 4-4.

Regularized Inversion Solution:

By regularizing C to $\tilde{C}$ via (2-21), we may insure the existence of an approximate inverse, $\tilde{C}^{-1}$, which we will use here. Then, by Theorem 4-1

$$\underline{\tilde{w}}^c = \underline{\tilde{w}} - \lambda\tilde{C}^{-1}\underline{e} \tag{4-74}$$

where $\underline{\tilde{w}}^c$ is the regularized constrained solution, and

$$\underline{\tilde{w}} = -\tilde{C}^{-1}\underline{b}$$

$$= -\frac{1}{(2+\varepsilon)}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{4-75}$$

is the unconstrained regularized solution already computed in (3-124). Also,

$$(\underline{e},\underline{\tilde{w}}) = 0 \qquad (4\text{-}76)$$

and, by (3-123),

$$\tilde{C}^{-1}\underline{e} = \frac{1}{2\ (2+\varepsilon)}\begin{pmatrix} 2+\varepsilon \\ -(2+\varepsilon) \end{pmatrix}$$

$$= \frac{1}{2\varepsilon}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad (4\text{-}77)$$

$$(\underline{e},\tilde{C}^{-1}e) = \frac{1}{\varepsilon} \qquad (4\text{-}78)$$

By (4-5.2), the complex composite Lagrange multiplier is

$$\lambda = -\frac{\tilde{d}^1 - (\underline{e},\underline{\tilde{w}})}{(\underline{e},\tilde{C}^{-1}\underline{e})}$$

$$= -\varepsilon \qquad (4\text{-}79)$$

Then, using (4-74), the desired regularized constrained solution is

$$\underline{\tilde{w}}^c = \underline{\tilde{w}} - \lambda\,\tilde{C}^{-1}\underline{e}$$

$$= -\frac{1}{(2+\varepsilon)}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \frac{1}{2(2+\varepsilon)}\begin{pmatrix} 2+\varepsilon-2 \\ -2-\varepsilon-2 \end{pmatrix}$$

$$= \frac{1}{2(2+\varepsilon)}\begin{pmatrix} \varepsilon \\ -4-\varepsilon \end{pmatrix} \qquad (4\text{-}80)$$

The validity of (4-80) may now be checked. Given that d=1, we note that $\underline{w}^c$ satisfies constraint (4-70.2); i.e.,

$$(\underline{e},\underline{\tilde{w}}) = \frac{1}{2(2+\varepsilon)}\begin{pmatrix}\varepsilon\\-4-\varepsilon\end{pmatrix}^T\begin{pmatrix}1\\-1\end{pmatrix}$$

$$= \frac{4+2\varepsilon}{2(2+\varepsilon)}$$

$$= 1 \tag{4-81}$$

We need to check that the regularized projected gradient

$$P\nabla_{\underline{w}}J(\underline{w}) = P(\tilde{C}\underline{w} + \underline{b}) \tag{4-82}$$

vanishes at $\underline{\tilde{w}}^c$. By (4-73) and (4-80), the regularized unconstrained gradient at $\underline{\tilde{w}}^c$ is

$$= \frac{1}{(2+\varepsilon)}\begin{pmatrix}1+\varepsilon & 1\\1 & 1+\varepsilon\end{pmatrix}\begin{pmatrix}\varepsilon\\-4-\varepsilon\end{pmatrix} \; 2\begin{pmatrix}1\\1\end{pmatrix}$$

$$= \frac{1}{(2+\varepsilon)}\begin{pmatrix}\varepsilon(1+\varepsilon) - (4+\varepsilon)\\\varepsilon - (1+\varepsilon)(4+\varepsilon)\end{pmatrix} + 2\begin{pmatrix}1\\1\end{pmatrix}$$

$$= \frac{1}{(2+\varepsilon)}\begin{pmatrix}\varepsilon^2 + \varepsilon - \varepsilon - 4 + 4 + 2\varepsilon\\\varepsilon - \varepsilon^2 - 5\varepsilon - 4 + 4 + 2\varepsilon\end{pmatrix}$$

$$= \frac{1}{(2+\varepsilon)}\begin{pmatrix}\varepsilon^2 + 2\varepsilon\\-\varepsilon^2 - 2\varepsilon\end{pmatrix}$$

$$= \varepsilon\begin{pmatrix}1\\-1\end{pmatrix} \tag{4-83}$$

clearly colinear to $\underline{e}$, or, equivalently, normal to hyperplane (4-70.2). Hence, its projection, (4-82), must vanish. Consequently, (4-80) is the correct regularized constrained solution.

Pseudoinversion Solution:

At this point one may naturally inquire as to whether it makes sense to use the pseudoinverse, C^+, in the Lagrange-multiplier solution of Theorem 4-1. That is,

$$\underline{\hat{w}}^c = \underline{\hat{w}} - \lambda C^+ \underline{e} \tag{4-84}$$

where

$$\begin{aligned} \underline{\hat{w}} &= - C^+ \underline{b} \\ &= - \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned} \tag{4-85}$$

is the unconstrained minimum-norm solution already computed in (3-130). Since

$$C^+ \underline{e} = \underline{0} \tag{4-86}$$

$(\underline{e}, C^+\underline{e}) = 0$, so that λ is not well defined by (4-5.2). Hence, (4-84) does not make sense. This simple example demonstrates that, in general, the pseudoinverse may not be used to derive the desired Lagrange multiplier solution.

CCG Solution:

We will use Theorem 4-8 to solve (4-70). As in the previous example, we choose as initial solution estimate

$$\begin{aligned} \underline{w}^0 &= \frac{d}{\|\underline{e}\|^2} \underline{e} \\ &= \frac{1}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \end{aligned} \tag{4-87}$$

with corresponding <u>initial projected gradient</u>

$$\underline{r}^0 = P(C\underline{w}^0 + \underline{b})$$

$$= P\left[\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]$$

$$= P\left[2\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]$$

$$= 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad (4\text{-}88)$$

which follows directly, since $(1 \quad 1)^T$ is normal to $\underline{e}$. Setting $\underline{p}^0 = \underline{r}^0$, we follow procedure (4-39)

$$\|\underline{r}^0\|^2 = 8 \qquad (4\text{-}89)$$

$$C\underline{p}^0 = 4\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= 8\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad (4\text{-}90)$$

$$PC\underline{p}^0 = 8\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad (4\text{-}91)$$

$$(\underline{p}^0, PC\underline{p}^0) = 32 \qquad (3\text{-}92)$$

$$\alpha^0 = \frac{1}{4} \qquad (4\text{-}93)$$

$$
\begin{aligned}
\underline{w}^1 &= \underline{w}^0 - \alpha_0 \underline{p}^0 \\
&= \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} - \frac{2}{4}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ -1 \end{pmatrix}
\end{aligned}
\tag{4-94}
$$

Noting that

$$
\begin{aligned}
\underline{r}^1 &= P(C\underline{w}^1 + \underline{b}) \\
&= P\left[2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} \right] \\
&= P\underline{0} \\
&= \underline{0}
\end{aligned}
\tag{4-95}
$$

the CCG process terminates. The desired solution is thus

$$
\underline{\hat{w}}^C = \underline{w}^1 = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{4-96}
$$

$\blacksquare$

It is interesting to note here that solution (4-96) is the <u>constrained minimum-norm solution</u> which happens to be the limit of the <u>constrained regularized solution</u> (4-80) as $\varepsilon \to 0$.

5.0 CONCLUDING REMARKS

The present memorandum has described the concept of Batch Covariance Relaxation (BCR) adaptive processing in the context of a baseband digitally-implementable sidelobe cancellation subsystem. Based on the numerically-stable and computationally-efficient conjugate gradients (CG) method, the BCR process constitutes an attractive alternative to conventional inversion approach particularly in large-scale applications. A variation of the BCR process has been introduced which addresses effectively the more general constrained adaptive array problem which includes the special case of adaptive beamforming.

More specifically, the memorandum begins by contrasting batch adaptive processing to dynamic processing, in general. Whereas the typical dynamic adaptive process exhibits an undesired transience to a nonstationary interference environment, a batch process is designed to circumvent this vulnerability.

With reference to adaptive array processing, the mathematical formulation of the batch adaptive process, based on a minimum-mean-square (MMS) criterion, leads to the special linear system

$$C\underline{w} + \underline{b} = \underline{0} \tag{5-1}$$

where C is a complex N x N Hermitian matrix, $\underline{b}$ is a complex N-vector in the eigenspace of C, and $\underline{w}$ is the unknown complex N-vector to be determined.

The conventional Sample Matrix Inversion (SMI) solution to (5-1) depends on the existence of the matrix inverse, C^{-1}. When C^{-1} does not exist, C may be regularized by adding to it a sufficient amount of additive diagonal variance, thereby guaranteeing the existence of an approximate inverse, $\tilde{C}^{-1}$, the regularized inverse of C. In so doing, the regularized inversion solution will incur an error and thus will not satisfy (5-1) exactly. Using the SVD method, however, it is possible to construct an inverse operator, C^{+}, the pseudoinverse of C, which may be used to derive the unique minimum-norm solution. Even so, with increasing system dimensionality, "open-loop" inversion approaches become numerically more and more sensitive, require comparatively greater computational precision and eventually reach a point where they are operationally unreliable.

The BCR approach offers a practical alternative to solving (5-1) without the need for explicit inversion of C. The general conjugate directions (CD) method begins with an arbitrary initial solution estimate, $\underline{w}^0$, and subsequently produces a sequence of improved estimates, $\underline{w}^1$, $\underline{w}^2$, ..., along specific relaxation directions, such that $\underline{w}^Q$ is the minimum-norm solution to (5-1) and Q does not exceed the system dimensionality N. From the numerical point of view, the iterative nature of the CD process may be exploited to minimize roundoff error. Replacing $\underline{w}^0$ by $\underline{w}Q$, the CD process may be executed a second time thereby refining the solution. This is a particularly useful mechanism for guaranteeing reliable numerical performance in practical large-scale applications.

The conjugate gradients (CG) method is a computationally-efficient special case of the general CD method. As a consequence, it is the recommended relaxation procedure for BCR processing.

A detailed mathematical development of the CD and CG methods is followed by two illustrative numerical examples which serve to demonstrate their special properties in direct contrast to appropriate inversion solutions.

The more general equality-constraint adaptive problem is considered next. A conventional Lagrange-multiplier solution to this problem is first derived. Subsequently, motivated by geometrical considerations, the BCR process is extended to apply effectively to this problem. The constrained CG (CCG) method follows directly from a detailed development of the general constrained CD (CCD) method. Two simple numerical examples are included to demonstrate the validity of the CCG method and to contrast it to the closed-form Lagrange-multiplier method which requires an explicit inverse of C. The second example which involves a singular matrix C is particularly interesting. Although a regularized inversion approach is useful for deriving an approximate Lagrange-multiplier solution, it is shown that a pseudoinverse cannot be used. In contrast, the CCG approach remains valid in this case.

BIBLIOGRAPHY

[1] Daniel, S. M., Aufderheide, M. B., Kuivila, R. P., King, T. M., McLane, M. L., Makovicka, T. J. and Katarski, E. P., "Advanced Digital Adaptive Array Batch Processor Development," Motorola IR&D Final Report 7263-001F, March 15, 1980.

[2] Hestenes, M. R. and Steifel, E., "Methods of Conjugate Gradients for Solving Linear Systems," Journal of Research of the National Bureau of Standards, Volume 49, No. 5, December 1952, pp. 409-536.

[3] Daniel, S. M., "BCR Adaptive Processing - Mathematical Model," Project Memorandum 8512-01, March 29, 1980.

[4] Daniel, S. M. and Myers, M. H., "Adaptive Array Processing Using Parallel and Sequential Gradient Control," 1980 Symposium on Adaptive Antennas, Coloroado Springs, Colorado.

[5] Daniel, S. M., "Stable Stochastic Steepest Descent Control for Adaptive Array Processing," Motorola Technical Memorandum No. M-172, 1977. Also, Correlations - An Engineering Bulletin from Motorola, Inc., Volume 1, No. 1, Winter 1978-79.

[6] Lawson, C. L. and Hanson, R. J., Solving Least Squares Problems, Prentice Hall, 1974.

[7] Fox, L., Huskey, H. D. and Wilkinson, J. H., "Notes on the Solution of Algebraic Linear Simultaneous Equations," Quarterly Journal of Mechanics and Applied Mathematics, pp. 149-173, 1948.

[8] Daniel, S. M., "Adaptive Beamforming in the Spectral Domain," 1973 URSI/GAP Conference, Boulder, Colorado.

[9] Daniel, S. M., "Closed Form Adaptive Array Processing with Null-Constrained Covariance Inversion," Motorola Technical Memorandum No. M-169, November 4, 1977.

[10] Roman, J. R., "Advanced Signal Processing Techniques," Motorola IR&D Final Report 7264-001, April 15, 1980.

COMPUTER SIMULATION OF BCR ADAPTIVE PROCESS

Prepared by: ______________________

S. M. Daniel
Project Leader
Advanced Technology and Systems Analysis

____

I. Kertesz
Advanced Technology and Systems Analysis

Reviewed by: ______________________

J. L. Bauck
Advanced Technology and Systems Analysis

Approved by: ______________________

G. V. Morris
Manager
Advanced Technology and Systems Analysis

____

D. L. Howell
Chief Engineer
Radar Systems Office

TABLE OF CONTENTS

TABLE OF CONTENTS
(continued)

1.0 INTRODUCTION

The present memorandum presents the modular analytical software developed for the purpose of analyzing the adaptive nulling performance of an adaptive array system employing the BCR adaptive process.

As stated previously, the system considered involves a Taylor-weighted main linear antenna array in combination with a set of weighted omnidirectional auxiliaries for the purpose of nulling a number of noise sources incident in the sidelobe region of the main antenna pattern. The adaptive weight adjustment is accomplished at baseband using the Batch Covariance Relaxation (BCR) approach.

The first section that follows discusses the general library-based software approach adopted for the analysis, employing the FORTRAN language. Included there is a detailed description of three major modular programs developed for the purpose. The first is SIGGEN, the latest version of the Signal Generation Program previously presented in Project Memorandum 8512-02. The second is BCRS, the BCR Simulation Program. The third is BCRP, the BCR Performance and Plotting Program.

The SIGGEN program utilizes system and scenario information from an input data file, SIGGEN:D, and produces an output data file, SIGGEN:O, containing sampled complex baseband signals received at the main and auxiliary ports specified. The input data file is written in the form of a menu that is flexible enough to allow a user to easily construct a case of interest by choosing RF bandwidth, single and multitap weighting, independent or multipath sources, etc.

Attaching SIGGEN:O to a small BCR header file, BCRS:DO, forms the input data file, BCRS:D, for the BCRS program. The BCRS program uses the port signal data in BCRS:D, exercises the BCR algorithm, produces the adaptive auxiliary weights and applies them on the port signals to produce the desired low-noise combined signal. Data produced by BCRS is transferred to output file, BCRS:O.

The input data file, BCRP:D, for program BCRP is formed by editing BCRS:O. Specifically, the actual number of iterations indicated in BCRS:O is entered in a reserved location of the header file, BCRS:DO, marked with an "X". Then all output between the end of the main-port signal and the beginning of the combined signal is deleted, except for the so-called composite weight-vector array. This augmentation procedure will become clear in the following section. The output from BCRP is in the form of an output file BCRP:O summarizing the BCR performance and its equivalent graphical representation via a TEKTRONIX plotting terminal.

The particular use of these three programs is explained in some detail in the next section with the aid of a benchtest example. In the subsequent section, the specific results of a number of interesting examples are presented with sufficient graphical information. Specific examples considered include a number of variations in system and scenario parameters. More general results obtained from families of related examples are also included. Finally, some comments are made on possible BCR processing alternatives.

In view of the variety of examples and general results included in this memorandum, it is clear that the software presented here constitutes a flexible and effective tool for analyzing BCR adaptive nulling performance for a specific adaptive array system in a variety of configurations and interference environments. The library-based approach used here is adaptable to analyzing even more complex and diverse adaptive systems with appropriate methodical modifications.

2.0 SIMULATION APPROACH AND PROGRAM DESCRIPTION

Discussed in this section is the structurally efficient and flexible computer simulation approach adopted for the evaluation of the BCR adaptive process applied to the adaptive array system described in Project Memorandum 8512-03. An introductory description of some important features of the general program structure is given in the first subsection. Each of the three subsequent subsections provides detailed descriptions of the structure and use of SIGGEN, BCRS and BCRP, the signal generation, BCR simulation and BCR performance and plotting programs, respectively.

2.1 General Program Structure

The three specific programs described in the next three sections share a common modular design and possess similar usage features. By way of introduction, the present section describes several of these common characteristics in the same order that the actual program details are presented subsequently.

2.1.1 Input Data File

The form of the input data file for these programs was designed with emphasis on clarity of presentation and ease of handling. The simple recurring format, used throughout the data file, clearly ties the input parameter description to its value. For a dynamic and versatile input process, key parameters are used to control the input of subsequent and related parameters. The structure of the data allows the user to focus on the important aspects of the problem description. Looking further at the details, the current system characteristics and the scenario become obvious. After the initial data construction, only the most simple editing tasks, such as character substitution, are required. This is possible partly through the leverage provided by the input control parameters, and partly through the use of multi-purpose parameters, both of which will be described later.

The input parameter names follow the implicit type convention of FORTRAN: If the name begins with a letter I-N, it is an integer. With the exception of the complex number POL(.), all other names represent real numbers. All the literals and special characters are treated as comments. The data are read with the following format:

TYPE	FORMAT	COLUMN WHERE DATA STARTS
Full-line comment	20A4	--
Comment + integer scalar	14A4,I12	57
Comment + integer vector	14A4,4X,10I2	61
Comment + real scalar	14A4,F12.5	57
Comment + complex scalar	14A4,2X,2F10.5	59

With the exceptions of ISC, ISP, ISW and POL(.), the variable type is obvious from its name, so the appropriate format is easily chosen. There is an alignment of the least significant digit in the scalar integer, real, and complex real-part, so potential errors in formatting can be avoided. The deletion and addition of lines, and changing of control parameters requires care, since they affect the input process.

2.1.2 Program Classifications

The program-modules in Section 2 will be discussed under two general classifications. One classification is based on their function within the next higher level of modularity. The other is according to the form of their representation. Some names and phrases, used in programming, may have restricted meanings in the context of this section. For this reason their definition will precede the ensuing discussion.

A program is a complete set of instructions which can be executed by a machine. It may require input data. It consists of one or more subprograms. The most important features of a program are completeness and executability.

A subprogram is a main-program or a set of one or more subroutines or functions. It is not executable because it is not complete. It depends on other subprograms to complete its control structure, and it may have external references.

A main-program is a program, if it is executable. Otherwise, it is a subprogram.

A module is a unit of program structure, which spans the program-subprogram boundaries. It may be equivalent to a subroutine, subprogram, or one or more programs. A module implies portability, or an ability to facilitate grouping or separation.

The informal classification of program-modules based on their function within the next higher level of modularity is used in program description. Three categories are identified.

An executive subprogram does little or no calculation, but calls other subprograms to do so.

A dedicated subprogram is devoted to performing tasks related to specific subjects. An example of this is the subroutine which computes the antenna weighting function, ANTWT. Although they can be called by more than one subprogram, their output will always be specifically dedicated to the same subject.

A general or utility subprogram performs the same task whenever called, but the subject of the task is general. Examples are the random number generator and FFT routines, as well as the input/output subprograms like RW and RWIRC.

A more formal classification is applied to the creation and subsequent execution of programs. It concerns the form of the program-modules, or how they are physically represented as file types.

A source module is a type of file which is created by the programmer, written in FORTRAN.

A binary or object module is created by a compiler. It constitutes a translation of the source program-module. Thus there is a one-to-one correspondence between a source module and its binary version. These modules are referenced and made available by a programmer, but they need not be edited or read.

The executable load module is created when all the binary modules which refer to one-another are linked together in such a way that they form a counterpart of the complete, executable source program. This type of file can be directly executed by the machine when appropriate input data is made available.

The naming convention (established for this project) makes use of a suffix to distinguish these and other file types. The following list defines some of these suffixes.

SUFFIX	MODULE FILE TYPE
:S	source
:B	binary
:L	load
:D	data-input
:O	data-output

It should be noted that S, B, and L suffix can be applied to the same program or subprogram, consistent with the definition and classification of "module". The suffix D and O can also be applied to the same program; however, they are not suffixes of program modules, but related to the program modules. The significance of this convention will be apparent when job control language (JCL) programs are discussed. There are other types of modules, such as a general information file, of which only one will be referenced here, and JCL types, which will be introduced later. The one information file of importance is RADAR:LIB. Its name is meant to associate a library of programs with radar signal processing. Its function is that of an index file. It shows name, author, and dates of origin and latest revision of members of the RADAR program library, which may be a subset of a larger program library. The yet unofficial rules of the library need to be pointed out here, since they have important effects on the use of the member programs.

1. Only programs and subprograms which meet a certain† standard may become members.

2. Only the latest revision of a member may exist. This implies that early versions are discarded, and that there is only one program or subprogram referred to by the same name, excluding suffixes to denote type.

3. Revisions and variations of members must meet requirements 1 and 2.

The RADAR:LIB information file is shown in Table 2-1.

2.1.3 Physical and Procedural Structures

In the attempt to produce efficient programs which are easy to use, understand, and maintain, certain priorities have to be established. There is always a compromise among these features, and room for improvement whenever the priorities shift.

The physical structure assigned to these programs is modular. The modules are separated by sometimes clearly defined subjects or functions, sometimes by physical or algorithmic necessities. The aim in each case is to produce modules or branches of modules in natural ways, rather than in contrived ways for the sake of "structuring".

The description of the following programs will be enhanced with the use of tree diagrams. The primary purpose of such diagrams is to show all the modules of the program, and their interrelationship. This ordered relationship is called the physical structure. The named blocks are subprograms, and the solid lines show exactly what other subprograms are needed by any subprogram. With some care, the tree diagram can also be made to convey a sense of timing related to the execution of the program, that is, to show its dynamic or procedural structure. This is possible because the modules are shown in the order in which they are first called during execution.

These program structures have no hidden complexity. They are as simple as they appear in the tree diagram. They are linear structures in the sense that adding or deleting a branch requires only a local change, no matter where the addition or deletion happens to be. This is one of the desirable features of modularity. Another important feature is the ability to change any module with confidence that other modules will not be affected by the change, as long as the local modular interface remains intact. The interface consists of common blocks or arguments passed in call statements, or both.

---

†The standard has not been specified formally.

Table 2-1. Radar Program Library, RADAR:LIB
Catalog of Source and JCL Programs

10:19 MAR 27,'81 DC/RADAR:LIB.1096

RADAR PROGRAM LIBRARY CATALOG

REVISION : MARCH 27, 1981

FILE	AUTHOR(S)	ORIGINATED	REVISED
ALLPLOT:S	S. M. DANIEL I. KERTESZ	8/23/80	2/ 3/81
AMPH:S	S. M. DANIEL I. KERTESZ	4/19/80	7/01/80
ANTWT:S	S. M. DANIEL I. KERTESZ	4/19/80	9/01/80
BCREXEC:S	S. M. DANIEL I. KERTESZ	8/10/80	1/06/81
BCRMAIN:S	S. M. DANIEL I. KERTESZ	7/23/80	8/12/80
BCRSET:S	S. M. DANIEL I. KERTESZ	7/23/80	1/24/81
BCRSIM:S	S. M. DANIEL I. KERTESZ	4/15/78	9/17/80
BCRPMAIN:S	S. M. DANIEL I. KERTESZ	7/23/80	2/ 3/81
BCRPSET:S	S. M. DANIEL I. KERTESZ	7/23/80	2/13/81
BLINK:S	I. KERTESZ S. M. DANIEL	6/12/80	6/16/80

42 -	42.000	CANDB:S	S. M. DANIEL	9/23/80	1/06/81
43 -	43.000		I. KERTESZ		
44 -	44.000				
45 -	45.000	CHANNEL:S	S. M. DANIEL	5/04/80	1/25/81
46 -	46.000		I. KERTESZ		
47 -	47.000				
48 -	48.000	CMBSGNL:S	S. M. DANIEL	9/06/80	9/13/80
49 -	49.000		I. KERTESZ		
50 -	50.000				
51 -	51.000	CMPCHNL:S	S. M. DANIEL	8/04/80	1/30/81
52 -	52.000		I. KERTESZ		
53 -	53.000				
54 -	54.000	FFT2:S	PROF. P. RANSOM	7/15/69	9/01/80
55 -	55.000		S. M. DANIEL		
56 -	56.000				
57 -	57.000	FIELD:S	S. M. DANIEL	8/04/80	1/31/81
58 -	58.000		I. KERTESZ		
59 -	59.000				
60 -	60.000	FILTER:S	S. M. DANIEL	4/19/80	6/23/80
61 -	61.000		I. KERTESZ		
62 -	62.000				
63 -	63.000	FORM:S	S. M. DANIEL	4/15/78	9/13/80
64 -	64.000		I. KERTESZ		
65 -	65.000				
66 -	66.000	IMPULS:S	S. M. DANIEL	5/11/80	6/07/80
67 -	67.000		I. KERTESZ		
68 -	68.000				
69 -	69.000	JCL:B	S. M. DANIEL	6/20/80	7/01/80
70 -	70.000		G. C. WANG		
71 -	71.000				
72 -	72.000	JCL:EX	S. M. DANIEL	6/22/80	6/23/80
73 -	73.000		G. C. WANG		
74 -	74.000				
75 -	75.000	JCLBCRP:BL	S. M. DANIEL	1/06/81	2/26/81
76 -	76.000				
77 -	77.000	JCLBCRS:BL	S. M. DANIEL	1/06/81	2/26/81
78 -	78.000				
79 -	79.000	JCLSIG:BL	S. M. DANIEL	6/21/80	2/26/81
80 -	80.000		G. C. WANG		
81 -	81.000				
82 -	82.000	PLOTS:S	S. M. DANIEL	9/ 6/80	2/ 2/81
83 -	83.000		I. KERTESZ		
84 -	84.000				

85 -	85.000	PWRSUP:S	S. M. DANIEL	9/11/80	9/12/80
86 -	86.000		I. KERTESZ		
87 -	87.000				
88 -	88.000	RCNORM:S	S. M. DANIEL	8/31/78	1/26/81
89 -	89.000		I. KERTESZ		
90 -	90.000				
91 -	91.000	RW:S	S. M. DANIEL	4/19/78	6/15/80
92 -	92.000		I. KERTESZ		
93 -	93.000				
94 -	94.000	RWIRC:S	S. M. DANIEL	4/19/80	8/13/80
95 -	95.000		I. KERTESZ		
96 -	96.000				
97 -	97.000	PRTSGNL:S	S. M. DANIEL	6/01/80	1/05/81
98 -	98.000		I. KERTESZ		
99 -	99.000				
100 -	100.000	SIGMAIN:S	S. M. DANIEL	4/19/80	1/05/81
101 -	101.000		I. KERTESZ		
102 -	102.000				
103 -	103.000	SIGNAL:S	S. M. DANIEL	6/08/80	9/01/80
104 -	104.000		I. KERTESZ		
105 -	105.000				
106 -	106.000	SIGSET:S	S. M. DANIEL	6/01/80	1/15/81
107 -	107.000		I. KERTESZ		
108 -	108.000				
109 -	109.000	SKIPR:S	S. M. DANIEL	2/23/81	2/23/81
110 -	110.000		I. KERTESZ		
111 -	111.000				
112 -	112.000	TTEKS:S	S. M. DANIEL	8/01/80	2/23/81
113 -	113.000		I. KERTESZ		
114 -	114.000		G. C. WANG		
115 -	115.000				
116 -	116.000	WITER:S	S. M. DANIEL	9/13/80	9/13/80
117 -	117.000		I. KERTESZ		
118 -	118.000				
119 -	119.000	------------	------------	------------	------------

2.1.4 Source Modules

In writing the programs, a serious attempt was made to make their use self-explanatory. Thus a short functional description, complete input and output identification and generous commentary are included with each subprogram. The heading of each module also shows the original and latest revision dates, and identities of the authors. Also, in the case of subroutines, the subroutine name is identical to the module name, except for suffixes, which are not used in the subroutine name.

2.1.5 Binary Modules

When working with a small or a fully checked out program, it is often desirable to submit it for execution in what appears to be a one-step job. Although in reality the job may consist of compile, link, and run steps, the user is not concerned with these details provided the program runs properly. While developing a large program like SIGGEN, however, this action may not be satisfactory.

Although SIGGEN and the other programs have been through several versions, the modular structure was there from the beginning. This structure enabled the semi-independent planning, implementation of models, and refinement of the resulting modules. When a module achieved a satisfactory form, its binary version, which is the product of a compile step, was saved. Therefore, when future jobs required this module, its binary version could be used directly without needs to be edited and subsequently compiled to recreate the new binary version.

An example of the job control language program to compile a source module is shown in Table 2-2. The JCL program, JCL:B, defines a source type file

Table 2-2. JCL Program, JCL:B

```
1.000 !JOB 298,DANIEL(8512),7,BLDG93
2.000 !LIMIT (TIME,1),(UO,10),(CO,16),(ACCOUNT)
3.000 !.......JCL:B...........................
4.000 !SET M:SI/NAME:S;IN;SAVE
5.000 !SET M:BO/NAME:B;OUT;SAVE
6.000 !FORTRAN LS,NS,BC,SI,BO
```

and a binary file, and makes them available as input and output to the compiler. To submit the job, the interactive command,

!BATCH JCL:B 'NAME'=XXXX

is entered, where XXXX following the equal sign stands for the desired source module name to be used. Thus program JCL:B is general.

It should be noted that the above example and other JCL programs to be shown in the following paragraphs apply specifically to the Honeywell CP-V computer system. The purpose and the general result of these programs, however, apply to any system.

2.1.6 Load Modules

The modularity of the programs makes it possible to use the program-library approach. Here are three of the most obvious advantages:

1. Reduction of file space because of shared member programs and subprograms.

2. Savings of time and effort in file manipulation, editing, and printing, because work is concentrated only on the modules affected, rather than the entire program.

3. Easier to maintain because there is only one copy of each member. In case of shared subprograms, this avoids the necessity of making the same change in several places.

A load module, NAME:L, an executable version of a complete source program, NAME:S, may be formed as follows. First, the binary versions of the set of modules comprising the program are concatenated into a composite binary module, NAME:B. Subsequently, this composite binary module is copied over the load module via a LYNX command. This process is carried out via the execution of a JCL program, JCLNAME:BL, listed in Table 2-3.†

Table 2-3. JCL Program, JCLNAME:BL

```
21:24 MAR 20,'81 DC/JCLNAME:BL.298

1.000 !JOB 298,DANIEL(8512),7,BLDG90
2.000 !LIMIT (TIME,1),(UO,2),(CO,16),(ACCOUNT)
3.000 !.....JCLNAME:BL.......................
4.000 !PCL
5.000 C      MAIN:B       OVER NAME:B
6.000 C      SUB1:B
7.000 C      SUB2:B
8.000 !LYNX NAME:B        OVER NAME:L
```

This JCL program is executed with the command.

!BATCH JCLNAME:BL

Note that after such an execution all library modules are returned to the library intact.

†An alternate method for forming NAME:L involves a direct linking of the individual binary without the need to create a composite binary module.

2.1.7 Generic Modular Program Structure

The modular program description discussed above is summarized clearly in Figure 2-1. Demonstrated there are the ideas and steps described, from the conception of the mathematical model to the desired output. The code "NAME" is used to indicate a general program name to be substituted upon execution. As such, this figure represents a procedure that is applicable to the three major programs to be discussed next; namely, SIGGEN, BCRS and BCRP.

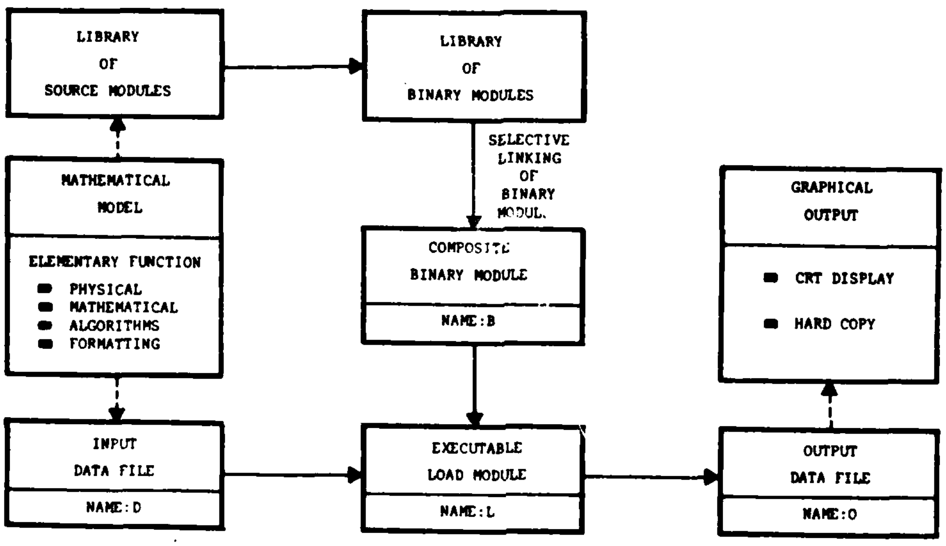


Figure 2-1. Generic Modular Program Structure

2.2 Signal Generation Program - SIGGEN

As already described in Project Memorandum 8512-02, SIGGEN is a modular program designed to generate received baseband signals at the main antenna array and specified auxiliary ports, of the adaptive array system under consideration, due to a number of incident wideband signal sources. In addition to its specific capabilities stated there, the latest version of SIGGEN included below is capable of generating baseband signals at specified port delayline taps from direct or multipath signals.

The SIGGEN program uses input file SIGGEN:D, a menu which is used to define the source scenario and system configuration. Upon execution, the output file SIGGEN:0 is formed which contains sampled baseband port signals stored in complex arrays whose dimensionality is equal to the desired number of samples, limited presently to 256.

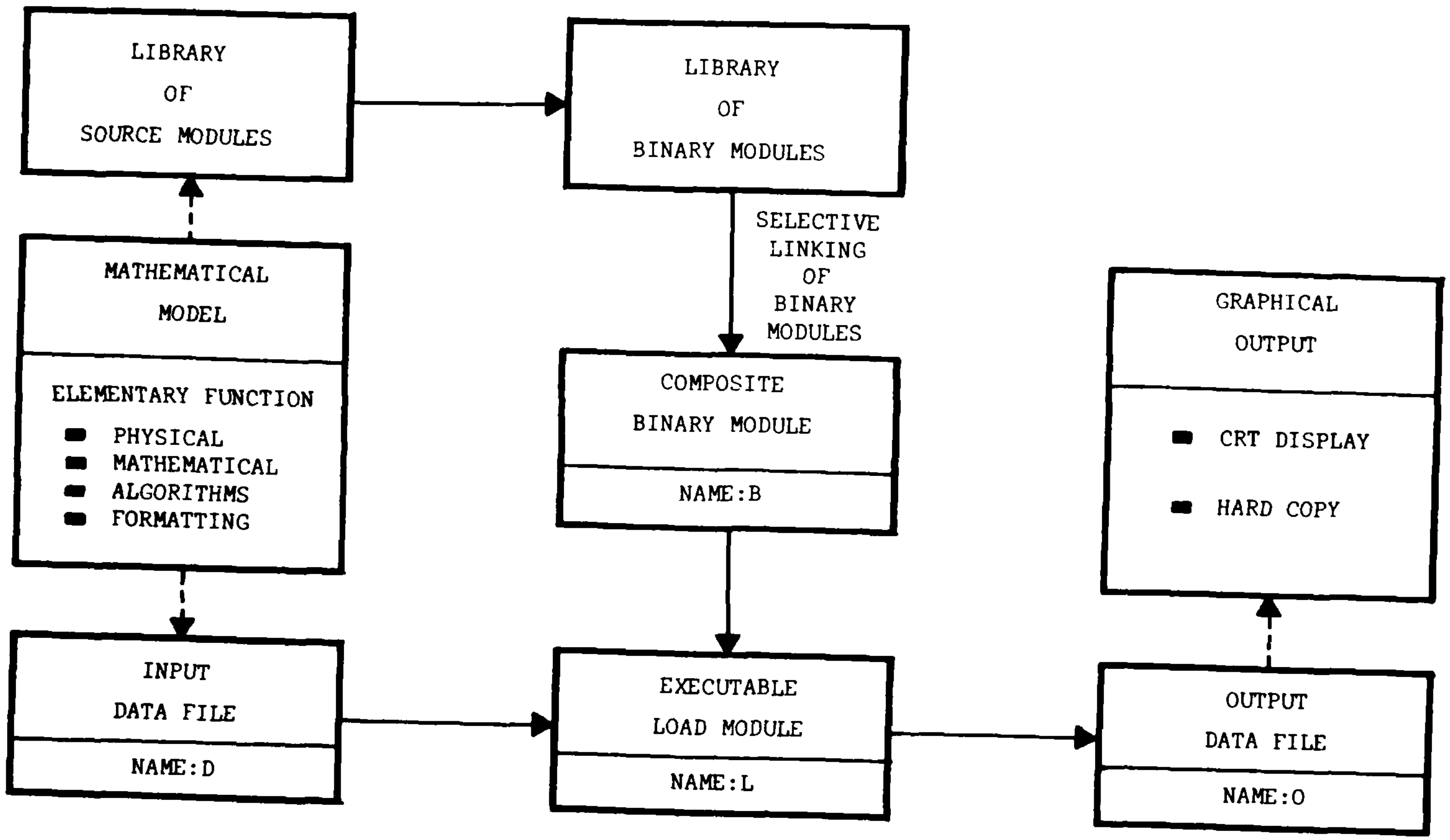


Figure 2-1. Generic Modular Program Structure

2.2.1 Input Data File - SIGGEN:D

The input data file called SIGGEN:D is listed in Table 2-4. It consists of four major parts: (1) Output control, (2) Filter, (3) Channel, and (4) Port descriptions. The first part contains the output options. These options do not affect the signal generation process; however, they can provide intermediately computed arrays for purposes of closer examination by the user. The second part of the input data file contains the description of the generic receiver in terms of its IF filter characteristics. This part, and the subsequent channel and port descriptions, completely determine the outcome of the signal general process. The third part of the data contains the channel description. An identical set of parameters is provided to describe each channel. The port parameters are included in the last part of the data file. Each port has a similar set of parameters.

A detailed description of each input parameter and the possible relationships among the parameters is indicated in Table 2-5. The parameters are listed in the same order in which they occur in the data file. This parameter description will be a useful reference for the following discussion of the benchtest scenario which is defined by the SIGGEN:D file in Table 2-4.

It can be noted that the print options are all set to the value 0, since the output controlled by them is not needed in this case.†

The receiver parameters show a 2-pole filter with a fractional bandwidth at the final IF of 10%, and a fractional bandwidth at RF of 0.1%. The normalized radian frequency range is 4.0, starting at -2.0 with a total of 32 frequency samples specified. The filter data is completed with the choice of the bandpass option.

The channel-data portion begins by indicating the number of channels to be 20. This means that the description of 20 channels will follow, although in this case not all will be used. In fact, the scenario and system description of SIGGEN:D constitutes the benchtest example illustrated in Figure 2-2. Shown there are two noise sources incident from 45° and 10° respectively. Source number 2 is given an intentional 0.1 sample-time delay. Looking at the channel parameters in Figure 2-2, it is obvious that channels 1 and 2 correspond to noise sources 1 and 2 in the scenario schematic, respectively. Channel 1 has an amplitude factor of 1.0, random number generator seed (IX0) of 1, and is incident from 45°. The first-time-on sample number is 1, and the blink duration exceeds NS (the number of signal samples, to be discussed later), so that this is a non-blinking source. The channel delay is set at 0.0. This describes noise source number 1. Channel 2 involves a distinct source with a seed that is different from that of channel 1. It is a continuous source with relative amplitude of 1.0, however, it shows a delay of 0.1 sample-time.

---

†The output data eliminated by choice is either not needed for the next program or recalculated as needed there for purposes of convenience.

Table 2-4. Input Data File of the Signal Generation Program, SIGGEN:D Benchtest Example (See Figure 2-2)

```
-------------------------------------------------------------------------------
                     SPECIFICATION OF SYSTEM PARAMETERS
                     ----------------------------------

-------------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWANT   -  MAIN ANTENNA ARRAY WEIGHTING             :        0
  IWRAP   -  RECEIVER AMPLITUDE AND PHASE             :        0
  IWRI    -  RECEIVER IMPULSE RESPONSE                :        0
  IWCAP   -  CHANNEL AMPLITUDE AND PHASE              :        0
  IWCI    -  CHANNEL IMPULSE RESPONSE                 :        0
  IWSC    -  INDIVIDUAL CHANNEL SIGNALS               :        0

FILTER PARAMETERS
-----------------
  NPOL    -  NUMBER OF LOWPASS PROTOTYPE POLES        :        2
             POL(1)                                   :  -.70700   -.70700
             POL(2)                                   :  -.70700    .70700
  FBIF    -  FRACTIONAL BANDWIDTH AT FINAL IF         :   .10000
  FBRF    -  FRACTIONAL BANDWIDTH AT RF               :   .00100
  RADRNG  -  NORMALIZED RADIAN FREQUENCY RANGE        :  4.00000
  RADIN   -  NORMALIZED INITIAL RADIAN FREQUENCY      : -2.00000
  NF      -  NUMBER OF FREQUENCY SAMPLES              :       32
  LPBP    -  LOWPASS/BANDPASS OPTION                  :        1
             0 :   LOWPASS
             1 :   BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS  -  NUMBER OF CHANNELS PER PORT              :       20
  ISC     -  CHANNEL SELECTOR ARRAY                   :  1 1 0 0 0 0 0 0 0 0
                                                      :  0 0 0 0 0 0 0 0 0 0

CHANNEL  1:
  AN      -  AMPLITUDE OF NOISE SOURCE                :  1.00000
  IX0     -  INITIAL RANDU SETTING                    :        1
  N1      -  FIRST-TIME-ON SAMPLE NUMBER              :        1
  NB      -  BLINK DURATION IN SAMPLES                :    10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)        : 45.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS       :   .00000

CHANNEL  2:
```

```
  AN      -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IX0     -  INITIAL RANDU SETTING                     :          11
  N1      -  FIRST-TIME-ON SAMPLE NUMBER               :           1
  NB      -  BLINK DURATION IN SAMPLES                 :       10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :    10.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :      .10000

CHANNEL  3:
  AN      -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IX0     -  INITIAL RANDU SETTING                     :         111
  N1      -  FIRST-TIME-ON SAMPLE NUMBER               :           1
  NB      -  BLINK DURATION IN SAMPLES                 :       10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :   -20.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :      .00000

CHANNEL  4:
  AN      -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IX0     -  INITIAL RANDU SETTING                     :        1111
  N1      -  FIRST-TIME-ON SAMPLE NUMBER               :           1
  NB      -  BLINK DURATION IN SAMPLES                 :       10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :   -35.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :      .00000

CHANNEL  5:
  AN      -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IX0     -  INITIAL RANDU SETTING                     :           3
  N1      -  FIRST-TIME-ON SAMPLE NUMBER               :           1
  NB      -  BLINK DURATION IN SAMPLES                 :       10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :    55.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :      .00000

CHANNEL  6:
  AN      -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IX0     -  INITIAL RANDU SETTING                     :          33
  N1      -  FIRST-TIME-ON SAMPLE NUMBER               :           1
  NB      -  BLINK DURATION IN SAMPLES                 :       10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :    30.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :      .00000

CHANNEL  7:
  AN      -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IX0     -  INITIAL RANDU SETTING                     :         333
  N1      -  FIRST-TIME-ON SAMPLE NUMBER               :           1
```

```
   NB     -  BLINK DURATION IN SAMPLES                   :       10000
   TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)           :   -50.00000
   CDEL   -  CHANNEL DELAY IN SAMPLE-TIME UNITS          :      .00000

CHANNEL  8:
   AN     -  AMPLITUDE OF NOISE SOURCE                   :     1.00000
   IX0    -  INITIAL RANDU SETTING                       :        3333
   N1     -  FIRST-TIME-ON SAMPLE NUMBER                 :           1
   NB     -  BLINK DURATION IN SAMPLES                   :       10000
   TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)           :    -5.00000
   CDEL   -  CHANNEL DELAY IN SAMPLE-TIME UNITS          :      .00000

CHANNEL  9:
   AN     -  AMPLITUDE OF NOISE SOURCE                   :     1.00000
   IX0    -  INITIAL RANDU SETTING                       :       33333
   N1     -  FIRST-TIME-ON SAMPLE NUMBER                 :           1
   NB     -  BLINK DURATION IN SAMPLES                   :       10000
   TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)           :     20.0000
   CDEL   -  CHANNEL DELAY IN SAMPLE-TIME UNITS          :      .00000

CHANNEL 10:
   AN     -  AMPLITUDE OF NOISE SOURCE                   :     1.00000
   IX0    -  INITIAL RANDU SETTING                       :         444
   N1     -  FIRST-TIME-ON SAMPLE NUMBER                 :           1
   NB     -  BLINK DURATION IN SAMPLES                   :       10000
   TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)           :   -40.00000
   CDEL   -  CHANNEL DELAY IN SAMPLE-TIME UNITS          :      .00000

CHANNEL 11:
   AN     -  AMPLITUDE OF NOISE SOURCE                   :     1.00000
   IX0    -  INITIAL RANDU SETTING                       :       10001
   N1     -  FIRST-TIME-ON SAMPLE NUMBER                 :           1
   NB     -  BLINK DURATION IN SAMPLES                   :       10000
   TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)           :   -29.00000
   CDEL   -  CHANNEL DELAY IN SAMPLE-TIME UNITS          :      .00000

CHANNEL 12:
   AN     -  AMPLITUDE OF NOISE SOURCE                   :     1.00000
   IX0    -  INITIAL RANDU SETTING                       :       10011
   N1     -  FIRST-TIME-ON SAMPLE NUMBER                 :           1
   NB     -  BLINK DURATION IN SAMPLES                   :       10000
   TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)           :   -24.00000
   CDEL   -  CHANNEL DELAY IN SAMPLE-TIME UNITS          :      .00000
```

```
CHANNEL 13:
  AN     -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IX0    -  INITIAL RANDU SETTING                     :       10111
  N1     -  FIRST-TIME-ON SAMPLE NUMBER               :           1
  NB     -  BLINK DURATION IN SAMPLES                 :       10000
  TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :   -17.00000
  CDEL   -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :      .00000

CHANNEL 14:
  AN     -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IX0    -  INITIAL RANDU SETTING                     :       11110
  N1     -  FIRST-TIME-ON SAMPLE NUMBER               :           1
  NB     -  BLINK DURATION IN SAMPLES                 :       10000
  TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :   -11.00000
  CDEL   -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :      .00000

CHANNEL 15:
  AN     -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IX0    -  INITIAL RANDU SETTING                     :       10003
  N1     -  FIRST-TIME-ON SAMPLE NUMBER               :           1
  NB     -  BLINK DURATION IN SAMPLES                 :       10000
  TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :     3.00000
  CDEL   -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :      .00000

CHANNEL 16:
  AN     -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IX0    -  INITIAL RANDU SETTING                     :       10033
  N1     -  FIRST-TIME-ON SAMPLE NUMBER               :           1
  NB     -  BLINK DURATION IN SAMPLES                 :       10000
  TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :    14.00000
  CDEL   -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :      .00000

CHANNEL 17:
  AN     -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IXC    -  INITIAL RANDU SETTING                     :       10333
  N1     -  FIRST-TIME-ON SAMPLE NUMBER               :           1
  NB     -  BLINK DURATION IN SAMPLES                 :       10000
  TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :    36.00000
  CDEL   -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :      .00000

CHANNEL 18:
  AN     -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
```

```
  IX0     -  INITIAL RANDU SETTING                       :      13300
  N1      -  FIRST-TIME-ON SAMPLE NUMBER                 :          1
  NB      -  BLINK DURATION IN SAMPLES                   :      10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)           :   40.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS          :     .00000

CHANNEL 19:
  AN      -  AMPLITUDE OF NOISE SOURCE                   :    1.00000
  IX0     -  INITIAL RANDU SETTING                       :      13000
  N1      -  FIRST-TIME-ON SAMPLE NUMBER                 :          1
  NB      -  BLINK DURATION IN SAMPLES                   :      10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)           :    44.0000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS          :     .00000

CHANNEL 20:
  AN      -  AMPLITUDE OF NOISE SOURCE                   :    1.00000
  IX0     -  INITIAL RANDU SETTING                       :      10444
  N1      -  FIRST-TIME-ON SAMPLE NUMBER                 :          1
  NB      -  BLINK DURATION IN SAMPLES                   :      10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)           :   50.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS          :     .00000

PORT PARAMETERS
---------------
  D0      -  ANTENNA-ELEMENT SEPARATION FACTOR           :    1.00000
  NS      -  NUMBER OF SIGNAL SAMPLES                    :        128
  NPORTS  -  NUMBER OF PORTS                             :         21
  ISP     -  PORT SELECTOR ARRAY                         :    1 1 1 1 0 0 0 0 0 0
                                                         :    0 0 0 0 0 0 0 0 0 0

PORT      0:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                  :        255
  LOC1    -  LOCATION OF THE FIRST ELEMENT               :          1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                  :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                     :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY          :     .00000

PORT      1:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                  :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT               :          1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                  :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                     :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY          :     .00000
```

```
PORT      2:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :          1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .10000

PORT      3:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :        255
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT      4:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :        255
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .10000

PORT      5:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :         11
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :    -.00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT      6:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :        202
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT      7:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :         31
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000
```

```
PORT     8:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :        163
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT     9:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :         59
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT    10:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :        227
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT    11:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :          1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT    12:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :          7
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT    13:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :        127
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT    14:
```

```
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :        255
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT    15:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :         11
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :    -.00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT    16:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :        202
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT    17:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :         31
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT    18:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :        163
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT    19:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :         59
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .00000

PORT    20:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
```

```
LOC1    -  LOCATION OF THE FIRST ELEMENT            :        227
BWFCTR  -  BANDWIDTH TOLERANCE FACTOR               :    1.00000
BWOFF   -  BANDWIDTH OFFSET FACTOR                  :     .00000
PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY       :     .00000
-------------------------------------------------------------------
```

Table 2-5. Description of Input Data for the Signal Generation Program, SIGGEN

```
--------------------------------------------------------------------------------
                SIGNAL GENERATION PROGRAM INPUT DATA DESCRIPTION
                ------------------------------------------------

--------------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWANT   -  MAIN ANTENNA ARRAY WEIGHTING

  IWRAP   -  RECEIVER AMPLITUDE AND PHASE

  IWRI    -  RECEIVER IMPULSE RESPONSE

  IWCAP   -  CHANNEL AMPLITUDE AND PHASE

  IWCI    -  CHANNEL IMPULSE RESPONSE

  IWSC    -  INDIVIDUAL CHANNEL SIGNALS

               ALL THE PRINT OPTION PARAMETERS WHEN SET TO  1 SERVE TO OUTPUT
               THE INDICATED DATA.  WHEN SET TO  0  THE OUTPUT IS SUPPRESSED.
               THIS IS RECOMMENDED WHEN THE OUTPUT DATA IS TO BE USED AS INPUT
               TO A SUBSEQUENT PROGRAM.

FILTER PARAMETERS
-----------------
  NPOL    -  NUMBER OF LOWPASS PROTOTYPE POLES LIMITED TO SIX.

  POL(1)  -  LOCATION OF POLE NUMBER 1. A COMLEX NUMBER
  POL(2)  -  LOCATION OF POLE NUMBER 2. A COMPLEX NUMBER
  .
  .
  POL(NPOL)
          -  LOCATION OF POLE NUMBER NPOL. A COMPLEX NUMBER. THE LAST ONE TO
             BE SPECIFIED.

  FBIF    -  FRACTIONAL BANDWIDTH AT FINAL IF.

  FBRF    -  FRACTIONAL BANDWIDTH AT RF.

  RADRNG  -  NORMALIZED RADIAN FREQUENCY RANGE.  TWICE THE NORMALIZED BANDWIDTH
             AT RF.  A GOOD VALUE IS 4.0.
```

RADIN - NORMALIZED INITIAL RADIAN FREQUENCY. THE FIRST FREQUENCY SAMPLE STARTS HERE, AND THE OTHERS FOLLOW AT INCREMENTS OF RADRNG/NF. IF RADRNG = 4.0, THEN A GOOD VALUE FOR RADIN IS -2.0.

NF - NUMBER OF FREQUENCY SAMPLES IN POWERS OF 2, RANGING UP TO 64.

LPBP - LOWPASS/BANDPASS OPTION
0 : LOWPASS
1 : BANDPASS
THE BANDPASS OPTION IS USED IN MOST PROBABLE IMPLEMENTATION.

CHANNEL PARAMETERS

NCHNLS - NUMBER OF CHANNELS PER PORT. THIS IS THE ACTUAL NUMBER OF CHANNELS WHOSE DESCRIPTION FOLLOWS. THE NUMBER OF CHANNELS DESCRIBED BELOW MUST BE EXACTLY NCHNLS. RANGE OF VALUES IS FROM 1 TO 20.

ISC - CHANNEL SELECTOR ARRAY. BY ASSIGNING 1 TO ANY ELEMENT OF THIS ARRAY, THE CHANNEL CORRESPONDING TO THE ELEMENT IS SELECTED FROM THE CHANNEL DESCRIPTIONS WHICH FOLLOW. THE EXAMPLE BELOW SELECTS CHANNELS 2,4,11, AND 19 FROM AN ASSUMED NUMBER OF 20 CHANNEL DESCRIPTIONS. NCHNLS ELEMENTS OF ISC MUST BE DEFINED.

ISC - CHANNEL SELECTOR ARRAY : 0 1 0 1 0 0 0 0 0 0
: 1 0 0 0 0 0 0 0 1 0

AN - AMPLITUDE OF NOISE SOURCE. ANY REASONABLE VALUE MAY BE ASSIGNED. A GOOD VALUE IS 1.0.

IX0 - INITIAL RANDU SETTING. THIS IS A SEED FOR THE RANDOM NUMBER GENERATOR. IT IS USED TO INSURE THE REPEATABILITY OF RANDOM SIGNALS, AND SHOULD BE A UNIQUE ODD INTEGER FOR EACH CHANNEL. FOR MULTIPATH SIMULATIONTHE SAME SEED SHOULD BE ASSIGNED TO ALL CHANNELS WHICH REPRESENT MULTIPLE PATHS OF THE SAME SIGNAL.

N1 - FIRST-TIME-ON SAMPLE NUMBER. THE SAMPLE NUMBER AT WHICH THE NOISE SOURCE IS FIRST "ON". SET N1 TO A VALUE BETWEEN 1 AND NS FOR A BLINKING SOURCE. SET N1 TO 1 TO REPRESENT A CONTINUOUS SOURCE.

NB - BLINK DURATION IN SAMPLES. NUMBER OF SAMPLES FOR WHICH TH BLINKING SOURCE IS ON. SET NB TO A VALUE BETWEEN 1 AND NS FOR A BLINKING SOURCE. SET NB TO A VALUE GREATER THAN NS TO REPRESENT A CONINUOUS SOURCE. NOTE THE INTERDEPENDENCE OF N1 AND NB.

TH - SOURCE ANGULAR DIRECTION OF INCIDENCE IN DEGREES.

CDEL - CHANNEL DELAY IN SAMPLE-TIME UNITS. USED TO REPRESENT MULTIPATH. SET TO ZERO FOR DISTINCT CHANNEL. NOTE THAT IN ADDITION TO RELATIVE DELAYS SPECIFIED BY CDEL, A GROUP OF MULTIPATH CHANNELS ARE CHARACTERIZED BY THE SAME RANDOM NUMBER GENERATOR SEED, IXU.

PORT PARAMETERS

D0 - ANTENNA-ELEMENT SEPARATION FACTOR. INTER-ELEMENT SPACING OF THE ANTENNA ARRAY. D0 = 1.0 FOR SPACING OF HALF WAVELENGTH.

NS - NUMBER OF SIGNAL SAMPLES RANGING UP TO 256. TO 256.

NPORTS - NUMBER OF PORTS RANGING UP TO 21. THIS IS THE ACTUAL NUMBER OF AUXILIARY PORTS PLUS THE MAIN PORT, WHOSE DESCRIPTION FOLLOWS. THE NUMBER OF PORTS DESCRIBED BELOW MUST BE EXACTLY NPORTS.

ISP - PORT SELECTOR ARRAY. BY ASSIGNING 1 TO ANY ELEMENT OF THIS ARRAY, THE AUXILIARY PORT CORRESPONDING TO THE ELEMENT IS SELECTED FROM THE PORT DESCRIPTIONS WHICH FOLLOW. THE EXAMPLE BELOW SELECTS PORTS 2,4,10 AND 20 FROM AN ASSUMED NUMBER OF 20 AUXILIARY PORTS. NPORTS - 1 ELEMENTS OF ISP MUST BE DEFINED.

ISP - PORT SELECTOR ARRAY : 0 1 0 1 0 0 0 0 0 1
: 0 0 0 0 0 0 0 0 0 1

NEL - NUMBER OF ANTENNA ELEMENTS. NUMBER OF ELEMENTS IN THE MAIN ARRAY. UP TO 1001 ELEMENTS MAY BE SPECIFIED.

LOC1 - LOCATION OF THE FIRST ELEMENT. LOC1 IS USUALLY 1 FOR PORT 0. FOR THE AUXILIARY PORTS LOC1 CAN BE ANY NUMBER BETWEEN 1 AND NEL. FOR TAP DELAY SIMULATION A PORT HAVING N-TAPS IS REPRESENTED BY N-PORTS WITH IDENTICAL VALUES ASSIGNED TO LOC1.

```
BWFCTR  -  BANDWIDTH TOLERANCE FACTOR RANGING FROM 1.0 TO 0.0.  THIS PARAME-
           TER REPRESENTS THE FRACTIONAL BANDWIDTH ACHIEVED IN THE DESIGN.
           WHEN BWFCTR=1.0 THE DESIGNED FILTER BANDWIDTH IS AS DESIRED.

BWOFF   -  BANDWIDTH OFFSET FACTOR RANGING FROM -1.0 TO 1.0.  THIS PARAMETER
           INDICATED THE FILTER CENTER FREQUENCY SHIFT FROM THE DESIRED VALUE
           AS A FRACTION OF BANDWIDTH.

PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY.  USED TO REPRESENT TAP-DELAY
           ON PORTS.
--------------------------------------------------------------------------------
```

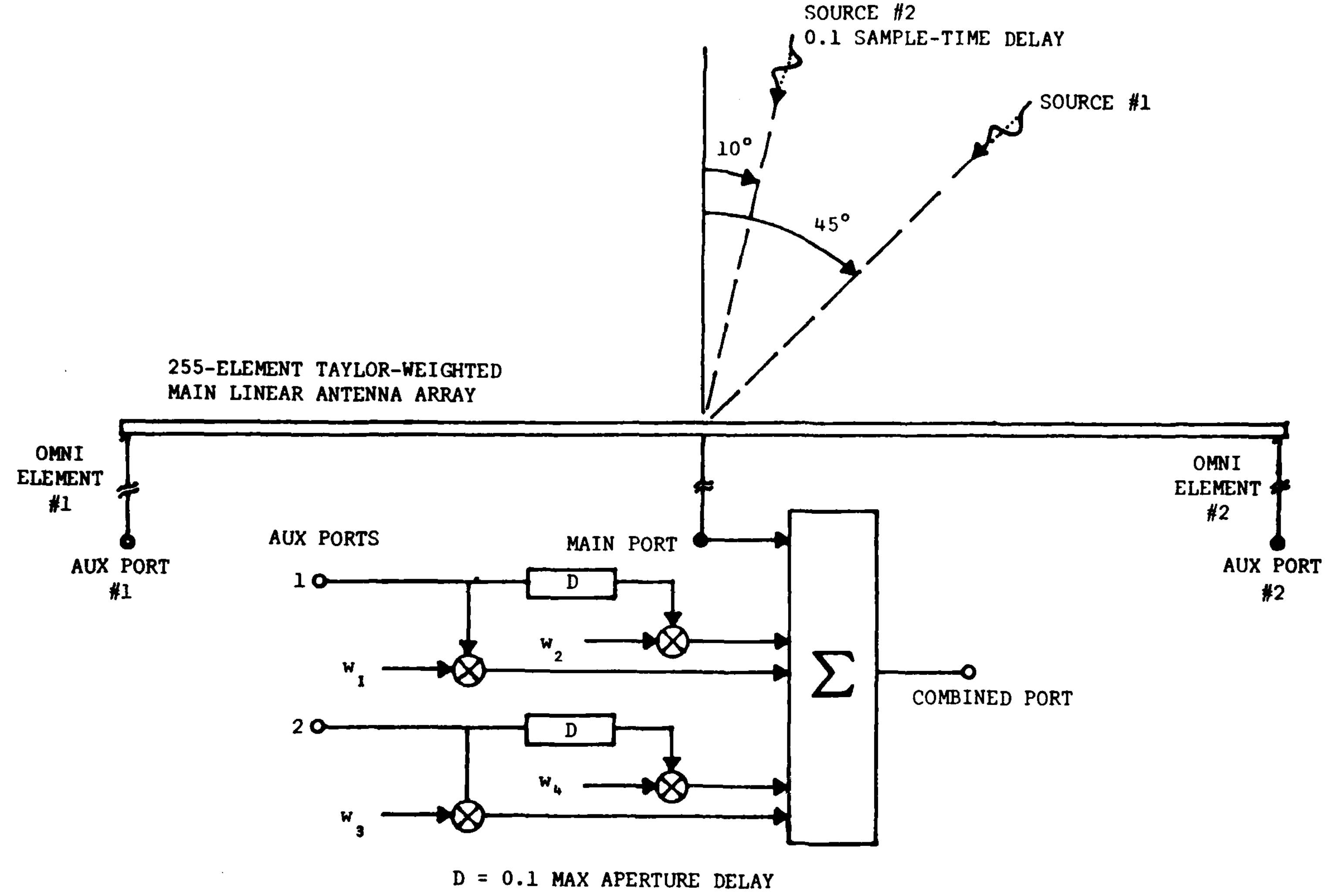


Figure 2-2. Benchtest Example. Scenario and System Description
0.1% RF Bandwidth
Benchtest Example

This delay is relative to some other signal which could be represented through the use of another channel with identical seed and zero delay, but possibly different angle and amplitude, as in a multipath situation. In this simple example, however, this was not done. For the current scenario the first two channels were selected by setting the first two elements of the selector array, ISC, to 1, and the next 18 elements to 0.† This completes the channel description.

The port-data portion begins by defining the element separation to be one-half of a wavelength at RF (i.e., DO = 1.0), the number of signal samples and ports, 128 and 21, respectively. Since the main port, PORT 0, is always selected, the port-selector array, ISP, need only be defined for the 20 auxiliary ports whose description follows that of the main port.

It is evident in Figure 2-2 that in addition to the main port, two auxiliary ports are also to be used. One auxiliary is at element number 1, the other is at element 255. Each auxiliary port has two taps, the 0-delay tap and one with delay D, chosen to be 0.1 of the maximum main antenna aperture delay. This situation is clearly reflected in the port parameter description of Table 2-4. Port 0, the main port, has 255 elements. Ports 1 and 2 are shown to share element one. They are distinguished, however, by their port delays, 0.0 and 0.1, respectively. As such, the combination may be viewed as a two-tap auxiliary port at element number one. In the same manner ports 3 and 4 represent another two-tap auxiliary port at element number 255.

2.2.2 Program Structure

The signal generation program was designed to have the modular and linear structure described in the introductory paragraphs. It makes use of the modularity by following the "library" approach, i.e., referencing subprograms which are members of a larger set of programs. Data input was intentionally restricted to one branch of the program for easier maintenance, and as a natural feature of the modular concept.

2.2.2.1 Tree Diagram

The structure of the signal generation program, SIGGEN, is best demonstrated by using a tree diagram as seen in Figure 2-3. Starting with module SIGMAIN and moving along the first horizontal line, the first substructure headed by SIGSET is encountered. During execution, the first subroutine call made by SIGMAIN is to SIGSET. SIGSET in turn calls RW, RWIRC, and eventually SKIPR. So, the parallel between structure and timing is becoming evident. However, in SIGSET several other calls were made to RW, RWIRC and SKIPR, and not necessarily in that order. This is not apparent from the structure. Each time a call is made to a related subroutine, and the program branches out, control eventually returns to the next statement in the subprogram in which the first call was made. This is when a branch of the structure has been traced or completed in space or time.

---

†Other channels could have been selected by setting their corresponding ISC elements to 1.

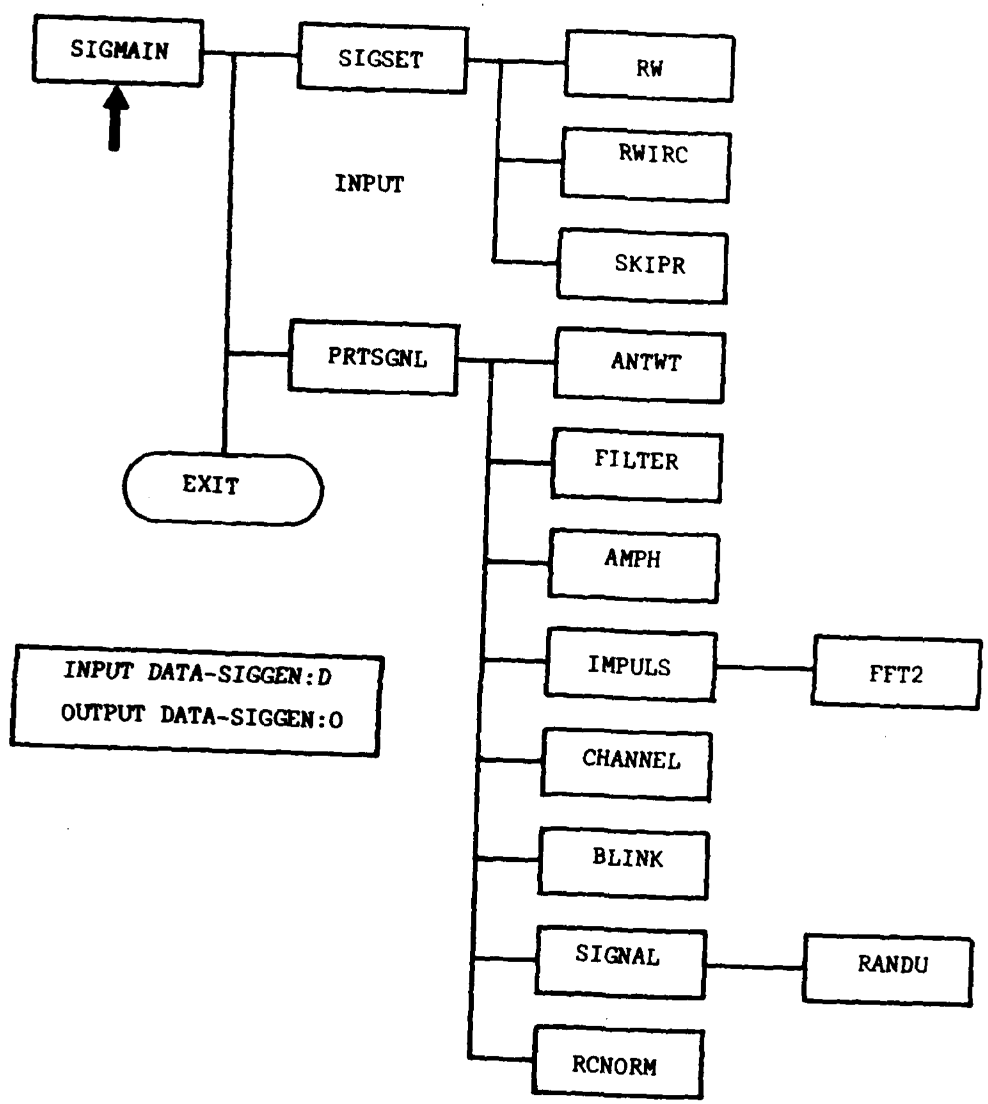


Figure 2-3. Tree Diagram of the Signal Generation Program, SIGGEN

Moving down the nearest untraced vertical line, the PRTSGNL substructure is encountered. It appears simple, and in the case of a 1-port/1-channel problem, it also represents the execution of this branch of the program. What the substructure does not show, is the iterative activity embedded into subprogram PRTSGNL. This activity is evident in the signal flow diagram of SIGGEN in Figure 2-4. There, the input data block is equivalent to the SIGSET branch discussed above. Then, at the beginning of the PRTSGNL branch the antenna array weighting is computed through ANTWT. For each execution of the port-loop the channel-loop is executed NCHNLS times, where NCHNLS is the number of channels. The port-loop is executed NPORTS times, where NPORTS is the number of ports. Once this is done, control again reverts to SIGMAIN, and EXIT is called.

Although the iterations occur on a modular level; that is, some branches are executed repeatedly, the control of the iterations is localized to the executive subprogram, PRTSGNL. Its job is simply accomplished by setting up a dual channel/port loop, making calls to the appropriate dedicated subroutines, and printing the results.

2.2.2.2 Source Modules

In the discussion of program structure, all the modules were referred to without a suffix. This was proper because of the one-to-one correspondence between source and binary modules. Thus a reference to any module applied to both types. This paragraph addresses source modules only, those with the suffix, :S.

A FORTRAN listing of all source modules comprising the signal generation source program, SIGGEN:S, is given in the next several pages under Table 2-6. A functional description of each source module of SIGGEN follows in Table 2-7.

2.2.2.3 Binary Modules

Each source module in the SIGGEN program has its binary version. Using the general JCL program introduced in Table 2-1, the binary module can be created by the method shown in paragraph 2.1.5.

2.2.2.4 Composite Binary and Load Modules

The creation of the executable load module for the SIGGEN program can be done in different ways. The JCL program, JCLSIG:BL, of Table 2-8 shows one of the methods. Here the individual binary modules are concatenated into a composite binary module, SIGGEN:B, in an order and indentation consistent with the tree-structure of Figure 2-3. This step accounts for the suffix B in the JCLSIG:BL-name. The next step of the program is to link this composite binary with the system library. This is done through the LYNX command. If all references in the linked programs are satisfied, then an executable load module, SIGGEN:L is created. This accounts for the suffix L in the JCL program name. This name, JCLSIG:BL is simply a convenient and expressive identifier, and it is not required to be in that form.

The execution of the concatenate-link JCL program is done with the interactive command

!BATCH JCLSIG:BL

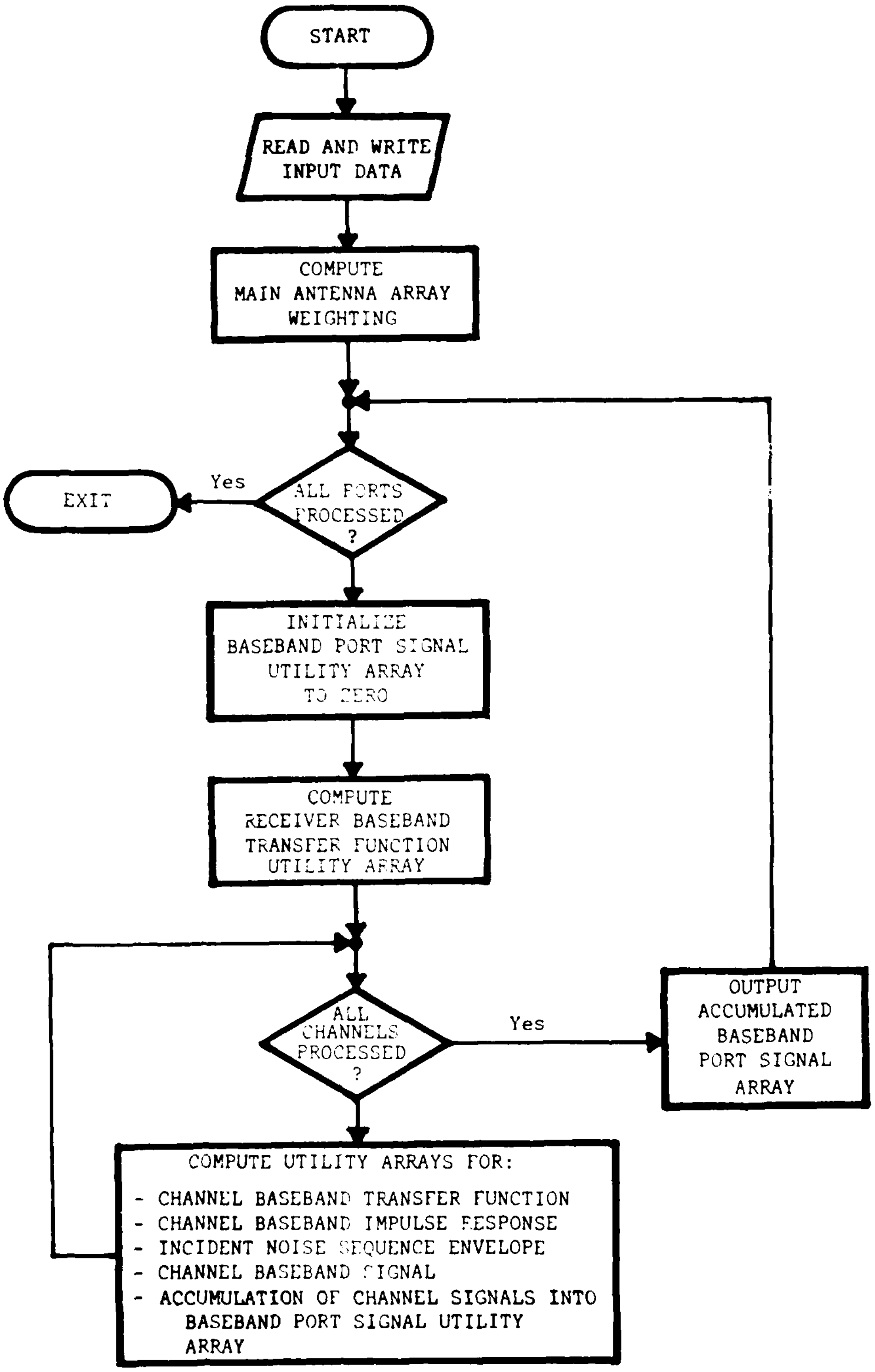


Figure 2-4. Functional Flow Diagram of the Signal Generation Program, SIGGEN

Table 2-6. FORTRAN Listing of Source Modules Comprising the Signal Generation Program, SIGGEN:S

```
      C ======================================================================
      C               SIGNAL  GENERATION  PROGRAM  :    SIGGEN:S
      C               ---------------------------------------
      C
      C                 COMPUTATION OF COMPLEX BASEBAND SIGNALS
      C                        AT MULTIPLE RECEIVER PORTS
      C                  CONNECTED TO SPECIFIED LINEAR ARRAYS
      C
      C              PROGRAM        :  SIGMAIN:S
      C              ORIGINAL       :  APRIL   19, 1980
      C              REVISION       :  JANUARY  5, 1981
      C
      C              PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
      C                                RADAR SYSTEMS ANALYSIS GROUP
      C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
      C                                TEMPE, ARIZONA  85282
      C ======================================================================
            CALL SIGSET
            CALL PRTSGNL
            CALL EXIT
            END
```

```
C ==================================================================
      SUBROUTINE SIGSET
C ==================================================================
C                   SPECIFICATION OF SYSTEM PARAMETERS
C
C             PROGRAM      :  SIGSET:S
C             ORIGINAL     :  JUNE      1, 1980
C             REVISION     :  JANUARY 15, 1981
C
C             PREPARED  BY :  S. M. DANIEL  &  I. KERTESZ
C                             RADAR SYSTEMS ANALYSIS GROUP
C                             MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                             TEMPE, ARIZONA  85282
C ------------------------------------------------------------------
C     INPUT  :    DATA FILE SIGGEN:D
C     OUTPUT :    DATA FILE SIGGEN:O
C ==================================================================
      COMPLEX POL,CDUM
      COMMON /SIG0/ IWANT,IWRAP,IWRI,IWCAP,IWCI,IWSC
      COMMON /SIG1/ POL(6),FBIF,FBRF,RADRNG,RADIN,DRAD,NPOL,LPBP,NF
      COMMON /SIG2/ AN(20),TH(20),CTAU(20),IX0(20),N1(20),NB(20)
     -,ISC(20),IDC(20),NCHNLS
      COMMON /SIG3/ BWFCTR(21),BWOFF(21),PTAU(21),D0,NEL(21),LOC1(21)
     -,ISP(20),IDP(21),NPORTS,NS
      DIMENSION AR14(14)
      DATA PI/3.14159265/
  100 FORMAT(1H1)
  200 FORMAT(14A4,4X,10I2)
      WRITE(108,100)
C ------------------------------------------------------------------
C     READ IN SYSTEM SPECIFICATIONS FROM  SIGGENT:D
C ------------------------------------------------------------------
C     READ IN PRINT OPTIONS
C ------------------------------------------------------------------
      CALL RW(7)
      CALL RWIRC(CDUM,RDUM,IWANT,0)
      CALL RWIRC(CDUM,RDUM,IWRAP,0)
      CALL RWIRC(CDUM,RDUM,IWRI,0)
      CALL RWIRC(CDUM,RDUM,IWCAP,0)
      CALL RWIRC(CDUM,RDUM,IWCI,0)
      CALL RWIRC(CDUM,RDUM,IWSC,0)
C ------------------------------------------------------------------
C     READ IN FILTER PARAMETERS
```

```
C -----------------------------------------------------------------------
      CALL RW(3)
      CALL RWIRC(CDUM,RDUM,NPOL,0)
      DO 10 I=1,NPOL
10    CALL RWIRC(POL(I),RDUM,IDUM,2)
      CALL RWIRC(CDUM,FBIF,IDUM,1)
      CALL RWIRC(CDUM,FBRF,IDUM,1)
      CALL RWIRC(CDUM,RADRNG,IDUM,1)
      CALL RWIRC(CDUM,RADIN,IDUM,1)
      CALL RWIRC(CDUM,RDUM,NF,0)
      CTAUX=2*PI/RADRNG
      DRAD=RADRNG/NF
      CALL RWIRC(CDUM,RDUM,LPBP,0)
C -----------------------------------------------------------------------
C     READ IN AND PACK CHANNEL PARAMETERS
C -----------------------------------------------------------------------
      CALL RW(5)
      CALL RWIRC(CDUM,RDUM,NCHNLS,0)
      NC1=1
      NC2=10
1000  READ (105,200) AR14,(ISC(IC),IC=NC1,NC2)
      WRITE(108,200) AR14,(ISC(IC),IC=NC1,NC2)
      NC1=NC1+10
      NC2=NC2+10
      IF(NC1.GT.NCHNLS) GOTO 2000
      IF(NC2.GE.NCHNLS) NC2=NCHNLS
      GOTO 1000
2000  IC=0
      DO 20 I=1,NCHNLS
      IF(ISC(I).EQ.0) GOTO 3000
      CALL RW(2)
      IC=IC+1
      IDC(IC)=I
      CALL RWIRC(CDUM,AN(IC),IDUM,1)
      CALL RWIRC(CDUM,RDUM,IX0(IC),0)
      CALL RWIRC(CDUM,RDUM,N1(IC),0)
      CALL RWIRC(CDUM,RDUM,NB(IC),0)
      CALL RWIRC(CDUM,TH(IC),IDUM,1)
      CALL RWIRC(CDUM,CDEL,IDUM,1)
      CTAU(IC)=CDEL*CTAUX
      GOTO 20
3000  CALL SKIPR(105,8)
20    CONTINUE
```

```
      NCHNLS=IC
C     ---------------------------------------------------------------
C     READ IN AND PACK PORT PARAMETERS
C     ---------------------------------------------------------------
      CALL RW(3)
      CALL RWIRC(CDUM,D0,IDUM,1)
      CALL RWIRC(CDUM,RDUM,NS,0)
      CALL RWIRC(CDUM,RDUM,NPORTS,0)
      NPM1=NPORTS-1
      NP1=1
      NP2=10
4000  READ (105,200) AR14,(ISP(IP),IP=NP1,NP2)
      WRITE(108,200) AR14,(ISP(IP),IP=NP1,NP2)
      NP1=NP1+10
      NP2=NP2+10
      IF(NP1.GT.NPM1) GOTO 5000
      IF(NP2.GE.NPM1) NP2=NPM1
      GOTO 4000
5000  IP=1
      CALL RW(2)
      CALL RWIRC(CDUM,RDUM,NEL(1),0)
      CALL RWIRC(CDUM,RDUM,LOC1(1),0)
      CALL RWIRC(CDUM,BWFCTR(1),IDUM,1)
      CALL RWIRC(CDUM,BWOFF(1),IDUM,1)
      CALL RWIRC(CDUM,PDEL,IDUM,1)
      PTAUX=PI*D0*NEL(1)
      PTAU(1)=PDEL*PTAUX
      DO 30 I=1,NPM1
      IF(ISP(I).EQ.0) GOTO 6000
      CALL RW(2)
      IP=IP+1
      IDP(IP)=I
      CALL RWIRC(CDUM,RDUM,NEL(IP),0)
      CALL RWIRC(CDUM,RDUM,LOC1(IP),0)
      CALL RWIRC(CDUM,BWFCTR(IP),IDUM,1)
      CALL RWIRC(CDUM,BWOFF(IP),IDUM,1)
      CALL RWIRC(CDUM,PDEL,IDUM,1)
      PTAU(IP)=PDEL*PTAUX
      GOTO 30
6000  CALL SKIPR(105,7)
30    CONTINUE
      NPORTS=IP
      IDP(1)=0
```

```
      CALL RW(1)
C -----------------------------------------------------------------------
      RETURN
      END
```

```
C   ======================================================================
      SUBROUTINE RW(N)
C   ======================================================================
C                 READING AND WRITING 80-CHARACTER DATA COMMENTS
C
C                 PROGRAM        :  RW:S
C                 ORIGINAL       :  APRIL 19, 1978
C                 REVISION       :  JUNE  15, 1980
C
C                 PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                   RADAR SYSTEMS ANALYSIS GROUP
C                                   MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                   TEMPE, ARIZONA  85282
C   ----------------------------------------------------------------------
C     INPUT  :    N         -  NUMBER OF LINES TO BE READ AND WRITTEN
C   ======================================================================
      DIMENSION AR20(20)
 100  FORMAT(20A4)
      DO 10 I=1,N
      READ (105,100) AR20
 10   WRITE(108,100) AR20
      RETURN
      END
```

```
C =====================================================================
      SUBROUTINE RWIRC(CX,RX,IX,IND)
C =====================================================================
C                   READING AND WRITING AN 80-CHARACTER LINE
C                                CONSISTING OF
C                           A 56-CHARACTER COMMENT
C                                     AND
C              A COMPLEX, REAL OR INTEGER SCALAR VARIABLE VALUE
C
C               PROGRAM        :  RWIRC:S
C               ORIGINAL       :  APRIL  19, 1980
C               REVISION       :  AUGUST 13, 1980
C
C               PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                 RADAR SYSTEMS ANALYSIS GROUP
C                                 MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                 TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C     INPUT  :    IND         -   OPTION INDICATOR
C                                  0  : READ & WRITE INTEGER
C                                  1  : READ & WRITE REAL
C                                  2  : READ & WRITE COMPLEX
C
C     OUTPUT :    CX          -   COMPLEX SCALAR EXCLUSIVE OF RX & IX
C                 RX          -   REAL SCALAR EXCLUSIVE OF CX & IX
C                 IX          -   INTEGER SCALAR EXCLUSIVE OF CX & RX
C =====================================================================
      COMPLEX CX
      DIMENSION AR14(14)
  100 FORMAT(14A4,I12)
  200 FORMAT(14A4,F12.5)
  300 FORMAT(14A4,2X,2(F10.5))
      GOTO (1,2),IND
      READ (105,100) AR14,IX
      WRITE(108,100) AR14,IX
      RETURN
    1 READ (105,200) AR14,RX
      WRITE(108,200) AR14,RX
      RETURN
    2 READ (105,300) AR14,CX
      WRITE(108,300) AR14,CX
      RETURN
      END
```

```
C =======================================================================
      SUBROUTINE SKIPR(NU,NS)
C =======================================================================
C SKIP NS LINES ON UNIT NU.
C
C                 PROGRAM          :  SKIPR:S
C                 ORIGINAL         :  FEBRUARY 23, 1981
C                 REVISION         :  FEBRUARY 23, 1981
C
C                 PREPARED  BY     :  S. M. DANIEL  &  I. KERTESZ
C                                     RADAR SYSTEMS ANALYSIS GROUP
C                                     MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                     TEMPE, ARIZONA  85282
C -----------------------------------------------------------------------
C     INPUT  :    NU          -  NUMBER OF UNIT OR DEVICE
C                 NS          -  NUMBER OF LINES TO SKIP
C
C     OUTPUT  :               -  NONE
C -----------------------------------------------------------------------
C     ENTRY POINTS             :  SKIPR
C     SUBROUTINES CALLED       :  NONE
C =======================================================================
100   FORMAT(1X)
C -----------------------------------------------------------------------
C     SKIP NS LINES ON UNIT NU
C -----------------------------------------------------------------------
      DO 10 I=1,NS
10    READ(NU,100)
      RETURN
      END
```

```
C ================================================================
      SUBROUTINE PRTSGNL
C ================================================================
C             GENERATION OF RECEIVED BASEBAND PORT SIGNALS
C
C              PROGRAM      :  PRTSGNL:S
C              ORIGINAL     :  JUNE      1, 1980
C              REVISION     :  JANUARY   5, 1981
C
C              PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                               RADAR SYSTEMS ANALYSIS GROUP
C                               MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                               TEMPE, ARIZONA  85282
C ----------------------------------------------------------------
C     INPUT  :    SYSTEM PARAMETERS IN COMMON
C
C     OUTPUT :    RECEIVED BASEBAND PORT SIGNALS
C ================================================================
      COMPLEX POL,HR,H,HIMPL,SC,SP,CDUM
      COMMON /SIG0/ IWANT,IWRAP,IWRI,IWCAP,IWCI,IWSC
      COMMON /SIG1/ POL(6),FBIF,FBRF,RADRNG,RADIN,DRAD,NPOL,LPBP,NF
      COMMON /SIG2/ AN(20),TH(20),CTAU(20),IX0(20),N1(20),NB(20)
     -,ISC(20),IDC(20),NCHNLS
      COMMON /SIG3/ BWFCTR(21),BWOFF(21),PTAU(21),D0,NEL(21),LOC1(21)
     -,ISP(20),IDP(21),NPORTS,NS
      DIMENSION HR(64),H(64),HIMPL(64),SC(1024),SP(1024),CDUM(1),RDUM(1)
     -,AM(64),PH(64),X(128),AL(1001),P(64),IBLNK(1056)
  100 FORMAT(1H1)
  200 FORMAT(1X,79('-'))
  300 FORMAT(37X,'PORT',I3,/,37X,7('-'))
  400 FORMAT(23X,'RECEIVER BASEBAND CHARACTERISTICS')
  500 FORMAT(29X,'PORT',I3,5X,'CHANNEL',I3,/,29X,22('-'))
  600 FORMAT(24X,'CHANNEL BASEBAND CHARACTERISTICS')
  700 FORMAT(24X,'RECEIVED BASEBAND CHANNEL SIGNAL')
  800 FORMAT(8F10.5)
  900 FORMAT(25X,'RECEIVED BASEBAND PORT SIGNAL',/,19X
     -,'NORMALIZATION CONSTANT    :  ',E13.6)
C ----------------------------------------------------------------
C     PROVIDE MAIN ANTENNA ARRAY ELEMENT WEIGHTING
C ----------------------------------------------------------------
      CALL ANTWT(NEL(1),LOC1(1),AL,IWANT)
C ----------------------------------------------------------------
C     DERIVE BASEBAND PORT SIGNALS
```

```
C ------------------------------------------------------------------
      DO 10 IP=1,NPORTS
      IPM1=IP-1
      IF(IWRAP.EQ.0.AND.IWRI.EQ.0) GOTO 1000
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,300) IDP(IP)
      WRITE(108,200)
      WRITE(108,400)
1000  CONTINUE
C ------------------------------------------------------------------
C     INITIALIZE RECEIVED BASEBAND PORT SIGNAL TO ZERO
C ------------------------------------------------------------------
      DO 20 IS=1,NS
20    SP(IS)=0
      IF(IPM1.GT.0) AL(1)=1
C ------------------------------------------------------------------
C     DERIVE RECEIVER CHARACTERISTICS
C ------------------------------------------------------------------
      CALL FILTER(NF,DRAD,RADIN,FBIF,LPBP,BWFCTR(IP),BWOFF(IP),NPOL,POL
     -,HR)
      IF(IWRAP.EQ.1) CALL AMPH(HR,NF,AM,PH)
      CALL IMPULS(NF,HR,X,HIMPL,IWRI)
C ------------------------------------------------------------------
C     DERIVE CHANNEL BASEBAND CHARACTERISTICS
C ------------------------------------------------------------------
      DO 30 IC=1,NCHNLS
      IF(IWCAP.EQ.0.AND.IWCI.EQ.0) GOTO 2000
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,500) IDP(IP),IDC(IC)
      WRITE(108,200)
      WRITE(108,600)
2000  CONTINUE
      CALL CHANNEL(NEL(IP),LOC1(IP),AL,D0,TH(IC),CTAU(IC),PTAU(IP)
     -,NF,RADIN,DRAD,BWFCTR(IP),BWOFF(IP),FBRF,HR,H)
      IF(IWCAP.EQ.1) CALL AMPH(H,NF,AM,PH)
      CALL IMPULS(NF,H,X,HIMPL,IWCI)
C ------------------------------------------------------------------
C     DERIVE RECEIVED BASEBAND CHANNEL SIGNALS AND THEIR ACCUMULATION
C     INTO A COMPOSITE PORT SIGNAL
C ------------------------------------------------------------------
      IX=IX0(IC)
```

```
      CALL RLINK(N1(IC),NB(IC),NF,NS,IBLNK)
      CALL SIGNAL(HIMPL,NF,IX,IBLNK,P,SC,NS)
      IF(IWSC.EQ.0) GOTO 3000
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,500) IDP(IP),IDC(IC)
      WRITE(108,200)
      WRITE(108,700)
      WRITE(108,200)
      WRITE(108,800) (SC(I),I=1,NS)
      WRITE(108,200)
3000  CONTINUE
      DO 40 IS=1,NS
40    SP(IS)=SP(IS)+AN(IC)*SC(IS)
30    CONTINUE
      CALL RCNORM(SP,RDUM,SPMAX,NS,2,1)
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,300) IDP(IP)
      WRITE(108,200)
      WRITE(108,900) SPMAX
      WRITE(108,200)
      WRITE(108,800) (SP(I),I=1,NS)
      WRITE(108,200)
10    CONTINUE
C     ----------------------------------------------------------------
      RETURN
      END
```

```
C ==================================================================
      SUBROUTINE ANTWT(NEL,LOC1,AL,)
C ==================================================================
C          COMPUTATION OF LINEAR ANTENNA ARRAY TAYLOR WEIGHTING
C
C               PROGRAM        :  ANTWT:S
C               ORIGINAL       :  APRIL      19, 1980
C               REVISION       :  SEPTEMBER  1, 1980
C
C               PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                 RADAR SYSTEMS ANALYSIS GROUP
C                                 MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                 TEMPE, ARIZONA  85282
C ------------------------------------------------------------------
C     INPUT  :    NEL       -  NUMBER OF ELEMENTS IN LINEAR ARRAY, IW
C                 LOC1      -  LOCATION (NUMBER) OF FIRST ELEMENT
C
C     OUTPUT :    AL        -  ARRAY ELEMENT WEIGHTING
C ==================================================================
      DIMENSION AL(1)
  100 FORMAT(1H1)
  200 FORMAT(1X,79('-'))
  300 FORMAT(20X,'TAYLOR WEIGHTING FOR MAIN ANTENNA ARRAY',/,20X
     -,39('-'))
  400 FORMAT(8F10.5)
      DATA PI/3.1415965/
      PI2=2*PI
      LOC=LOC1-1
      CON2=0.5*(NEL+1)
      CON1=PI2/NEL
      DO 10 I=1,NEL
      LOC=LOC+1
      ARG=CON1*(LOC-CON2)
   10 AL(I)=1+0.5*COS(ARG)
      IF(IW.EQ.0) RETURN
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,300)
      WRITE(108,400) (AL(I),I=1,NEL)
      WRITE(108,200)
      RETURN
      END
```

```
C ==================================================================
    SUBROUTINE FILTER(NF,DRAD,RADIN,FBIF,LPBP,BWFCTR,BWOFF,NPOL,POL
   -,H)
C ==================================================================
C EVALUATION OF NORMALIZED RECEIVER BASEBAND SAMPLED TRANSFER FUNCTION
C                      AT DISCRETE RADIAN FREQUENCIES
C                                  USING
C         LOWPASS PROTOTYPE FILTER OR ITS BANDPASS EQUIVALENT
C
C            PROGRAM       :  FILTERIS
C            ORIGINAL      :  APRIL 19, 1980
C            REVISION      :  JUNE  23, 1980
C
C            PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                             RADAR SYSTEMS ANALYSIS GROUP
C                             MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                             TEMPE, ARIZONA  85282
C ------------------------------------------------------------------
C     INPUT  :     NF        -  NUMBER OF FREQUENCY SAMPLES
C                  DRAD      -  NORMALIZED RADIAN FREQUENCY INCREMENT
C                  RADIN     -  INITIAL RADIAN FREQUENCY
C                  FBIF      -  FRACTIONAL BANDWIDTH AT FINAL IF
C                  LPBP      -  LOWPASS / BANDPASS OPTION
C                               0  :  LOWPASS
C                               1  :  BANDPASS
C                  BWFCTR    -  BANDWIDTH FACTOR
C                  BWOFF     -  BANDWIDTH OFFSET FACTOR
C                  NPOL      -  NUMBER OF POLES
C                  POL       -  POLE LOCATIONS
C
C     OUTPUT :     H         -  SAMPLED TRANSFER FUNCTION
C ==================================================================
    COMPLEX POL,S,DEN,H
    DIMENSION POL(1),H(1)
C ------------------------------------------------------------------
C   ADJUST INCREMENT AND INITIAL RADIAN FREQUENCY
C   COMPUTE SAMPLED TRANSFER FUNCTION
C ------------------------------------------------------------------
    DRADL=0.5*DRAD/BWFCTR
    RAD1=(RADIN/2-BWOFF)/BWFCTR-DRADL
    DO 10 I=1,NF
    RAD=RAD1+I*DRADL
    H(I)=1
```

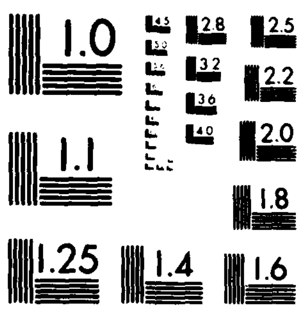
```
      S=CMPLX(0,RAD)
      IF(LPBP.EQ.1) S=S*(1+1/(1+RAD*FBIF))
      DO 10 J=1,NPOL
      DEN=S-POL(J)
10    H(I)=H(I)*POL(J)/DEN
C ----------------------------------------------------------------------
      RETURN
      END
```

```
C ====================================================================
      SUBROUTINE AMPH(H,NF,AM,PH)
C ====================================================================
C                    EVALUATION OF AMPLITUDE AND PHASE
C                                  FROM
C                    COMPLEX SAMPLED TRANSFER FUNCTION
C
C              PROGRAM        :  AMPH:S
C              ORIGINAL       :  APRIL 19, 1980
C              REVISION       :  JULY   1, 1980
C
C              PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C --------------------------------------------------------------------
C     INPUT  :    H          -   SAMPLED TRANSFER FUNCTION
C                 NF         -   NUMBER OF FREQUENCY SAMPLES
C
C     OUTPUT :    AM         -   SAMPLED AMPLITUDE RESPONSE
C                 PH         -   SAMPLED PHASE RESPONSE
C ====================================================================
      COMPLEX H
      DIMENSION H(1),AM(1),PH(1)
  100 FORMAT(1X,79('-'))
  200 FORMAT(35X,'AMPLITUDE',/,35X,9('='),/,(8F10.5))
  300 FORMAT(37X,'PHASE',/,37X,5('='),/,(8F10.5))
C --------------------------------------------------------------------
C     COMPUTE AMPLITUDE AND PHASE
C --------------------------------------------------------------------
      DO 10 I=1,NF
      AM(I)=CABS(H(I))
      PH(I)=ACOS(REAL(H(I))/AM(I))
   10 IF(AIMAG(H(I)).LT.0) PH(I)=-PH(I)
C --------------------------------------------------------------------
      WRITE(108,100)
      WRITE(108,200) (AM(I),I=1,NF)
      WRITE(108,100)
      WRITE(108,300) (PH(I),I=1,NF)
      WRITE(108,100)
      RETURN
      END
```

```
      C ======================================================================
            SUBROUTINE IMPULS(NF,H,X,HIMPL,)
      C ======================================================================
      C                     EVALUATION OF SAMPLED IMPULSE RESPONSE
      C                        FROM A SAMPLED TRANSFER FUNCTION
      C
      C              PROGRAM         :  IMPULSIS
      C              ORIGINAL        :  MAY 11, 1980
      C              REVISION        :  JUNE 7, 1980
      C
      C              PREPARED  BY    :  S. M. DANIEL  &  I. KERTESZ
      C                                 RADAR SYSTEMS ANALYSIS GROUP
      C                                 MOTOROLA GOVERNMENT ELECTRONICS DIV.
      C                                 TEMPE, ARIZONA 85282
      C ----------------------------------------------------------------------
      C     INPUT  :     NF         -  NUMBER OF FREQUENCY SAMPLES, IW
      C                  H          -  SAMPLED TRANSFER FUNCTION
      C
      C     OUTPUT :     HIMPL      -  COMPLEX IMPULSE RESPONSE
      C                  X          -  AUXILIARY REAL ARRAY
      C ======================================================================
            COMPLEX H,HIMPL
            DIMENSION H(1),HIMPL(1),X(1)
      100   FORMAT(1X,79('-'))
      200   FORMAT(32X,'IMPULSE RESPONSE',/,32X,16('-'),/,(8F10.5))
            NF2=2*NF
            X(1)=X(2)=0
            IH=2
            DO 10 I=3,NF2,2
            X(I)=REAL(H(IH))
            X(I+1)=AIMAG(H(IH))
      10    IH=IH+1
            CALL FFT2(X,NF,1)
            DO 20 I=1,NF
            I2=2*I
            I1=I2-1
      20    HIMPL(I)=CMPLX(X(I1),X(I2))
            IF(IW.EQ.0) RETURN
            WRITE(108,100)
            WRITE(108,200) (HIMPL(I),I=1,NF)
            WRITE(108,100)
            RETURN
            END
```

AD-A109 927 MOTOROLA INC TEMPE AZ GOVERNMENT ELECTRONICS DIV F/G 20/14
BATCH COVARIANCE RELAXATION (BCR) ADAPTIVE PROCESSING.(U)
AUG 81 S M DANIEL, I KERTESZ F30602-80-C-0031
UNCLASSIFIED 8512-F RADC-TR-81-212 NL

4 of 7
AD A109927



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

```
C ======================================================================
      SUBROUTINE FFT2(X,NX,IF)
C ======================================================================
C               COMPUTATION OF BASE-2 DISCRETE FOURIER TRANSFORM
C                     OF A COMPLEX-VALUED TIME SERIES
C                  REPRESENTED BY REAL AUXILIARY ARRAY X
C
C                PROGRAM       :  FFT2:S
C                ORIGINAL      :  JULY       15, 1969
C                REVISION      :  SEPTEMBER  1, 1980
C
C                PREPARED BY   :  PROF. P. RANSOM & S. M. DANIEL
C                                 ELECTRICAL ENGINEERING DEPARTMENT
C                                 UNIVERSITY OF ILLINOIS
C                                 CHAMPAIGN-URBANA, ILLINOIS
C ----------------------------------------------------------------------
C     INPUT  :    X          -   AUXILIARY REAL ARRAY WHOSE ODD AND EVEN-
C                                NUMBERED ELEMENTS ARE THE REAL AND THE
C                                IMAGINARY PARTS OF THE ORIGINAL COMPLEX
C                                INPUT ARRAY
C                 NX         -   NUMBER OF COMPLEX SAMPLES
C                 IF         -   FOURIER TRANSFORM OPTION
C                                -1  : FOREWARD DFT
C                                +1  : INVERSE  DFT
C
C     OUTPUT :    X          -   TRANSFORMED AUXILIARY ARRAY
C ======================================================================
      DIMENSION X(1)
      DATA PI/3.14159265/
      PI2=2*PI
      NX2=2*NX
      SNXI=1
      IF(IF.EQ.1) SNXI=1.0/NX
      DO 10 I=1,NX
      IMOD=MOD(I-1,2)
      I2=2*I
      I2M1=I2-1
      FCTR=SNXI
      IF(IMOD.EQ.1) FCTR=-SNXI
      X(I2M1)=FCTR*X(I2M1)
      X(I2)=FCTR*X(I2)
   10 CONTINUE
      J=1
```

```
      DO 20 I=1,NX2,2
      IP1=I+1
      JP1=J+1
      IF(I-J.GE.0) GOTO 1000
      XTR=X(J)
      XTI=X(JP1)
      X(J)=X(I)
      X(JP1)=X(IP1)
      X(I)=XTR
      X(IP1)=XTI
1000  K=NX
2000  IF(J-K.LE.0) GOTO 20
      J=J-K
      K=K/2
      IF(K-2.GE.0) GOTO 2000
20    J=J+K
      KMAX=2
3000  IF(KMAX-NX2.GE.0) GOTO 4000
      ISTEP=2*KMAX
      TH=IF*PI2/KMAX
      STH=SIN(TH/2)
      WSR=-2*STH**2
      WSI=SIN(TH)
      WR=1
      WI=0
      DO 40 K=1,KMAX,2
      DO 30 I=K,NX2,ISTEP
      J=I+KMAX $ JP1=J+1 $ IP1=I+1
      WXTR=WR*X(J)-WI*X(JP1)
      WXTI=WR*X(JP1)+WI*X(J)
      X(J)=X(I)-WXTR
      X(JP1)=X(IP1)-WXTI
      X(I)=X(I)+WXTR
      X(IP1)=X(IP1)+WXTI
30    CONTINUE
      WXTR=WR
      WR=WR*(1+WSR)-WI*WSI
      WI=WI*(1+WSR)+WXTR*WSI
40    CONTINUE
      KMAX=ISTEP
      GOTO 3000
4000  CONTINUE
      DO 50 I=1,NX
```

```
      IMOD=MOD(I-1,2)
      IF(IMOD.EQ.0) GOTO 50
      I1=2*I-1
      I2=I1+1
      X(I1)=-X(I1)
      X(I2)=-X(I2)
50    CONTINUE
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE CHANNEL(NEL,LOC1,AL,D0,TH,CTAU,PTAU,NF,RADIN,DRAD
     -,BWFCTR,BWOFF,FBRF,HR,H)
C ======================================================================
C EVALUATION OF NORMALIZED CHANNEL BASEBAND SAMPLED TRANSFER FUNCTION
C                     AT DISCRETE RADIAN FREQUENCIES
C                                INCLUDING
C            LINEAR ANTENNA ARRAY AND RECEIVER CHARACTERISTICS
C                 WITH SPEIFIED CHANNEL AND PORT DELAYS
C
C            PROGRAM       :  CHANNEL:S
C            ORIGINAL      :  MAY       4, 1980
C            REVISION      :  JANUARY 25, 1981
C
C            PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                             RADAR SYSTEMS ANALYSIS GROUP
C                             MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                             TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     INPUT  :     NEL       -  NUMBER OF ELEMENTS IN LINEAR ARRAY
C                  LOC1      -  LOCATION (NUMBER) OF FIRST ELEMENT
C                  AL        -  ARRAY ELEMENT WEGHTING
C                  D0        -  ELEMENT SEPARATION FACTOR
C                  TH        -  AZIMUTH ANGLE (DEG) OF INCIDENCE
C                  CTAU      -  NORMALIZED CHANNEL DELAY
C                  PTAU      -  NORMALIZED PORT DELAY
C                  NF        -  NUMBER OF FREQUENCY SAMPLES
C                  RADIN     -  INITIAL RADIAN FREQUENCY
C                  DRAD      -  NORMALIZED RADIAN FREQUENCY INCREMENT
C                  BWFCTR    -  BANDWIDTH FACTOR
C                  BWOFF     -  BANDWIDTH OFFSET FACTOR
C                  FBRF      -  FRACTIONAL BANDWIDTH AT RF
C                  HR        -  RECEIVER SAMPLED TRANSFER FUNCTION
C
C     OUTPUT :     H         -  CHANNEL SAMPLED TRANSFER FUNCTION
C ======================================================================
      DOUBLE COMPLEX  CH
      COMPLEX HR,H
      DIMENSION HR(1),H(1),AL(1)
      DATA PI/3.14159265/
      RATIO=FBRF/BWFCTR
      DRADL=RATIO*DRAD/2
      RAD1=1+RATIO*(RADIN/2-BWOFF)-DRADL
```

```
      PDSTH=PI*D0*SIN(TH*PI/180)
C -----------------------------------------------------------------
C     EVALUATION OF BASEBAND CHANNEL SAMPLED TRANSFER FUNCTION
C -----------------------------------------------------------------
      DO 10 I=1,NF
      RAD=RAD1+I*DRADL
      ARG0=PDSTH*RAD
      CH=0
      LOC=LOC1-1
      DO 20 J=1,NEL
      LOC=LOC+1
      ARG=ARG0*LOC
      CH=CH+AL(J)*DCMPLX(DCOS(ARG),DSIN(ARG))
   20 CONTINUE
      ARG=-RAD*(CTAU+PTAU)
      CH=CH*DCMPLX(DCOS(ARG),DSIN(ARG))
   10 H(I)=CH*HR(I)
C -----------------------------------------------------------------
      RETURN
      END
```

```
C     =================================================================
      SUBROUTINE BLINK(N1,NB,NF,NS,IBLNK)
C     =================================================================
C                     ON/OFF PERIODIC BLINK CONTOL GENERATOR
C
C                PROGRAM       :  BLINK:S
C                ORIGINAL      :  JUNE 12, 1980
C                REVISION      :  JUNE 16, 1980
C
C                PREPARED  BY  :  I. KERTESZ   &   S. M. DANIEL
C                                 RADAR SYSTEMS ANALYSIS GROUP
C                                 MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                 TEMPE, ARIZONA  85282
C     -----------------------------------------------------------------
C         INPUT  :     N1        -   FIRST-TIME-ON SAMPLE NUMBER
C                      NB        -   BLINK DURATION IN SAMPLES
C                      NF        -   NUMBER OF FREQUENCY SAMPLES
C                      NS        -   NUMBER OF SIGNAL SAMPLES
C
C         OUTPUT :     IBLNK     -   BLINK CONTROL ARRAY HAVING (NS+NF/2)
C                                    SAMPLES
C     =================================================================
      DIMENSION IBLNK(1)
      J1=1
      J2=0
      N1M1=N1-1
      DO 10 I=1,N1M1
   10 IBLNK(I)=0
      J=0
      NSPH=NS+NF/2
      DO 20 I=N1,NSPH
      J=J+1
      IBLNK(I)=J1
      IF(MOD(J,NB).NE.0) GOTO 20
      J3=J2
      J2=J1
      J1=J3
   20 CONTINUE
      RETURN
      END
```

```
20    RX(I)=RX(I)/XMAX
      RETURN
1000  DO 30 I=1,NX
      ABSX=ABS(REAL(CX(I)))
      IF(ABSX.GT.XMAX) XMAX=ABSX
      ABSX=ABS(AIMAG(CX(I)))
30    IF(ABSX.GT.XMAX) XMAX=ABSX
      IF(XMAX.EQ.0) GOTO 9999
      IF(IOPTN.EQ.0) RETURN
      DO 40 I=1,NX
40    CX(I)=CX(I)/XMAX
C     -----------------------------------------------------------------
      RETURN
9999  WRITE(108,999)
      STOP
      END
```

```
C ====================================================================
      SUBROUTINE SIGNAL(XIMPL,NX,IX,P,S,NS)
C ====================================================================
C               COMPUTATION OF RECEIVED BASEBAND SAMPLED SIGNALS
C                                    FROM
C                    A WIDEBAND NOISE INTERFERENCE SOURCE
C
C                PROGRAM       :  SIGNAL01S
C                ORIGINAL      :  JUNE        8, 1980
C                REVISION      :  SEPTEMBER   1, 1980
C
C                PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                                 RADAR SYSTEMS ANALYSIS GROUP
C                                 MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                 TEMPE, ARIZONA  85282
C --------------------------------------------------------------------
C     INPUT  :    XIMPL      -    COMPLEX BASEBAND SAMPLED IMPULSE
C                                 RESPONSE
C                 NX         -    NUMBER OF SAMPLES IN XIMPL
C                 IX         -    INITIAL RANDU SETTING FOR NOISE SOURCE, IBLINK
C                 P          -    NX-ELEMENT SLIDING-WINDOW NOISE ARRAY
C                 NS         -    NUMBER OF SIGNAL SAMPLES
C
C     OUTPUT :    S          -    NS-ELEMENT RECEIVED COMPLEX BASEBAND
C                                 SAMPLED SIGNAL ARRAY
C ====================================================================
      COMPLEX XIMPL,S,SI
      DIMENSION XIMPL(1),P(1),S(1),IBLNK(1)
      NXH=NX/2
      NXHP1=NXH+1
      NXM1=NX-1
      NXP1=NX+1
      DO 10 I=1,NXH
   10 P(I)=0
      IB=1
      DO 20 I=NXHP1,NX
      CALL RANDU(IX,IY,Z)
      P(I)=IBLNK(IB)*(Z-0.5)
   20 IB=IB+1
      DO 30 I=1,NS
      SI=0
      DO 40 J=1,NX
   40 SI=SI+P(J)*XIMPL(NXP1-J)
```

```
      S(I)=SI
      DO 50 J=1,NXM1
   50 P(J)=P(J+1)
      CALL RANDU(IX,IY,Z)
      P(NX)=IBLNK(IB)*(Z-0.5)
   30 IB=IB+1
      RETURN
      END
```

```
C =====================================================================
      SUBROUTINE RCNORM(CX,RX,XMAX,NX,IND,IOPTN)
C =====================================================================
C                   NORMALIZATION OF REAL OR COMPLEX ARRAY
C
C              PROGRAM       :  RCNORM:S
C              ORIGINAL      :  AUGUST  31, 1980
C              REVISION      :  JANUARY 26, 1981
C
C              PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                               RADAR SYSTEMS ANALYSIS GROUP
C                               MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                               TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C     INPUT  :     CX         -  COMPLEX ARRAY TO BE NORMALIZED
C                  RX         -  REAL ARRAY TO BE NORMALIZED
C                  NX         -  ARRAY DIMENSIONALITY
C                  IND        -  ARRAY-TYPE INDICATOR
C                                 1  : REAL
C                                 2  : COMPLEX
C                  IOPTN      -  NORMALIZING OPTION
C                                 0  : BYPASS  NORMALIZATION
C                                 1  : PERFORM NORMALIZATION
C
C     OUTPUT :     CX         -  NORMALIZED COMPLEX ARRAY
C                  RX         -  NORMALIZED REAL ARRAY
C                  XMAX       -  NORMALIZATION CONSTANT
C =====================================================================
      COMPLEX CX
      DIMENSION CX(1),RX(1)
999   FORMAT(' *** WARNING  :  YOU ARE NORMALIZING A NULL ARRAY ***')
C ---------------------------------------------------------------------
C     DETERMINE MAXIMUM REAL OR IMAGINARY COMPONENT MAGNITUDE
C     OF GIVEN ARRAY AND PERFORM NORMALIZATION IF  IOPTN=1
C ---------------------------------------------------------------------
      XMAX=0
      IF(IND.EQ.2) GOTO 1000
      DO 10 I=1,NX
      ABSX=ABS(RX(I))
10    IF(ABSX.GT.XMAX) XMAX=ABSX
      IF(XMAX.EQ.0) GOTO 9999
      IF(IOPTN.EQ.0) RETURN
      DO 20 I=1,NX
```

Table 2-7. Functional Description of the Modules Comprising SIGGEN

FUNCTIONAL DESCRIPTION OF SIGGEN SOURCE MODULES

SIGMAIN:S	-	MAIN EXECUTIVE PROGRAM. IT CALLS OTHER EXECUTIVE PROGRAMS WHICH TOGETHER PERFORM THE SIGNAL GENERATION TASK.
SIGSET:S	-	EXECUTIVE SUBPROGRAM. IT IS DESIGNED TO READ FROM THE INPUT DATA FILE, SIGGEN:D, USING THE GENERAL INPUT AND OUTPUT SUBROUTINES RW AND RWIRC IN ACCORDANCE TO PRESET FORMAT. ALL DATA IS MADE AVAILABLE TO APPROPRIATE SUBROUTINES THROUGH COMMON BLOCKS.
RW:S	-	GENERAL SUBPROGRAM. IT IS DESIGNED TO READ AND WRITE AN 80-CHARACTER LITERAL DATA STRING (COMMENT). NO DATA IS RETURNED TO THE CALLING PROGRAM.
RWIRC:S	-	GENERAL SUBPROGRAM. IT IS DESIGNED TO READ AND WRITE A 56-CHARACTER LITERAL DATA STRING FOLLOWED BY NUMERICAL DATA. THE NUMERICAL DATA MAY BE INTEGER, REAL, OR COMPLEX SCALAR. THE NUMERICAL DATA IS RETURNED TO THE CALLING PROGRAM.
SKIPR:S	-	GENERAL SUBPROGRAM. IT IS USED TO SKIP ONE OR MORE LINES OF THE INPUT DATA. NO DATA IS RETURNED.
PRTSGNL:S	-	EXECUTIVE SUBPROGRAM. IT USES THE INPUT DATA TO GENERATE THE PORT SIGNALS BY CALLING A NUMBER OF DEDICATED SUBROUTINES. THIS PROGRAM MAY BE CONSIDERED THE CENTRAL EXECUTIVE PROGRAM OF SIGGEN.
ANTWT:S	-	DEDICATED SUBPROGRAM. IT IS DESIGNED TO PROVIDE A TAYLOR WEIGHTING SEQUENCE FOR A LINEAR ARRAY GIVEN THE NUMBER OF ELEMENTS IN THE ARRAY.
FILTER:S	-	DEDICATED SUBPROGRAM. IT IS DESIGNED TO GENERATE A DISCRETIZATION OF THE BASEBAND EQUIVALENT TRANSFER FUNCTION OF EITHER THE FINAL IF BANDPASS FILTER OR A LOWPASS ALTERNATIVE.

AMPH:S - DEDICATED SUBPROGRAM. IT IS DESIGNED TO COMPUTE AND PRINT THE AMPLITUDE AND PHASE OF AN INPUT TRANSFER FUNCTION.

IMPULS:S - DEDICATED SUBPROGRAM. IT IS DESIGNED TO COMPUTE A SAMPLED BASEBAND IMPULSE RESPONSE FROM A SAMPLED BASEBAND TRANSFER FUNCTION.

FFT2:S - GENERAL SUBPROGRAM. IT IS DESIGNED TO COMPUTE THE FORWARD OR INVERSE DISCRETE FOURIER TRANSFORM OF A COMPLEX SEQUENCE WHOSE NUMBER OF SAMPLES IS A POWER OF 2.

CHANNEL:S - DEDICATED SUBPROGRAM. IT USES THE ANTENNA AND RECEIVER CHARACTERISTICS TO COMPUTE A BASEBAND TRANSFER FUNCTION ASSOCIATED WITH A PARTICULAR ANGLE OF ARRIVAL OF A SPECIFIED NOISE SOURCE.

BLINK:S - DEDICATED SUBPROGRAM. ITS PURPOSE IS TO GENERATE A DISCRETIZED PERIODIC ON/OFF SWITCHING FUNCTION CONSISTING OF AN APPROPRIATE SEQUENCE OF 0'S AND 1'S.

SIGNAL:S - DEDICATED SUBPROGRAM. IT USES THE ON/OFF SWITCHING SEQUENCE TO INITIALLY TURN ON AND SUBSEQUENTLY PERIODICALLY BLINK AN INPUT NOISE SEQUENCE PROVIDED BY A RANDOM NUMBER GENERATOR. IT THEN USES THIS BLINKED NOISE SEQUENCE TO PRODUCE A DESIRED NUMBER OF RECEIVED BASEBAND SAMPLES THROUGH ITS ASSOCIATED CHANNEL VIA DISCRETE CONVOLUTION WITH ITS IMPULSE RESPONSE. ITS OUTPUT IS THE INDIVIDUAL CHANNEL BASEBAND SIGNAL.

RANDU:S - GENERAL SUBPROGRAM. ITS PURPOSE IS TO PROVIDE A REPEATABLE SEQUENCE OF NUMBERS WHICH ARE UNIFORMLY DISTRIBUTED BETWEEN 0 AND 1 AND HAVE THE CHARACTERISTICS OF RANDOM NUMBERS.

RCNORM:S - GENERAL SUBPROGRAM. IT IS USED TO NORMALIZE REAL AND COMPLEX ARRAYS. COMPLEX ARRAYS ARE NORMALIZED WITH THE LARGEST REAL OR IMAGINARY ELEMENT.

Table 2-8. JCL Program JCLSIG:BL

```
!JOB 1269,DANIEL(8512),7,BLDG90
!LIMIT (TIME,1),(UO,2),(CO,16),(ACCOUNT)
!.....JCLSIG:BL.......................H
!PCL
C      SIGMAIN:B,1269          OVER SIGGEN:B
C         SIGSET:B,1269
C            RW:B,1269
C            RWIRC:B,1269
C         PRTSGNL:B,1269
C            ANTWT:B,1269
C            FILTER:B,1269
C            AMPH:B,1269
C            IMPULS:B,1269
C               FFT2:B,1269
C            CHANNEL:B,1269
C            BLINK:B,1269
C            SIGNAL:B,1269
C            RCNORM:B,1269
!LYNX SIGGEN:B                 OVER SIGGEN:L
```

Table 2-9. Execution Program, JCL:X

```
1.000 !JOB 298,DANIEL(8512),7,BLDG90
2.000 !LIMIT (TIME,5),(UO,50),(CO,24),(ACCOUNT)
3.000 !........JCL:X.........................
4.000 !SET F:105/NAME:D
5.000 !SET F:108/NAME:O
6.000 !RUN (LMN,NAME:L,1269)
```

Upon execution of this job, SIGGEN:B and SIGGEN:L are created.

2.2.3 Program Execution

The load module described in the previous paragraph can be executed upon assigning the proper input and, as an option, output files. The input file for this case is SIGGEN:D, the one discussed in paragraph 2.2.1. The output file can be the line printer output, or a disc file similar to the input file. Since the output of this program needs to be used as input to other programs, an output disc file will be generated.

One of the methods of file assignments and program execution is shown in the JCL program, JCL:X , in Table 2-9. This is also a general program in which name substitution is used for output data file (NAME:O), load module (NAME:L), and input data file (NAME:D). Upon entering the interactive command

!BATCH JCL:X 'NAME' = SIGGEN

the job is entered into the queue. When completed, the output will be in file SIGGEN:O.

2.2.4 Output File - SIGGEN:O

The output data file, SIGGEN:O, is shown in Table 2-10. It was produced by executing SIGGEN:L with input data in SIGGEN:D. The beginning of this output file is a replica of the beginning of the input file, read and printed through subroutines RW and RWIRC. Thus the output also completely describes the current system and scenario. The list of channel and port parameters this time contains only those which were indicated by the channel and port selector arrays in the benchtest example considered. The output actually produced (rather than echoed) by SIGGEN begins with the PORT 0 signal, and is completed with the printout of the four auxiliary port signals.

2.3 BCR Simulation Program - BCRS

The BCR simulation program applies the Batch Covariance Relaxation technique to signals generated by the SIGGEN program. In particular, BCRS computes the covariance of a batch of the auxiliary port signals, and the cross correlation of the main and auxiliary port signals. From these it estimates an adaptive weight vector which is then applied to the original signals to produce a minimum-noise combined signal. The output of the program is the adaptive weight vector, the combined signal, and power suppression achieved.

2.3.1 Input Data File - BCRS:D

The input data for the BCR simulation program is consistent with the conventions and format established for the signal generation program. The name of this data file is BCRS:D. For the benchtest example, this file is given in Table 2-11. It has two major parts, the BCR specification and the signal generation data. The first part is a special header data file, BCRS:DO, shown in Table 2-12 with entries appropriate to the example considered.

Table 2-10. Output Data File of the Signal Generation Program, SIGGEN:0 Benchtest Example (See Figure 2-2)

```
------------------------------------------------------------------------------
                     SPECIFICATION OF SYSTEM PARAMETERS
                     ----------------------------------

------------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWANT   -  MAIN ANTENNA ARRAY WEIGHTING            :          0
  IWRAP   -  RECEIVER AMPLITUDE AND PHASE            :          0
  IWRI    -  RECEIVER IMPULSE RESPONSE               :          0
  IWCAP   -  CHANNEL AMPLITUDE AND PHASE             :          0
  IWCI    -  CHANNEL IMPULSE RESPONSE                :          0
  IWSC    -  INDIVIDUAL CHANNEL SIGNALS              :          0

FILTER PARAMETERS
-----------------
  NPOL    -  NUMBER OF LOWPASS PROTOTYPE POLES       :          2
             POL(1)                                  :   -.70700   -.70700
             POL(2)                                  :   -.70700    .70700
  FBIF    -  FRACTIONAL BANDWIDTH AT FINAL IF        :    .10000
  FBRF    -  FRACTIONAL BANDWIDTH AT RF              :    .00100
  RADRNG  -  NORMALIZED RADIAN FREQUENCY RANGE       :   4.00000
  RADIN   -  NORMALIZED INITIAL RADIAN FREQUENCY     :  -2.00000
  NF      -  NUMBER OF FREQUENCY SAMPLES             :         32
  LPBP    -  LOWPASS/BANDPASS OPTION                 :          1
             0  :   LOWPASS
             1  :   BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS  -  NUMBER OF CHANNELS PER PORT             :        20
  ISC     -  CHANNEL SELECTOR ARRAY                  :  1 1 0 0 0 0 0 0 0 0
                                                     :  0 0 0 0 0 0 0 0 0 0

CHANNEL  1:
  AN      -  AMPLITUDE OF NOISE SOURCE               :   1.00000
  IX0     -  INITIAL RANDU SETTING                   :          1
  N1      -  FIRST-TIME-ON SAMPLE NUMBER             :          1
  NB      -  BLINK DURATION IN SAMPLES               :      10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)       :  45.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS      :    .00000
```

```
CHANNEL   2:
  AN      -  AMPLITUDE OF NOISE SOURCE                  :     1.00000
  IX0     -  INITIAL RANDU SETTING                      :          11
  N1      -  FIRST-TIME-ON SAMPLE NUMBER                :           1
  NB      -  BLINK DURATION IN SAMPLES                  :       10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)          :    10.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS         :      .10000

PORT PARAMETERS
---------------
  D0      -  ANTENNA-ELEMENT SEPARATION FACTOR          :     1.00000
  NS      -  NUMBER OF SIGNAL SAMPLES                   :         128
  NPORTS  -  NUMBER OF PORTS                            :          21
  ISP     -  PORT SELECTOR ARRAY                        :     1 1 1 1 0 0 0 0 0 0
                                                        :     0 0 0 0 0 0 0 0 0 0

PORT      0:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :         255
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                    :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY         :      .00000

PORT      1:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                    :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY         :      .00000

PORT      2:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                    :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY         :      .10000

PORT      3:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :         255
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                    :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY         :      .00000
```

```
PORT     4:
  NEL      -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1     -  LOCATION OF THE FIRST ELEMENT             :        255
  BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF    -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY        :     .10000
-----------------------------------------------------------------------
```

PORT 0

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .359902E+00

.05464	.03818	-.27935	-.18506	-.80850	-.80400	-.96805	-1.00000
-.89117	-.93809	-.86581	-.91809	-.51389	-.71681	-.01191	-.37046
.51795	.07067	.69954	.48863	.33405	.37157	-.72323	-.56527
-.48070	-.23752	-.28359	.13504	-.05666	.10339	.17505	.00267
-.06350	-.11284	-.27573	-.25835	-.11408	.03863	.07952	.27616
-.00055	-.11476	.45160	.31795	-.25093	-.05661	-.53158	-.23006
.10718	-.05535	.29687	.03986	-.22163	-.13246	-.32173	-.228[illegible]4
-.37581	.00988	-.03880	.09207	.23237	.18048	-.03938	.26499
-.60312	-.16596	-.22783	-.27798	.15059	.04556	.49784	.37388
.05924	.20007	-.67014	-.26716	-.51796	-.17555	.41518	.42498
.46477	.55190	-.24890	.00095	-.22332	-.22121	-.17883	-.20944
-.14506	-.08280	-.48174	-.33351	-.15840	-.23864	.03017	.12233
-.24552	.07159	.07142	.10982	.79052	.48894	.42138	.42662
-.56041	-.25491	-.64554	-.51348	.04592	.16574	.45397	.33689
-.15339	-.20463	-.60183	-.43150	-.23293	.15092	-.05293	.03788
.70177	.26965	.66569	.35406	.76548	.50369	.33144	.11207
.34198	.16070	-.22887	.04206	-.10226	.16876	.08978	.24863
.07344	-.10328	.46115	.36154	.51861	.34287	.17532	.33440
-.01579	.20464	.13438	.29128	-.17331	-.08766	.00878	.12494
-.21701	-.41771	.06459	-.23581	-.57297	-.59389	-.25658	-.38524
-.71101	-.39013	-.17867	-.31976	.16877	.45003	-.15970	.15444
.40744	.45485	.68224	.67182	.68668	.85575	-.08121	.25855
.30583	.30440	.19326	.26502	-.49589	-.22002	-.67978	-.41590
-.30489	-.53854	.35300	.05499	-.29986	.01299	-.08097	-.02180
.53392	.59253	-.08945	.13345	-.56345	-.37763	-.77421	-.36072
-.67038	-.15848	-.09812	.07619	-.07994	.11141	-.19309	-.00092
-.09846	-.05803	-.32363	-.11191	-.70204	-.51644	-.42421	-.15956
.35976	.29671	.55861	.31601	.34769	-.05135	.29570	.17155
-.02046	.16898	-.40331	-.40163	-.33261	-.31393	.02263	.05234
-.29163	-.15834	.20011	.26848	.21790	.33672	.14524	.09034
.12943	.14092	-.11299	.08524	.11257	.27772	.18866	.44657
.08273	.17843	.09462	.15594	-.23630	-.08516	.11599	-.09951

PORT 1

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .610091E+00

.03302	.02712	-.09132	-.17755	-.18820	-.83060	-.29433	-1.00000
-.21299	-.9A208	-.24348	-.93500	.045A3	-.82961	.37524	-.58496
.69A15	-.25163	.44716	.32677	.25227	.27871	-.37896	-.48880
-.63976	.03568	-.61052	.42171	-.26588	.28932	.34864	-.15242
.05773	-.16416	-.08271	-.27327	-.37270	.21433	-.13248	.38092
.09187	-.14752	.32176	.21072	-.18946	-.02783	-.67211	.07588
.21314	-.11392	.48308	-.20447	-.13321	-.11077	-.26331	-.15314
-.66599	.31100	-.21420	.24147	.14301	.13614	-.34743	.42636
-.71764	.15784	-.08099	-.23739	.13595	-.00945	.29930	.28787
-.02352	.19411	-.72186	.01725	-.74155	.16904	.00674	.52368
.18736	.50627	-.36548	.17965	-.07136	-.19976	-.04465	-.21834
-.09107	-.06771	-.31924	-.23553	-.04164	-.23201	-.08992	.14818
-.48834	.30318	-.12398	.22782	.61549	.29267	.28063	.30504
-.47300	-.08395	-.49339	-.34048	-.27158	.33822	.38861	.21674
.17691	-.35199	-.53938	-.24903	-.614A2	.44777	-.16926	.17435
.76491	-.02179	.61187	.10926	.60848	.28806	.45776	-.09097
.31446	.02369	-.35141	.16868	-.53609	.46701	-.04419	.31528
.17626	-.17624	.25267	.29613	.40248	.20129	-.07790	.37641
-.37641	.43022	-.07058	.36290	-.20013	.00489	-.11260	.20098
.22704	-.54907	.41214	-.47726	-.07624	-.65830	-.02222	-.44125
-.53181	-.22691	-.10767	-.27365	-.22590	.56261	-.51941	.44397
.01805	.53026	.17133	.70150	.10187	.89872	-.42416	.49915
.00668	.38858	.11727	.21994	-.4A694	-.03700	-.53686	-.20884
.10807	-.61208	.48876	-.19993	-.42084	.16529	-.26779	.13871
.10717	.61734	-.15058	.20583	-.41848	-.23829	-.80477	-.06036
-.97474	.28751	-.27616	.24423	-.24091	.22343	-.30947	.16292
-.0A795	-.02728	-.29755	.00077	-.45309	-.39201	-.63687	.13347
.15087	.30367	.57666	.09911	.65897	-.36033	.20237	.06469
-.16686	.26395	-.06628	-.42963	-.25135	-.22688	.01462	.03858
-.28190	-.05992	-.13892	.38777	.01027	.37187	.09397	.07921
.05936	.11497	-.31679	.23686	-.24357	.43610	-.24705	.62380
-.07275	.25A36	-.03243	.18642	-.22215	.01502	.20369	-.18164

PORT 2

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .610608E+00

-.02491	.03355	.16263	-.08818	.81748	-.18453	1.00000	-.29458
.98087	-.21286	.93707	-.24589	.83185	.03664	.59328	.36582
.26113	.69540	-.31089	.45527	-.29870	.25877	.48329	-.35991
-.01577	-.64259	-.42033	-.60788	-.29770	-.28708	.14425	.34693
.16590	.06461	.27090	-.07696	-.19333	-.36938	-.39666	-.14192
.14848	.08694	-.20708	.31965	.02145	-.16564	-.07124	-.67903
.10375	.18729	.20995	.49534	.10654	-.12410	.16072	-.25671
-.30212	-.66093	-.24916	-.23378	-.13129	.14652	-.42227	-.33021
-.17230	-.72136	.23572	-.09746	.01897	.13590	-.28427	.29409
-.19967	-.00354	-.01921	-.71096	-.15962	-.74948	-.51749	-.01636
-.51148	.20221	-.18992	-.36335	.19277	-.07815	.22331	-.04254
.06712	-.08752	.23033	-.31654	.23836	-.05146	-.13878	-.07756
-.30424	-.48622	-.22803	-.14235	-.29011	.60396	-.30762	.30404
.06820	-.46383	.34942	-.49497	-.32462	-.28447	-.23160	.37586
.34197	.19756	.26476	-.53024	-.43914	-.61555	-.18685	-.19341
.02313	.75344	-.10330	.61841	-.28966	.60655	.08191	.46258
-.01371	.32165	-.16907	-.33224	-.45422	-.54541	-.33628	-.05353
.18200	.17323	-.29086	.24971	-.20186	.40144	-.37266	-.05740
-.42744	-.38361	-.37236	-.07101	-.00402	-.19901	-.20547	-.11646
.53097	.21369	.48674	.42032	.64545	-.07112	.46042	-.01821
.21547	-.52287	.29255	-.12527	-.55336	-.20650	-.44781	-.52980
-.52516	.01045	-.69464	.16639	-.89979	.11444	-.50962	-.42002
-.38674	-.01006	-.22910	.12950	.03470	-.47597	.19819	-.54335
.60872	.08529	.21755	.50033	-.16688	-.40291	-.13090	-.28446
-.61051	.10815	-.22541	-.14262	.23738	-.41077	.06817	-.79295
-.28067	-.98174	-.24945	-.29299	-.21914	-.23018	-.17132	-.31848
.03016	-.08478	-.00962	-.29434	.39153	-.44412	-.11776	-.64398
-.30695	.12837	-.10783	.57398	.35383	.65887	-.04413	.22125
-.27802	-.16981	.42009	-.06146	.23955	-.25383	-.04047	.01365
.06365	-.27440	-.37577	-.14819	-.38166	.01140	-.08118	.08872
-.11255	.06829	-.23277	-.31185	-.42853	-.24660	-.62527	-.24683
-.26828	-.07985	-.18350	-.02597	-.02687	-.22517	.18985	.19357

PORT 3

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .779343E+00

.03553	.03825	-.23307	-.25001	-.74354	-.87491	-.86642	-1.00000
-.84147	-.96906	-.79776	-.91628	-.62521	-.77081	-.31204	-.45886
.05329	-.07371	.44374	.46365	.20395	.22604	-.48456	-.53220
-.09156	.00943	.13003	.30762	.10785	.17217	-.04291	-.13119
-.10665	-.13830	-.23519	-.26839	.13020	.23388	.17714	.26432
-.06153	-.12531	.25730	.25908	-.09900	-.06139	-.12132	-.02551
-.06191	-.13662	-.00140	-.09031	-.14191	-.131[illegible]	-.15835	-.14524
.06242	.22468	.09327	.14617	.16672	.16[illegible]29	.22804	.37299
-.14449	-.03258	-.20566	-.26463	.06339	.05043	.32116	.33779
.11695	.17053	-.20065	-.10117	-.05091	.07172	.42132	.50242
.41170	.49108	-.00008	.06620	-.19522	-.24028	-.16501	-.19502
-.09367	-.07952	-.29634	-.29418	-.16157	-.20566	.10885	.17313
.08267	.20812	.14721	.18026	.41812	.37577	.29959	.33705
-.25749	-.21473	-.35798	-.36526	.21317	.31954	.21523	.19404
-.25396	-.33762	-.27480	-.24231	.17093	.35754	.04293	.06770
.21649	.09150	.29874	.23991	.39797	.36847	.05161	-.03802
.15369	.12049	.02681	.12367	.25114	.40101	.13566	.18775
-.04054	-.11894	.31294	.34029	.28543	.26222	.27731	.37342
.24385	.35734	.21012	.28148	-.03584	-.02366	.08202	.13720
-.40433	-.56414	-.22577	-.35089	-.57194	-.67218	-.27968	-.33856
-.39289	-.33852	-.15000	-.20281	.36619	.54276	.19071	.32112
.43369	.51311	.62765	.71943	.70947	.86662	.23836	.36384
.31599	.35595	.15954	.18996	-.17788	-.11882	-.36966	-.34193
-.44087	-.59280	.04145	-.02978	-.01252	.10422	.07955	.11803
.51118	.62030	.03943	.09123	-.30200	-.29875	-.26235	-.14836
-.05444	.13452	.07886	.14670	.10642	.18723	.00257	.06106
-.03515	-.03093	-.13717	-.08959	-.42822	-.43797	-.03624	.08101
.26072	.28348	.20613	.13930	-.08240	-.25361	.18838	.19791
.09070	.16655	-.37476	-.45706	-.20103	-.20717	.01002	.03062
-.10695	-.07620	.29852	.38727	.24513	.31122	.09329	.07568
.11462	.13322	.10823	.19735	.29595	.40479	.39115	.53786
.15041	.18361	.14103	.17837	-.09906	-.07480	-.03096	-.10842

PORT 4

RECEIVED BASEBAND PORT SIGNAL

NORMALIZATION CONSTANT : .780274E+00

-.03678	.03492	.23329	-.21909	.86342	-.73378	1.00000	-.86610
.96786	-.84023	.91825	-.80002	.77471	-.63001	.46831	-.32138
.08469	.04396	-.45162	.43556	-.24909	.22376	.52855	-.47955
.00875	-.10555	-.30803	.13039	-.17855	.10913	.12626	-.03892
.13781	-.10485	.26768	-.23405	-.21584	.11663	-.28070	.18853
.13078	-.06747	-.25776	.25729	.05354	-.08949	.02853	-.12514
.13186	-.06230	.09489	-.00227	.12508	-.13571	.15364	-.16373
-.21955	.05893	-.15048	.09321	-.16307	.16264	-.37372	.23197
.01814	-.13456	.26797	-.21060	-.04088	.05573	-.33448	.31802
-.17906	.12709	.09810	-.19682	-.06110	-.06102	-.49374	.41082
-.49855	.42044	-.07721	.00836	.23598	-.19245	.20013	-.16858
.07728	-.09084	.28895	-.29244	.21472	-.17014	-.16661	.10609
-.21063	.08431	-.17727	.14133	-.37081	.41300	-.34407	.30951
.19766	-.24299	.37754	-.36810	-.30777	.20201	-.20702	.22364
.32705	-.24216	.25794	-.28607	-.35328	.16801	-.07539	.04373
-.08686	.21189	-.23436	.29481	-.37201	.40113	.03077	.05726
-.11226	.14949	-.12633	.03116	-.38935	.24063	-.20771	.14987
.12903	-.04933	-.33737	.31059	-.26064	.28383	-.37398	.28123
-.35380	.23941	-.29183	.21894	.02728	-.03921	-.14539	.08896
.55285	-.39874	.35714	-.22750	.66054	-.56345	.35608	-.29137
.32431	-.38152	.22754	-.17236	-.54062	.36826	-.32227	.18842
-.50683	.42798	-.71107	.61984	-.87153	.71631	-.37348	.24541
-.35185	.31092	-.20023	.17013	.11499	-.17318	.33096	-.36196
.59618	-.44777	.04543	.03237	-.11078	-.00558	-.10588	.06664
-.61690	.50981	-.11093	.05577	.29847	-.30015	.15461	-.26520
-.12893	-.06044	-.14835	.07754	-.18465	.10630	-.06845	.00663
.03464	-.03712	.07825	-.12736	.43991	-.42871	-.06727	-.04842
-.28342	.25692	-.14837	.21305	.25127	-.08094	-.18200	.17960
-.18334	.10331	.45062	-.36908	.21937	-.21103	-.03344	.01272
.08065	-.11038	-.37613	.28883	-.32143	.25304	-.07556	.09229
-.13175	.11534	-.19421	.10610	-.39669	.28910	-.54147	.39412
-.19048	.15429	-.17685	.14169	.06323	-.09104	.12127	-.04268

The second part is identical to the signal generation program output, SIGGEN:0. Thus BCRS:D may be created by concatenating the files BCRS:DO and SIGGEN:0. Note that the SIGGEN:0 data used here must be created with all print option parameters set to zero. Since all parameters involved in SIGGEN:0 were already discussed in the previous section only the new parameters of BCRS:DO are defined in Table 2-13.

Referring again to Table 2-11, the print options parameters are self-explanatory. Since the subsequent program, BCRP, will need the main port signal, it is necessary to set IWSO to 1 if BCRP is to be run next.

The BCR parameter NWX refers to the maximum number of weights and is thus limited to 20, the maximum number of auxiliary ports. Its associated weight-selector array ISW is used to indicate which of the ports selected by ISP are to be weighted. As such, ISW is a subset of ISP; that is, when a component of ISP is 0, the corresponding component of ISW may not be 1. The opposite condition is allowed.

The BCR program is allowed to go through at most 20 iterations, the maximum number of ports. So, unless fewer iterations are intended, ITERX, the maximum number of iterations, need not be changed. No other BCR parameter need be changed.

2.3.2 Program Structure

The BCR simulation program was designed with the same methods as the signal generation program. It has a similar modular-linear structure with the data input restricted to one executive program branch. The use of the "library" approach is demonstrated more convincingly by referencing subprograms which were first introduced for the SIGGEN program. Details of the program are shown in the following paragraphs.

2.3.2.1 Tree Diagram

The structure and modular order of execution of the BCRS program are clearly demonstrated by the tree diagram of Figure 2-5. The data input is done in the BCRSET branch, again using the utility subprograms RW, RWIRC, and SKIPR. This is a method similar to the one used in SIGGEN.

After input, the central executive subprogram takes over in the BCREXEC branch. The first task here is the computation of the covariance matrix C and forcing vector $\underline{b}$ via subprogram CANDB. Using the computed C and B, the iterative BCR process is applied in subprogram BCRSIM to estimate the adaptive weight vector, $\underline{w}$. The final task is completed in the CMBSGNL branch where the main port signal and the weighted auxiliary port signals are accumulated as the combined signal. The power suppression ratio is also calculated here.

2.3.2.2 Source Modules

The source program list of BCRS is shown in Table 2-14. To each subprogram in this list, there is a corresponding named module in the BCRS tree diagram. Since all of these subprograms are members of RADAR:LIB, those member-programs (i.e., RW, RWIRC, etc.) already employed in connection with SIGGEN are not listed. The functional description of the modules comprising BCRS, excluding those common to SIGGEN, is given in Table 2-15.

Table 2-11. Input Data File of the BCR Simulation Program, BCRS:D Benchtest Example (See Figure 2-2)

```
-----------------------------------------------------------------------------
                    DIGITAL ADAPTIVE ARRAY PROCESSING
                    ---------------------------------
                                  USING
                                  -----
                       BATCH COVARIANCE RELAXATION
                       ---------------------------
-----------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWS0    -   OPTIONAL OUTPUT OF S0                 :          1
  IWS1    -   OPTIONAL OUTPUT OF  S1                :          0
  IWCB    -   OPTIONAL OUTPUT OF  C   AND   B       :          1
BCR PARAMETERS
--------------
  NSX     -   MAXIMUM NUMBER OF SIGNAL SAMPLES      :        256
  IW0     -   SIZE OF BAND DIAGONAL                 :          0
  NWX     -   MAXIMUM NUMBER OF WEIGHTS             :         20
  ISW     -   WEIGHT SELECTOR ARRAY                 :   1 1 1 1 0 0 0 0 0 0
                                                    :   0 0 0 0 0 0 0 0 0 0
  ITERX   -   MAXIMUM NUMBER OF ITERATIONS          :         20
-----------------------------------------------------------------------------
  ITER    -   ACTUAL NUMBER OF ITERATIONS           :          X
-----------------------------------------------------------------------------
```

```
--------------------------------------------------------------------------------
                     SPECIFICATION OF SYSTEM PARAMETERS
                     ----------------------------------

--------------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWANT   -  MAIN ANTENNA ARRAY WEIGHTING               :        0
  IWRAP   -  RECEIVER AMPLITUDE AND PHASE               :        0
  IWRI    -  RECEIVER IMPULSE RESPONSE                  :        0
  IWCAP   -  CHANNEL AMPLITUDE AND PHASE                :        0
  IWCI    -  CHANNEL IMPULSE RESPONSE                   :        0
  IWSC    -  INDIVIDUAL CHANNEL SIGNALS                 :        0

FILTER PARAMETERS
-----------------
  NPOL    -  NUMBER OF LOWPASS PROTOTYPE POLES          :        2
             POL(1)                                     :  -.70700   -.70700
             POL(2)                                     :  -.70700    .70700
  FBIF    -  FRACTIONAL BANDWIDTH AT FINAL IF           :   .10000
  FBRF    -  FRACTIONAL BANDWIDTH AT RF                 :   .00100
  RADRNG  -  NORMALIZED RADIAN FREQUENCY RANGE          :  4.00000
  RADIN   -  NORMALIZED INITIAL RADIAN FREQUENCY        : -2.00000
  NF      -  NUMBER OF FREQUENCY SAMPLES                :       32
  LPBP    -  LOWPASS/BANDPASS OPTION                    :        1
             0  :   LOWPASS
             1  :   BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS  -  NUMBER OF CHANNELS PER PORT                :       20
  ISC     -  CHANNEL SELECTOR ARRAY                     :  1 1 0 0 0 0 0 0 0 0
                                                        :  0 0 0 0 0 0 0 0 0 0

CHANNEL  1:
  AN      -  AMPLITUDE OF NOISE SOURCE                  :  1.00000
  IX0     -  INITIAL RANDU SETTING                      :        1
  N1      -  FIRST-TIME-ON SAMPLE NUMBER                :        1
  NB      -  BLINK DURATION IN SAMPLES                  :    10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)          : 45.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS         :   .00000
```

```
CHANNEL  2:
  AN      -  AMPLITUDE OF NOISE SOURCE                :     1.00000
  IX0     -  INITIAL RANDU SETTING                    :          11
  N1      -  FIRST-TIME-ON SAMPLE NUMBER              :           1
  NB      -  BLINK DURATION IN SAMPLES                :       10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)        :    10.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS       :      .10000

PORT PARAMETERS
---------------
  D0      -  ANTENNA-ELEMENT SEPARATION FACTOR        :     1.00000
  NS      -  NUMBER OF SIGNAL SAMPLES                 :         128
  NPORTS  -  NUMBER OF PORTS                          :          21
  ISP     -  PORT SELECTOR ARRAY                      :   1 1 1 1 0 0 0 0 0 0
                                                      :   0 0 0 0 0 0 0 0 0 0

PORT    0:
  NEL     -  NUMBER OF ANTENNA ELEMENTS               :         255
  LOC1    -  LOCATION OF THE FIRST ELEMENT            :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR               :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                  :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY       :      .00000

PORT    1:
  NEL     -  NUMBER OF ANTENNA ELEMENTS               :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT            :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR               :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                  :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY       :      .00000

PORT    2:
  NEL     -  NUMBER OF ANTENNA ELEMENTS               :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT            :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR               :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                  :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY       :      .10000

PORT    3:
  NEL     -  NUMBER OF ANTENNA ELEMENTS               :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT            :         255
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR               :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                  :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY       :      .00000
```

```
PORT      4:
  NEL     -  NUMBER OF ANTENNA ELEMENTS              :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT           :        255
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR              :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                 :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY      :     .10000
---------------------------------------------------------------------------
```

PORT 0

RECEIVED BASEBAND PORT SIGNAL

NORMALIZATION CONSTANT : .359902E+00

.05464	.03818	-.27935	-.18506	-.80850	-.80400	-.96805	-1.00000
-.89117	-.93809	-.86581	-.91809	-.51389	-.71681	-.01191	-.37046
.51795	.07067	.69954	.48863	.33405	.37157	-.72323	-.56527
-.48070	-.23752	-.28359	.13504	-.05666	.10339	.17505	.00267
-.06350	-.11284	-.27573	-.25835	-.11408	.03863	.07952	.27616
-.00055	-.11476	.45160	.31795	-.25093	-.05661	-.53158	-.23006
.10718	-.05535	.29687	.03986	-.22163	-.13246	-.32173	-.22894
-.37581	.00988	-.03880	.09207	.23237	.18048	-.03938	.26499
-.60312	-.16596	-.22783	-.27798	.15059	.04556	.49784	.37388
.05924	.20007	-.67014	-.26716	-.51796	-.17555	.41518	.42498
.46477	.55190	-.24890	.00095	-.22332	-.22121	-.17883	-.20944
-.14506	-.08280	-.48174	-.33351	-.15840	-.23864	.03017	.12233
-.24552	.07159	.07142	.10982	.79052	.48894	.42138	.42662
-.56041	-.25491	-.64554	-.51348	.04592	.16574	.45397	.33689
-.15339	-.20463	-.60183	-.43150	-.23293	.15092	-.05293	.03788
.70177	.26965	.66569	.35406	.76548	.50369	.33144	.11207
.34198	.16070	-.22887	.04206	-.10226	.16876	.08978	.24863
.07344	-.10328	.46115	.36154	.51861	.34287	.17532	.33440
-.01579	.20464	.13438	.29128	-.17331	-.08766	.00878	.12494
-.21701	-.41771	.06459	-.23581	-.57297	-.59389	-.25658	-.38524
-.71101	-.39013	-.17867	-.31976	.16877	.45003	-.15970	.15444
.40744	.45485	.68224	.67182	.68668	.85575	-.08121	.25855
.30583	.30440	.19326	.26502	-.49589	-.22002	-.67978	-.41590
-.30489	-.53854	.35300	.05499	-.29986	.01299	-.08097	-.02180
.53392	.59253	-.08945	.13345	-.56345	-.37763	-.77421	-.36072
-.67038	-.15848	-.09812	.07619	-.07994	.11141	-.19309	-.00092
-.09846	-.05803	-.32363	-.11191	-.70204	-.51644	-.42421	-.15956
.35976	.29671	.55861	.31601	.34769	-.05135	.29570	.17155
-.02046	.16898	-.40331	-.40163	-.33261	-.31393	.02263	.05234
-.29163	-.15834	.20011	.26848	.21790	.33672	.14524	.09034
.12943	.14092	-.11299	.08524	.11257	.27772	.18866	.44657
.08273	.17843	.09462	.15594	-.23630	-.08516	.11599	-.09951

PORT 1

RECEIVED BASEBAND PORT SIGNAL

NORMALIZATION CONSTANT : .610091E+00

.03302	.02712	-.09132	-.17755	-.18820	-.83060	-.29433	-1.00000
-.21299	-.98208	-.24348	-.93500	.04583	-.82961	.37524	-.58496
.69815	-.25163	.44716	.32677	.25227	.27871	-.37896	-.48880
-.63976	.03568	-.61052	.42171	-.26588	.28932	.34864	-.15242
.05773	-.16416	-.08271	-.27327	-.37270	.21433	-.13248	.38092
.09187	-.14752	.32176	.21072	-.18946	-.02783	-.67211	.07588
.21314	-.11392	.48308	-.20447	-.13321	-.11077	-.26331	-.15314
-.66599	.31100	-.21420	.24147	.14301	.13614	-.34743	.42636
-.71764	.15784	-.08099	-.23739	.13595	-.00945	.29930	.28787
-.02352	.19411	-.72186	.01725	-.74155	.16904	.00674	.52368
.18736	.50627	-.36548	.17965	-.07136	-.19976	-.04465	-.21834
-.09107	-.06771	-.31924	-.23553	-.04164	-.23201	-.08992	.14818
-.48834	.30318	-.12398	.22782	.61549	.29267	.28063	.30504
-.47300	-.08395	-.49339	-.34048	-.27158	.33822	.38861	.21674
.17691	-.35199	-.53938	-.24903	-.61482	.44777	-.16926	.17435
.76491	-.02179	.61187	.10926	.60848	.28806	.45776	-.09097
.31446	.02369	-.35141	.16868	-.53609	.46701	-.04419	.31528
.17626	-.17624	.25267	.29613	.40248	.20129	-.07790	.37641
-.37641	.43022	-.07058	.36290	-.20013	.00489	-.11260	.20098
.22704	-.54907	.41214	-.47726	-.07624	-.65830	-.02222	-.44125
-.53181	-.22691	-.10767	-.27365	-.22590	.56261	-.51941	.44397
.01805	.53026	.17133	.70150	.10187	.89872	-.42416	.49915
.00668	.38858	.11727	.21994	-.48694	-.03700	-.53686	-.20884
.10807	-.61208	.48876	-.19993	-.42084	.16529	-.26779	.13871
.10717	.61734	-.15058	.20583	-.41848	-.23829	-.80477	-.06036
-.97474	.28751	-.27616	.24423	-.24091	.22343	-.30947	.16292
-.08795	-.02728	-.29755	.00077	-.45309	-.39201	-.63687	.13347
.15087	.30367	.57666	.09911	.65897	-.36033	.20237	.06469
-.16686	.26395	-.06628	-.42963	-.25135	-.22688	.01462	.03858
-.28190	-.05992	-.13892	.38777	.01027	.37187	.09397	.07921
.05936	.11497	-.31679	.23686	-.24357	.43610	-.24705	.62380
-.07275	.25836	-.03243	.18642	-.22215	.01502	.20369	-.18164

PORT 2

RECEIVED BASEBAND PORT SIGNAL

NORMALIZATION CONSTANT : .610608E+00

-.02491	.03355	.16263	-.08818	.81748	-.18453	1.00000	-.29458
.98087	-.21286	.93707	-.24589	.83185	.03664	.59328	.36582
.26113	.69540	-.31089	.45527	-.29870	.25877	.48329	-.35991
-.01577	-.64259	-.42033	-.60788	-.29770	-.28708	.14425	.34693
.16590	.06461	.27090	-.07696	-.19333	-.36938	-.39666	-.14192
.14848	.08694	-.20708	.31965	.02145	-.16564	-.07124	-.67903
.10375	.18729	.20995	.49534	.10654	-.12410	.16072	-.25671
-.30212	-.66093	-.24916	-.23378	-.13129	.14652	-.42227	-.33021
-.17230	-.72136	.23572	-.09746	.01897	.13590	-.28427	.29409
-.19967	-.00354	-.01921	-.71096	-.15962	-.74948	-.51749	-.01636
-.51148	.20221	-.18992	-.36335	.19277	-.07815	.22331	-.04254
.06712	-.08752	.23033	-.31654	.23836	-.05146	-.13878	-.07756
-.30424	-.48622	-.22803	-.14235	-.29011	.60396	-.30762	.30404
.06820	-.46383	.34942	-.49497	-.32462	-.28447	-.23160	.37586
.34197	.19756	.26476	-.53024	-.43914	-.61555	-.18685	-.19341
.02313	.75344	-.10330	.61841	-.28966	.60655	.08191	.46258
-.01371	.32165	-.16907	-.33224	-.45422	-.54541	-.33628	-.05353
.18200	.17323	-.29086	.24971	-.20186	.40144	-.37266	-.05740
-.42744	-.38361	-.37236	-.07101	-.00402	-.19901	-.20547	-.11646
.53097	.21369	.48674	.42032	.64545	-.07112	.46042	-.01821
.21547	-.52287	.29255	-.12527	-.55336	-.20650	-.44781	-.52980
-.52516	.01045	-.69464	.16639	-.89979	.11444	-.50962	-.42002
-.38674	-.01006	-.22910	.12950	.03470	-.47597	.19819	-.54335
.60872	.08529	.21755	.50033	-.16688	-.40291	-.13090	-.28446
-.61051	.10815	-.22541	-.14262	.23738	-.41077	.06817	-.79295
-.28067	-.98174	-.24945	-.29299	-.21914	-.23018	-.17132	-.31848
.03016	-.08478	-.00962	-.29434	.39153	-.44412	-.11776	-.64398
-.30695	.12837	-.10783	.57398	.35383	.65887	-.04413	.22125
-.27802	-.16981	.42009	-.06146	.23955	-.25383	-.04047	.01365
.06365	-.27440	-.37577	-.14819	-.38166	.01140	-.08118	.08872
-.11255	.06829	-.23277	-.31185	-.42853	-.24660	-.62527	-.24683
-.26828	-.07985	-.18350	-.02597	-.02687	-.22517	.18985	.19357

PORT 3

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .779343E+00

.03553	.03825	-.23307	-.25001	-.74354	-.87491	-.86642	-1.00000
-.84147	-.96906	-.79776	-.91628	-.62521	-.77081	-.31204	-.45886
.05329	-.07371	.44374	.46365	.20395	.22604	-.48456	-.53220
-.09156	.00943	.13003	.30762	.10785	.17217	-.04291	-.13119
-.10665	-.13830	-.23519	-.26839	.13020	.23388	.17714	.26432
-.06153	-.12531	.25730	.25908	-.09900	-.06139	-.12132	-.02551
-.06191	-.13662	-.00140	-.09031	-.14191	-.13168	-.15835	-.14524
.06242	.22468	.09327	.14617	.16672	.16929	.22804	.37299
-.14449	-.03258	-.20566	-.26463	.06339	.05043	.32116	.33779
.11695	.17053	-.20065	-.10117	-.05091	.07172	.42112	.50242
.41170	.49108	-.00008	.06620	-.19522	-.24028	-.16501	-.19502
-.09367	-.07952	-.29634	-.29418	-.16157	-.20566	.10885	.17313
.08267	.20812	.14721	.18026	.41812	.37577	.29959	.33705
-.25749	-.21473	-.35798	-.36526	.21317	.31954	.21523	.19404
-.25396	-.33762	-.27480	-.24231	.17093	.35754	.04293	.06770
.21649	.09150	.29874	.23991	.39797	.36847	.05161	-.03802
.15369	.12049	.02681	.12367	.25114	.40101	.13566	.18775
-.04054	-.11894	.31294	.34029	.28543	.26222	.27731	.37342
.24385	.35734	.21012	.28148	-.03584	-.02366	.08202	.13720
-.40433	-.56414	-.22577	-.35089	-.57194	-.67218	-.27968	-.33856
-.39289	-.33852	-.15000	-.20281	.36619	.54276	.19071	.32112
.43369	.51311	.62765	.71943	.70947	.86662	.23836	.36384
.31599	.35595	.15954	.18996	-.17788	-.11882	-.36966	-.34193
-.44087	-.59280	.04145	-.02978	-.01252	.10422	.07955	.11803
.51118	.62030	.03943	.09123	-.30200	-.29875	-.26235	-.14836
-.05444	.13452	.07886	.14670	.10642	.18723	.00257	.06106
-.03515	-.03093	-.13717	-.08959	-.42822	-.43797	-.03624	.08101
.26072	.28348	.20613	.13930	-.08240	-.25361	.18838	.19791
.09070	.16655	-.37476	-.45706	-.20103	-.20717	.01002	.03062
-.10695	-.07620	.29852	.38727	.24513	.31122	.09329	.07568
.11462	.13322	.10823	.19735	.29595	.40479	.39115	.53786
.15041	.18361	.14103	.17837	-.09906	-.07480	-.03096	-.10842

PORT 4

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .780274E+00

-.03678	.03492	.23329	-.21909	.85342	-.73378	1.00000	-.86610
.96786	-.84023	.91825	-.80002	.77471	-.63001	.46831	-.32138
.08469	.04396	-.45162	.43556	-.24909	.22376	.52855	-.47955
.00875	-.10555	-.30803	.13039	-.17855	.10913	.12626	-.03892
.13781	-.10485	.26768	-.23405	-.21584	.11663	-.28070	.18853
.13078	-.06747	-.25776	.25729	.05354	-.08949	.02853	-.12514
.13186	-.06230	.09489	-.00227	.12508	-.13571	.15364	-.16373
-.21955	.05893	-.15048	.09321	-.16307	.16264	-.37372	.23197
.01814	-.13456	.26797	-.21060	-.04088	.05573	-.33448	.31802
-.17906	.12709	.09810	-.19682	-.06110	-.06102	-.49374	.41082
-.49855	.42044	-.07721	.00836	.23598	-.19245	.20013	-.16858
.07728	-.09084	.28895	-.29244	.21472	-.17014	-.16661	.10609
-.21063	.08431	-.17727	.14133	-.37081	.41300	-.34407	.30951
.19766	-.24299	.37754	-.36810	-.30777	.20201	-.20702	.22364
.32705	-.24216	.25794	-.28607	-.35328	.16801	-.07539	.04373
-.08686	.21189	-.23436	.29481	-.37201	.40113	.03077	.05726
-.11226	.14949	-.12633	.03116	-.38935	.24063	-.20771	.14987
.12903	-.04933	-.33737	.31059	-.26064	.28383	-.37398	.28123
-.35380	.23941	-.29183	.21894	.02728	-.03921	-.14539	.08896
.55285	-.39874	.35714	-.22750	.66054	-.56345	.35608	-.29137
.32431	-.38152	.22754	-.17236	-.54062	.36826	-.32227	.18842
-.50683	.42798	-.71107	.61984	-.87153	.71631	-.37348	.24541
-.35185	.31092	-.20023	.17013	.11499	-.17318	.33096	-.36196
.59618	-.44777	.04543	.03237	-.11078	-.00558	-.10588	.06664
-.61690	.50981	-.11093	.05577	.29847	-.30015	.15461	-.26520
-.12893	-.06044	-.14835	.07754	-.18465	.10630	-.06845	.00663
.03464	-.03712	.07825	-.12736	.43991	-.42871	-.06727	-.04842
-.28342	.25692	-.14837	.21305	.25127	-.08094	-.18200	.17960
-.18334	.10331	.45062	-.36908	.21937	-.21103	-.03344	.01272
.08065	-.11038	-.37613	.28883	-.32143	.25304	-.07556	.09229
-.13175	.11534	-.19421	.10610	-.39669	.28910	-.54147	.39412
-.19048	.15429	-.17685	.14169	.06323	-.09104	.12127	-.04268

Table 2-12. BCR Specification Auxiliary Data File, BCRS:0

```
--------------------------------------------------------------------------------
                    DIGITAL ADAPTIVE ARRAY PROCESSING
                    ---------------------------------
                                  USING
                                  -----
                        BATCH COVARIANCE RELAXATION
                        ---------------------------
--------------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWS0    -  OPTIONAL OUTPUT OF S0                :          1
  IWS1    -  OPTIONAL OUTPUT OF  S1               :          0
  IWCB    -  OPTIONAL OUTPUT OF  C  AND  B        :          1

BCR PARAMETERS
--------------
  NSX     -  MAXIMUM NUMBER OF SIGNAL SAMPLES     :        256
  IWO     -  SIZE OF BAND DIAGONAL                :          0
  NWX     -  MAXIMUM NUMBER OF WEIGHTS            :         20
  ISW     -  WEIGHT SELECTOR ARRAY                :    1 1 1 1 0 0 0 0 0 0
                                                  :    0 0 0 0 0 0 0 0 0 0
  ITERX   -  MAXIMUM NUMBER OF ITERATIONS         :         20
--------------------------------------------------------------------------------
  ITER    -  ACTUAL NUMBER OF ITERATIONS          :          X
--------------------------------------------------------------------------------
```

Table 2-13. Input Data Description of the BCR Simulation Program, BCRS

```
-------------------------------------------------------------------------------
                      DIGITAL ADAPTIVE ARRAY PROCESSING
                                   USING
                        BATCH COVARIANCE RELAXATION
                        INPUT PARAMETER DEFINITION
                        --------------------------

PRINT OPTIONS
-------------
 IWS0    -  OPTIONAL OUTPUT OF S0

 IWS1    -  OPTIONAL OUTPUT OF  S1

 IWCB    -  OPTIONAL OUTPUT OF  C  AND  B
              SET ANY OF THESE PARAMETERS TO 0 FOR NO-PRINT, OR 1 FOR PRINT.
              THE PRINTING OF S0 IS NECESSARY IF THE DATA IS TO BE USED FOR
              INPUT TO THE BCRP PROGRAM.
```

BCR PARAMETERS

```
NSX      -  MAXIMUM NUMBER OF SIGNAL SAMPLES.  THIS PARAMETER SETS CERTAIN
            ARRAY LIMITS, AND HAS NO EFFECT ON THE PROGRAM AS LONG AS THE
            LIMITS  NS < NSX < 256 ARE OBSERVED.

IWO      -  SIZE OF BAND DIAGONAL.  THIS PARAMETER HAS NO EFFECT ON THE PROG-
            RAM.

NWX      -  MAXIMUM NUMBER OF WEIGHTS.  THIS PARAMETER MUST HAVE THE SAME
            VALUE AS THE NUMBER OF AUXILIARY PORTS IN THE SIGGEN:D DEFINITION.
            NWX = NPORTS - 1 .

ISW      -  WEIGHT SELECTOR ARRAY.  FOR EACH WEIGHT SELECTED BY ISW, THERE MUST
            BE A PORT SELECTED BY ISP.  THE CONVERSE IS NOT TRUE.  THERE MUST
            BE NWX-ELEMENTS OF ISW DEFINED.

ITERX    -  MAXIMUM NUMBER OF ITERATIONS.  RANGE OF VALUES IS 1 TO NPORTS-1.

ITER     -  ACTUAL NUMBER OF ITERATIONS.  THIS IS CONSIDERED AS A COMMENT IN
            THIS PROGRAM.
```

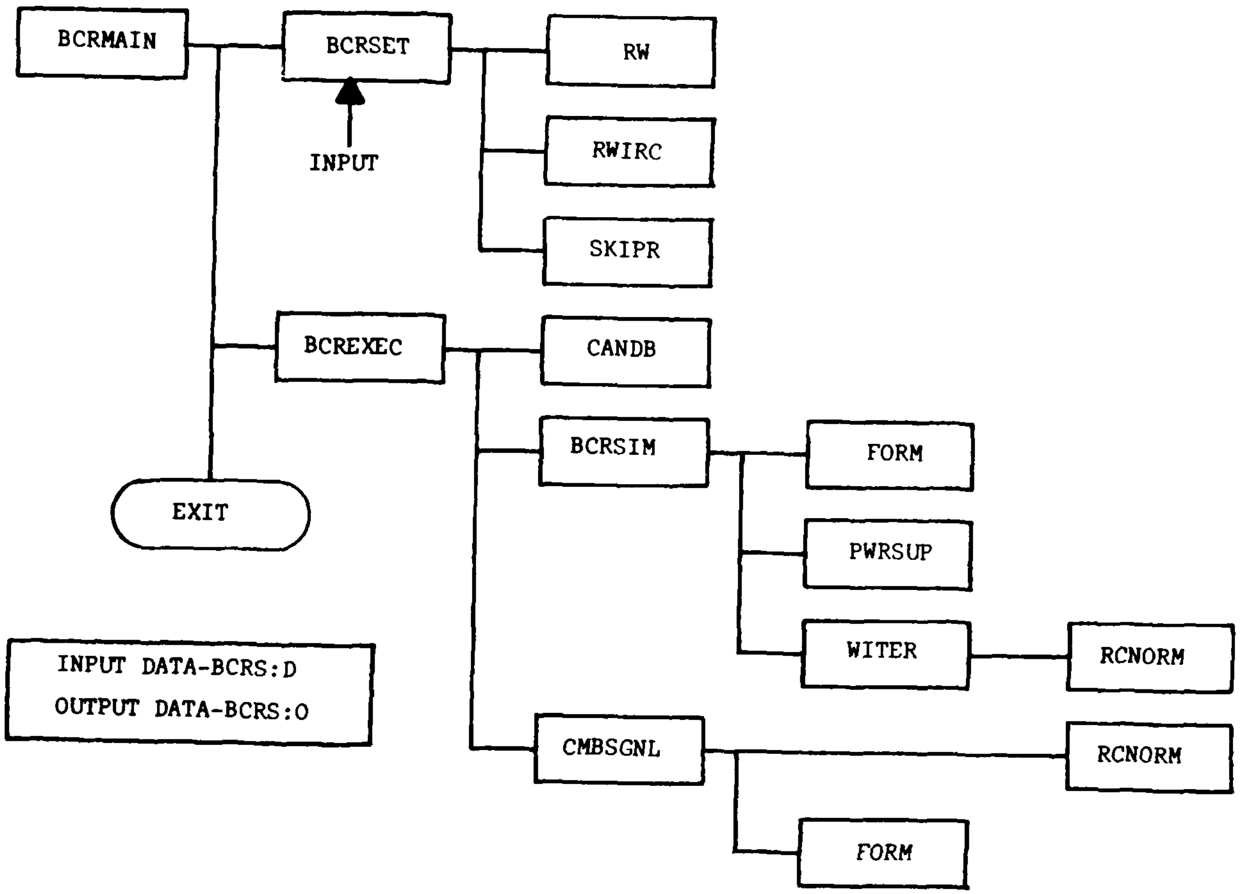


Figure 2-5. Tree Diagram of the BCR Simulation Program, BCRS

Table 2-14. FORTRAN Listing of Source Modules Comprising the BCR Simulation Program, BCRS:S

```
C ===================================================================
C                    BCR SIMULATION PROGRAM :  BCRS:S
C                    -------------------------------
C
C                    DIGITAL ADAPTIVE ARRAY PROCESSING
C                                 USING
C                       BATCH COVARIANCE RELAXATION
C
C            PROGRAM      :  BCRMAIN:S
C            ORIGINAL     :  JULY   23, 1980
C            REVISION     :  AUGUST 12, 1980
C
C            PREPARED  BY :  S. M. DANIEL  &  I. KERTESZ
C                            RADAR SYSTEMS ANALYSIS GROUP
C                            MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                            TEMPE, ARIZONA  85282
C ===================================================================
      CALL BCRSET
      CALL BCREXEC
      CALL EXIT
      END
```

```
C     ==================================================================
      SUBROUTINE BCRSET
C     ==================================================================
C                       SPECIFICATION OF SYSTEM PARAMETERS,
C                  MAIN-PORT AND AUXILIARY BASEBAND SIGNALS
C
C                PROGRAM        :  BCRSET:S
C                ORIGINAL       :  JULY    23, 1980
C                REVISION       :  JANUARY 24, 1981
C
C                PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                  RADAR SYSTEMS ANALYSIS GROUP
C                                  MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                  TEMPE, ARIZONA  85282
C     ------------------------------------------------------------------
C         INPUT  :    DATA FILE BCRSP:D
C         OUTPUT :    DATA FILE BCRSP:O
C     ==================================================================
      COMPLEX CDUM,S1,S0
      COMMON /BCR0/ IWS0,IWS1,IWCB
      COMMON /BCR1/ S0(256),S1(256,20),NSX,NS
      COMMON /BCR2/ ISW(20),NWX,IW0,NPM1,ITERX
      DIMENSION ISC(20),ISP(20),AR14(14),AR12(12)
100   FORMAT(14A4,4X,1012)
200   FORMAT(12A4,E13.6)
300   FORMAT(8F10.5)
999   FORMAT('****  SELECTED WEIGHTS ARE NOT COMPATIBLE WITH DESIGNATED
     -PORTS  *****')
C     ------------------------------------------------------------------
C         READ IN SYSTEM SPECIFICATIONS FROM
C     ------------------------------------------------------------------
      CALL RW(12)
      CALL RWIRC(CDUM,RDUM,IWS0,0)
      CALL RWIRC(CDUM,RDUM,IWS1,0)
      CALL RWIRC(CDUM,RDUM,IWCB,0)
      CALL RW(3)
      CALL RWIRC(CDUM,RDUM,NSX,0)
      CALL RWIRC(CDUM,RDUM,IW0,0)
      CALL RWIRC(CDUM,RDUM,NWX,0)
C     ------------------------------------------------------------------
C         READ IN WEIGHT SELECTOR ARRAY OVER DESIGNATED PORTS
C     ------------------------------------------------------------------
      NW1=1
```

```
      NW2=10
 1000 READ (105,100) AR14,(ISW(IW),IW=NW1,NW2)
      WRITE(108,100) AR14,(ISW(IW),IW=NW1,NW2)
      NW1=NW1+10
      NW2=NW2+10
      IF(NW1.GT.NWX) GOTO 1500
      IF(NW2.GE.NWX) NW2=NWX $ GOTO 1000
 1500 CALL RWIRC(CDUM,RDUM,ITERX,0)
      CALL RW(20)
      CALL RWIRC(CDUM,RDUM,NPOL,0)
      CALL RW(NPOL+11)
C ---------------------------------------------------------------------
C     READ IN CHANNEL SELECTOR ARRAY AND ASSOCIATED CHANNEL DESCRIPTIONS
C ---------------------------------------------------------------------
      CALL RWIRC(CDUM,RDUM,NCHNLS,0)
      NC1=1
      NC2=10
 2000 READ(105,100) AR14,(ISC(IC),IC=NC1,NC2)
      WRITE(108,100) AR14,(ISC(IC),IC=NC1,NC2)
      NC1=NC1+10
      NC2=NC2+10
      IF(NC1.GT.NCHNLS) GOTO 2500
      IF(NC2.GE.NCHNLS) NC2=NCHNLS
      GOTO 2000
 2500 ICSUM=0
      DO 30 IC=1,NCHNLS
 30   ICSUM=ICSUM+ISC(IC)
      CALL RW(4+8*ICSUM)
      CALL RWIRC(CDUM,RDUM,NS,0)
C ---------------------------------------------------------------------
C     READ IN PORT SELECTOR ARRAY AND ASSOCIATED PORT DESCRIPTIONS
C ---------------------------------------------------------------------
      CALL RWIRC(CDUM,RDUM,NPORTS,0)
      NPM1=NPORTS-1
      NP1=1
      NP2=10
 3000 READ (105,100) AR14,(ISP(IP),IP=NP1,NP2)
      WRITE(108,100) AR14,(ISP(IP),IP=NP1,NP2)
      NP1=NP1+10
      NP2=NP2+10
      IF(NP1.GT.NPM1) GOTO 3500
      IF(NP2.GE.NPM1) NP2=NPM1
      GOTO 3000
```

```
3500  IPSUM=0
      DO 50 IP=1,NPM1
      IPIW=ISP(IP)*ISW(IP)
      IF(IPIW.NE.ISW(IP)) GOTO 9999
50    IPSUM=IPSUM+ISP(IP)
      CALL RW(7*(IPSUM+1))
C ------------------------------------------------------------------
C     READ IN MAIN AND AUXILIARY BASEBAND SIGNALS
C ------------------------------------------------------------------
      IF(IWS0.EQ.0) GOTO 4000
C ------------------------------------------------------------------
C     READ AND WRITE S0
C ------------------------------------------------------------------
      CALL RW(7)
      READ (105,200) AR12,SMAX
      WRITE(108,200) AR12,SMAX
      CALL RW(1)
      READ (105,300) (S0(IS),IS=1,NS)
      WRITE(108,300) (S0(IS),IS=1,NS)
      GOTO 4500
C ------------------------------------------------------------------
C     READ BUT DO NOT WRITE S0
C ------------------------------------------------------------------
4000  CALL SKIPR(105,7)
      READ (105,200) AR12,SMAX
      CALL SKIPR(105,1)
      READ (105,300) (S0(IS),IS=1,NS)
C ------------------------------------------------------------------
C     DENORMALIZE S0
C ------------------------------------------------------------------
4500  DO 60 IS=1,NS
60    S0(IS)=SMAX*S0(IS)
C ------------------------------------------------------------------
C     READ ONLY THOSE OF THE DESIGNATED PORTS SELECTED BY ISW
C ------------------------------------------------------------------
      IIP=0
      NROWS=(NS-1)/4+1
      DO 80 IP=1,NPM1
      IF(ISP(IP).EQ.0) GOTO 80
      IF(ISW(IP).EQ.0) GOTO 5000
      IIP=IIP+1
      IF(IWS1.EQ.0) GOTO 5500
C ------------------------------------------------------------------
```

```
C     READ AND WRITE S1
C ----------------------------------------------------------------------
      CALL RW(7)
      READ (105,200) AR12,SMAX
      WRITE(108,200) AR12,SMAX
      CALL RW(1)
      READ (105,300) (S1(IS,IIP),IS=1,NS)
      WRITE(108,300) (S1(IS,IIP),IS=1,NS)
      GOTO 6000
C ----------------------------------------------------------------------
C     IF ISP(IP)=1 AND ISW(IP)=0 USE SKIPREAD
C ----------------------------------------------------------------------
5000  CALL SKIPR(105,9+NROWS)
      GOTO 80
C ----------------------------------------------------------------------
C     READ BUT NOT WRITE S1
C ----------------------------------------------------------------------
5500  CALL SKIPR(105,7)
      READ (105,200) AR12,SMAX
      CALL SKIPR(105,1)
      READ (105,300) (S1(IS,IIP),IS=1,NS)
C ----------------------------------------------------------------------
C     DENORMALIZE S1
C ----------------------------------------------------------------------
6000  DO 70 IS=1,NS
70    S1(IS,IIP)=SMAX*S1(IS,IIP)
80    CONTINUE
      NPM1=IIP
      CALL RW(1)
C ----------------------------------------------------------------------
      RETURN
9999  WRITE(108,999)
      STOP
      END
```

```
C ========================================================================
      SUBROUTINE RW(N)
C ========================================================================
C               READING AND WRITING 80-CHARACTER DATA COMMENTS
C
C               PROGRAM        :  RW:S
C               ORIGINAL       :  APRIL 19, 1978
C               REVISION       :  JUNE  15, 1980
C
C               PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                 RADAR SYSTEMS ANALYSIS GROUP
C                                 MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                 TEMPE, ARIZONA  85282
C ------------------------------------------------------------------------
C     INPUT  :    N          -  NUMBER OF LINES TO BE READ AND WRITTEN
C ========================================================================
      DIMENSION AR20(20)
  100 FORMAT(20A4)
      DO 10 I=1,N
      READ (105,100) AR20
   10 WRITE(108,100) AR20
      RETURN
      END
```

```
C     ==========================================================================
      SUBROUTINE RWIRC(CX,RX,IX,IND)
C     ==========================================================================
C                        READING AND WRITING AN 80-CHARACTER LINE
C                                       CONSISTING OF
C                                 A 56-CHARACTER COMMENT
C                                            AND
C                 A COMPLEX, REAL OR INTEGER SCALAR VARIABLE VALUE
C
C                  PROGRAM        :  RWIRC:S
C                  ORIGINAL       :  APRIL  19, 1980
C                  REVISION       :  AUGUST 13, 1980
C
C                  PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                    RADAR SYSTEMS ANALYSIS GROUP
C                                    MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                    TEMPE, ARIZONA  85282
C     --------------------------------------------------------------------------
C     INPUT  :    IND        -   OPTION INDICATOR
C                                 0  : READ & WRITE INTEGER
C                                 1  : READ & WRITE REAL
C                                 2  : READ & WRITE COMPLEX
C
C     OUTPUT :    CX         -   COMPLEX SCALAR EXCLUSIVE OF RX & IX
C                 RX         -   REAL SCALAR EXCLUSIVE OF CX & IX
C                 IX         -   INTEGER SCALAR EXCLUSIVE OF CX & RX
C     ==========================================================================
      COMPLEX CX
      DIMENSION AR14(14)
  100 FORMAT(14A4,I12)
  200 FORMAT(14A4,F12.5)
  300 FORMAT(14A4,2X,2(F10.5))
      GOTO (1,2),IND
      READ (105,100) AR14,IX
      WRITE(108,100) AR14,IX
      RETURN
    1 READ (105,200) AR14,RX
      WRITE(108,200) AR14,RX
      RETURN
    2 READ (105,300) AR14,CX
      WRITE(108,300) AR14,CX
      RETURN
      END
```

```
C ==================================================================
      SUBROUTINE SKIPR(NU,NS)
C ==================================================================
C SKIP NS LINES ON UNIT NU.
C
C                PROGRAM        :  SKIPRIS
C                ORIGINAL       :  FEBRUARY 23, 1981
C                REVISION       :  FEBRUARY 23, 1981
C
C                PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                  RADAR SYSTEMS ANALYSIS GROUP
C                                  MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                  TEMPE, ARIZONA  85282
C ------------------------------------------------------------------
C     INPUT  :    NU          -    NUMBER OF UNIT OR DEVICE
C                 NS          -    NUMBER OF LINES TO SKIP
C
C     OUTPUT  :               -    NONE
C ------------------------------------------------------------------
C     ENTRY POINTS            :    SKIPR
C     SUBROUTINES CALLED      :    NONE
C ==================================================================
100   FORMAT(1X)
C ------------------------------------------------------------------
C     SKIP NS LINES ON UNIT NU
C ------------------------------------------------------------------
      DO 10 I=1,NS
10    READ(NU,100)
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE BCREXEC
C ======================================================================
C                              BCR EXECUTION PROGRAM
C
C                 PROGRAM       :  BCREXEC:S
C                 ORIGINAL      :  AUGUST  10,1980
C                 REVISION      :  JANUARY  6, 1981
C
C                 PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                                  RADAR SYSTEMS ANALYSIS GROUP
C                                  MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                  TEMPE, ARIZONA  85282
C ======================================================================
      COMPLEX S0,S1,C,B,R,P,CP,W,SC,WIT
      COMMON /BCR0/ IWS0,IWS1,IWCB
      COMMON /BCR1/ S0(256),S1(256,20),NSX,NS
      COMMON /BCR2/ ISW(20),NWX,IW0,NPM1,ITERX
      COMMON /BCR3/ W(20),SC(256),WIT(21,20)
      DIMENSION C(20,20),B(20),R(20),P(20),CP(20)
      CALL CANOB(S0,S1,C,B,CMAX,BMAX,P0,NSX,NS,NWX,NPM1,IWCB)
      ITERX1=ITERX+1
      CALL BCRSIM(C,B,R,P,CP,W,WIT,CMAX,BMAX,P0,IW0,NWX,NPM1,ITERX
     -,ITERX1)
      CALL CMBSGNL(S0,S1,W,SC,CMAX,BMAX,NSX,NS,NPM1)
      RETURN
      END
```

```
C  ======================================================================
      SUBROUTINE CANDB(S0,S1,C,B,CMAX,BMAX,P0,NSX,NS,NWX,NPM1,IWCB)
C  ======================================================================
C               COMPUTATION OF THE BATCH COVARIANCE MATRIX
C                         AND THE FORCING VECTOR
C
C                 PROGRAM        :  CANDB:S
C                 ORIGINAL       :  JULY     23, 1980
C                 REVISION       :  JANUARY   6, 1981
C
C                 PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                   RADAR SYSTEMS ANALYSIS GROUP
C                                   MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                   TEMPE, ARIZONA  85282
C  ----------------------------------------------------------------------
C     INPUT  :       S0          -  MAIN-PORT SIGNAL
C                    S1          -  AUXILIARY SIGNAL VECTOR
C                    NSX         -  MAXIMUM NUMBER OF SIGNAL SAMPLES
C                    NS          -  ACTUAL NUMBER OF SIGNAL SAMPLES
C                    NWX         -  MAXIMUM NUMBER OF WEIGHTS
C                    NPM1        -  NUMBER OF AUXILIARY PORTS
C                    IWCB        -  PRINT OPTION FOR  C  AND  B
C                                    0  : NO OUTPUT
C                                    1  : OUTPUT
C
C     OUTPUT :       P0          -  MAIN-PORT SIGNAL POWER
C                    C           -  COVARIANCE OF AUXILIARY SIGNAL VECTOR
C                    B           -  CROSS-CORRELATION VECTOR BETWEEN
C                                   MAIN AND AUXILIARY SIGNALS
C                    CMAX        -  NORMALIZATION CONSTANT FOR  C
C                    BMAX        -  NORMALIZATION CONSTANT FOR  B
C  ======================================================================
      COMPLEX S0,S1,C,B,CS1
      DIMENSION S0(1),S1(NSX,1),C(NWX,1),B(1)
  100 FORMAT(1H1)
  200 FORMAT(1X,79('-'))
  300 FORMAT(/,31X,'COVARIANCE MATRIX  C',/,19X
     -,'NORMALIZATION CONSTANT     :  ',E13.6,/)
  400 FORMAT(8F10.5)
  500 FORMAT(/,33X,'FORCING VECTOR  B',/,19X
     -,'NORMALIZATION CONSTANT     :  ',F13.5,/)
C  ----------------------------------------------------------------------
C     COMPUTE MAIN-PORT SIGNAL POWER
```

```
C -----------------------------------------------------------------
      P0=0
      DO 10 IS=1,NS
10    P0=P0+CABS(S0(IS))**2
C -----------------------------------------------------------------
C     INITIALIZE C AND B TO ZERO
C -----------------------------------------------------------------
      DO 20 IP=1,NPM1
      B(IP)=0
      DO 20 JP=1,NPM1
20    C(IP,JP)=0
C -----------------------------------------------------------------
C     COMPUTE C AND B FOR SELECTED PORTS
C -----------------------------------------------------------------
      DO 30 IS=1,NS
      DO 30 IP=1,NPM1
      CS1=CONJG(S1(IS,IP))
      B(IP)=B(IP)+CS1*S0(IS)
      DO 30 JP=1,NPM1
      C(IP,JP)=C(IP,JP)+CS1*S1(IS,JP)
30    CONTINUE
C -----------------------------------------------------------------
C     NORMALIZE  C  AND  B  INDEPENDENTLY
C -----------------------------------------------------------------
      CMAX=BMAX=0
      DO 40 IP=1,NPM1
      ABSX=ABS(REAL(C(IP,IP)))
      IF(ABSX.GT.CMAX) CMAX=ABSX
      ABSX=ABS(AIMAG(C(IP,IP)))
      IF(ABSX.GT.CMAX) CMAX=ABSX
      ABSX=ABS(REAL(B(IP)))
      IF(ABSX.GT.BMAX) BMAX=ABSX
      ABSX=ABS(AIMAG(B(IP)))
40    IF(ABSX.GT.BMAX) BMAX=ABSX
      DO 50 IP=1,NPM1
      B(IP)=B(IP)/BMAX
      DO 50 JP=1,NPM1
50    C(IP,JP)=C(IP,JP)/CMAX
C -----------------------------------------------------------------
C     GET OPTIONAL OUTPUT OF  C  AND  B
C -----------------------------------------------------------------
      IF(IWCB.EQ.0) RETURN
      WRITE(108,100)
```

```
      WRITE(108,200)
      WRITE(108,300) CMAX
      DO 60 IP=1,NPM1
60    WRITE(108,400) (C(IP,JP),JP=1,NPM1)
      WRITE(108,500) BMAX
      WRITE(108,400) (B(IP),IP=1,NPM1)
      WRITE(108,200)
C ---------------------------------------------------------------------------
      RETURN
      END
```

```
C ==================================================================
      SUBROUTINE BCRSIM(C,B,R,P,CP,W,WIT,CMAX,BMAX,P0,IW0,NWX,NPM1
     -,ITERX,ITERX1)
C ==================================================================
C                      ESTIMATION OF ADAPTIVE WEIGHT VECTOR
C                                      VIA
C                                BCR SIMULATION
C
C                 PROGRAM       :  BCRSIM:S
C                 ORIGINAL      :  APRIL      15, 1978
C                 REVISION      :  FEBRUARY  27, 1981
C
C                 PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                                  RADAR SYSTEMS ANALYSIS GROUP
C                                  MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                  TEMPE, ARIZONA  85282
C ------------------------------------------------------------------
C     INPUT  :     C          -  COVARIANCE MATRIX OF AUXILIARY SIGNAL
C                                VECTOR
C                  B          -  CROSS-CORRELATION VECTOR OF MAIN AND
C                                AUXILIARY SIGNALS
C                  P0         -  MAIN-PORT SIGNAL POWER
C                  CMAX       -  NORMALIZATION CONSTANT FOR  C
C                  BMAX       -  NORMALIZATION CONSTANT FOR  B
C                  IW0        -  WEIGHT VECTOR INITIALIZATION INDICATOR
C                                  0  : SETS INITIAL W TO ZERO
C                                  1  : USES PREVIOUS W VALUE
C                  NWX        -  MAXIMUM NUMBER OF WEIGHTS
C                  ITERX      -  MAXIMUM NUMBER OF ITERATIONS
C                  ITERX1     -  MAXIMUM NUMBER OF ITERATIONS PLUS ONE
C                  NPM1       -  NUMBER OF AUXILIARY PORTS
C
C     OUTPUT :     W          -  ADAPTIVE WEIGHT VECTOR
C ==================================================================
      COMPLEX C,B,R,P,CP,W,CARR,WIT
      DIMENSION C(NWX,1),B(1),R(1),P(1),CP(1),W(1),CARR(1),RARR(1)
     -,IARR(1),WIT(ITERX1,1)
  100 FORMAT(1H1)
  200 FORMAT(1X,109('-'))
  300 FORMAT(41X,'PERFORMANCE SUMMARY OF BCR SIMULATION',/,41X,37('-'))
  400 FORMAT(2X,'INITIAL CONDITIONS'/2X,18('-'))
  500 FORMAT(/,2X,' C                            :',8F10.5,/
     -,(28X,8F10.5))
```

```
600   FORMAT(27X,':',8F10.5,/,(28X,8F10.5))
700   FORMAT(//,2X,'MAIN PROCESS',/,2X,12('-'))
C     ----------------------------------------------------------------
C     BCR SIMULATION OVER SELECTED PORTS
C     ----------------------------------------------------------------
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,300)
      WRITE(108,200)
C     ----------------------------------------------------------------
C     INITIAL BCR CONDITIONS
C     ----------------------------------------------------------------
      BDB=RDR=WDW=ITER=0
      IF(IWO.EQ.1) GOTO 1000
      DO 10 IP=1,NPM1
10    W(IP)=0
1000  DO 20 IP=1,NPM1
      R(IP)=B(IP)
      DO 30 JP=1,NPM1
30    R(IP)=R(IP)+C(IP,JP)*W(JP)
      P(IP)=R(IP)
      BDB=BDB+CABS(B(IP))**2
      RDR=RDR+CABS(R(IP))**2
20    WDW=WDW+CABS(W(IP))**2
      BNORM=SQRT(BDB)
      RNORM=SQRT(RDR)
      WNORM=SQRT(WDW)
      ROB=RDR/BDB
      ROBDB=-100
      IF(ROB.GT.1.0E-10) ROBDB=10*ALOG10(ROB)
      WRITE(108,400)
      WRITE(108,500) (C(1,JP),JP=1,NPM1)
      DO 40 IP=2,NPM1
40    WRITE(108,600) (C(IP,JP),JP=1,NPM1)
      CALL FORM('  B                        :',B,RARR,IARR,NPM1,1,2)
      RARR(1)=BNORM
      CALL FORM('  BNORM                    :',CARR,RARR,IARR,1,0,1)
      CALL FORM('  R                        :',R,RARR,IARR,NPM1,0,2)
      RARR(1)=RNORM
      CALL FORM('  RNORM                    :',CARR,RARR,IARR,1,0,1)
      RARR(1)=ROBDB
      CALL FORM('  P                        :',P,RARR,IARR,NPM1,0,2)
      CALL FORM('  W                        :',W,RARR,IARR,NPM1,0,2)
```

```
      RARR(1)=WNORM
      CALL FORM('  WNORM                          :',CARR,RARR,IARR,1,0,1)
      CALL PWRSUP(B,R,W,CMAX,BMAX,P0,P0C,NPM1)
      RARR(1)=P0C
      CALL FORM('  P0C   (DB)                     :',CARR,P0C,IARR,1,0,1)
      WRITE(108,700)
      CALL WITER(W,WIT,CMAX,BMAX,ITERX1,NPM1,IW0,ITER,0)
C ---------------------------------------------------------------------
C     MAIN BCR PROCESS
C ---------------------------------------------------------------------
2000  ITER=ITER+1
      PCP=0
      DO 50 IP=1,NPM1
      CP(IP)=0
      DO 60 JP=1,NPM1
60    CP(IP)=CP(IP)+C(IP,JP)*P(JP)
50    PCP=PCP+REAL(CONJG(P(IP))*CP(IP))
      ALPHA=RDR/PCP
      RDR0=RDR
      RDR=WDW=0
      DO 70 IP=1,NPM1
      W(IP)=W(IP)-ALPHA*P(IP)
      R(IP)=R(IP)-ALPHA*CP(IP)
      RDR=RDR+CABS(R(IP))**2
70    WDW=WDW+CABS(W(IP))**2
      RNORM=SQRT(RDR)
      WNORM=SQRT(WDW)
      ROB=RDR/BDB
      ROBDB=-100
      IF(ROB.GT.1.0E-10) ROBDB=10*ALOG10(ROB)
      BETA=RDR/RDR0
      DO 80 IP=1,NPM1
80    P(IP)=R(IP)+BETA*P(IP)
      IARR(1)=ITER
      CALL FORM('  ITER                           :',CARR,RARR,IARR,1,1,0)
      CALL FORM('  CP                             :',CP,RARR,IARR,NPM1,0,2)
      RARR(1)=PCP
      CALL FORM('  PCP                            :',CARR,RARR,IARR,1,0,1)
      RARR(1)=ALPHA
      CALL FORM('  ALPHA                          :',CARR,RARR,IARR,1,0,1)
      CALL FORM('  W                              :',W,RARR,IARR,NPM1,0,2)
      RARR(1)=WNORM
      CALL FORM('  WNORM                          :',CARR,RARR,IARR,1,0,1)
```

```
      CALL FORM('  R                              :',R,RARR,IARR,NPM1,0,2)
      RARR(1)=RNORM
      CALL FORM('  RNORM                          :',CARR,RARR,IARR,1,0,1)
      RARR(1)=ROBDB
      CALL FORM('  ROBDB                          :',CARR,RARR,IARR,1,0,1)
      RARR(1)=BETA
      CALL FORM('  BETA                           :',CARR,RARR,IARR,1,0,1)
      CALL FORM('  P                              :',P,RARR,IARR,NPM1,0,2)
      CALL PWRSUP(B,R,W,CMAX,BMAX,P0,POC,NPM1)
      RARR(1)=POC
      CALL FORM('  POC   (DB)                     :',CARR,POC,IARR,1,0,1)
      CALL WITER(W,WIT,CMAX,BMAX,ITERX1,NPM1,IW0,ITER,0)
      IF(ROB.LT.1.0E-6) GOTO 3000
      IF(ITER.LT.ITERX) GOTO 2000
 3000 WRITE(108,200)
      CALL WITER(W,WIT,CMAX,BMAX,ITERX1,NPM1,IW0,ITER,1)
C ------------------------------------------------------------------
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE FORM(LABEL,CX,RX,IX,NX,ISKIP,IND)
C ======================================================================
C                          SPECIALLY-FORMATTED OUTPUT
C                                     FOR
C                       COMPLEX, REAL OR INTEGER ARRAYS
C
C             PROGRAM      :  FORM:S
C             ORIGINAL     :  APRIL      15, 1978
C             REVISION     :  SEPTEMBER 13, 1980
C
C             PREPARED  BY :  S. M. DANIEL  &  I. KERTESZ
C                             RADAR SYSTEMS ANALYSIS GROUP
C                             MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                             TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     INPUT  :     LABEL      -  IDENTIFYING LABEL
C                  CX         -  COMPLEX-VALUED ARRAY
C                  RX         -  REAL-VALUED ARRAY
C                  IX         -  INTEGER-VALUED ARRAY
C                  NX         -  ARRAY DIMENSIONALITY
C                  ISKIP      -  SINGLE-LINE SKIP OPTION
C                                 0  : NO SKIP
C                                 1  : SKIP
C                  IND        -  ARRAY-TYPE INDICATOR
C                                 0  : INTEGER
C                                 1  : REAL
C                                 2  : COMPLEX
C ======================================================================
      COMPLEX CX
      DIMENSION LABEL(7),CX(1),RX(1),IX(1)
  100 FORMAT(/)
  200 FORMAT(7A4,8I10)
  300 FORMAT(28X,8I10)
  400 FORMAT(7A4,8F10.5)
  500 FORMAT(28X,8F10.5)
      IND1=IND+3
      NX1=1
      NX2=4
      IF(NX2.GT.NX) NX2=NX
      IF(ISKIP.EQ.1) WRITE(108,100)
      GOTO (1,2),IND1
      WRITE(108,200) LABEL,(IX(I),I=NX1,NX2)
```

```
      GOTO 1000
 1    WRITE(108,400) LABEL,(RX(I),I=NX1,NX2)
      GOTO 1000
 2    WRITE(108,400) LABEL,(CX(I),I=NX1,NX2)
1000  NX1=NX1+4
      IF(NX1.GT.NX) RETURN
      NX2=NX2+4
      IF(NX2.GT.NX) NX2=NX
      GOTO (1,2,3,4,5),IND1
 3    WRITE(108,300) (IX(I),I=1,NX1,NX2)
      GOTO 1000
 4    WRITE(108,500) (RX(I),I=NX1,NX2)
      GOTO 1000
 5    WRITE(108,500) (CX(I),I=NX1,NX2)
      GOTO 1000
2000  RETURN
      END
```

```
C =====================================================================
      SUBROUTINE PWRSUP(B,R,W,CMAX,BMAX,P0,P0C,NPM1)
C =====================================================================
C            COMPUTATION OF RELATIVE COMBINED POWER SUPPRESSION
C                         AT SPECIFIED BCR ITERATION
C
C                PROGRAM        :  PWRSUP:S
C                ORIGINAL       :  SEPTEMBER 11, 1980
C                REVISION       :  SEPTEMBER 12, 1980
C
C                PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                  RADAR SYSTEMS ANALYSIS GROUP
C                                  MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                  TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C     INPUT  :      B          -   FORCING VECTOR
C                   R          -   RESIDUAL VECTOR
C                   W          -   ADAPTIVE WEIGHT VECTOR
C                   P0         -   MAIN-PORT SIGNAL POWER
C                   CMAX       -   NORMALIZATION CONSTANT FOR  C
C                   BMAX       -   NORMALIZATION CONSTANT FOR  B
C                   NPM1       -   NUMBER OF AUXILIARY PORTS
C
C
C    OUTPUT :      P0C        -   MAIN/COMBINED SIGNAL POWER  IN  DB
C =====================================================================
      COMPLEX B,R,W
      DIMENSION B(1),R(1),W(1)
      WBRMAX=BMAX**2/CMAX
      PC=P0
      DO 10 IP=1,NPM1
10    PC=PC+WBRMAX*REAL(CONJG(W(IP))*(B(IP)+R(IP)))
      PC=ABS(PC)
      P0C=PC/P0
      IF(P0C.LT.1.0E-10) P0C=1.0E-10
      P0C=-10*ALOG10(P0C)
      RETURN
      END
```

```
C =======================================================================
      SUBROUTINE WITER(W,WIT,CMAX,BMAX,ITERX1,NPM1,IW0,ITER,IND)
C =======================================================================
C                         FORMATION OF COMPOSITE WEIGHT ARRAY
C                                    CONTAINING
C                       ADAPTIVE WEIGHTS AT EACH BCR ITERATION
C
C                 PROGRAM        :  WITERIS
C                 ORIGINAL       :  SEPTEMBER 13, 1980
C                 REVISION       :  FEBRUARY  26, 1981
C
C                 PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                   RADAR SYSTEMS ANALYSIS GROUP
C                                   MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                   TEMPE, ARIZONA  85282
C -----------------------------------------------------------------------
C     INPUT  :     W          -  ADAPTIVE WEIGHT VECTOR
C                  ITERX1     -  MAXIMUM NUMBER OF ITERATIONS PLUS ONE
C                  CMAX       -  NORMALIZATION CONSTANT FOR C
C                  BMAX       -  NORMALIZATION CONSTANT FOR B
C                  NPM1       -  NUMBER OF AUXILIARY PORTS
C                  IW0        -  WEIGHT VECTOR INITIALIZATION INDICATOR
C                                 0  : SETS INITIAL W TO ZERO
C                                 1  : USES PREVIOUS W VALUE
C                  ITER       -  ITERATION NUMBER
C                  IND        -  BCR STATUS INDICATOR
C                                 0  : BCR PROCESS IN PROGRESS
C                                 1  : BCR PROCESS COMPLETE
C
C     OUTPUT :     WIT        -  TWO DIMENSIONAL COMPOSITE COMPLEX
C                                WEIGHT VECTOR
C =======================================================================
      COMPLEX W,WIT
      DIMENSION W(1),WIT(ITERX1,1),RARR(1)
  100 FORMAT(1H1)
  200 FORMAT(1X,79('-'))
  300 FORMAT(26X,'COMPOSITE WEIGHT-VECTOR ARRAY'/26X,29('-'),/
     -,16X,'ADAPTIVE WEIGHT VECTOR AT INDICATED BCR ITERATION',/
     -,19X,'NORMALIZATION CONSTANT     :  ',E13.6)
  400 FORMAT(8F10.5)
C -----------------------------------------------------------------------
C     COMPUTE GLOBAL SCALE FACTOR FOR ALL WEIGHT-VECTOR ITERATES
C -----------------------------------------------------------------------
```

```
      IF(IND.EQ.1) GOTO 1000
      IT1=ITER+1
      WITMAX=WMAX=0
      IF(IW0.EQ.0.AND.ITER.EQ.0) GOTO 2000
      CALL RCNORM(W,RARR,WMAX,NPM1,2,0)
2000  DO 10 IP=1,NPM1
10    WIT(IP,IT1)=W(IP)
      IF(WMAX.GT.WITMAX) WITMAX=WMAX
      IF(IND.EQ.0) RETURN
1000  WSCL=WITMAX*RMAX/CMAX
      DO 20 IT=1,IT1
      DO 20 IP=1,NPM1
20    WIT(IP,IT)=WIT(IP,IT)/WITMAX
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,300) WSCL
      WRITE(108,200)
      DO 30 IT=1,IT1
30    WRITE(108,400) (WIT(IP,IT),IP=1,NPM1)
      WRITE(108,200)
      RETURN
      END
```

```
C ============================================================================
      SUBROUTINE RCNORM(CX,RX,XMAX,NX,IND,IOPTN)
C ============================================================================
C                       NORMALIZATION OF REAL OR COMPLEX ARRAY
C
C               PROGRAM       :  RCNORM:S
C               ORIGINAL      :  AUGUST  31, 1980
C               REVISION      :  JANUARY 26, 1981
C
C               PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------------
C     INPUT  :      CX         -  COMPLEX ARRAY TO BE NORMALIZED
C                   RX         -  REAL ARRAY TO BE NORMALIZED
C                   NX         -  ARRAY DIMENSIONALITY
C                   IND        -  ARRAY-TYPE INDICATOR
C                                  1  : REAL
C                                  2  : COMPLEX
C                   IOPTN      -  NORMALIZING OPTION
C                                  0  : BYPASS  NORMALIZATION
C                                  1  : PERFORM NORMALIZATION
C
C     OUTPUT :      CX         -  NORMALIZED COMPLEX ARRAY
C                   RX         -  NORMALIZED REAL ARRAY
C                   XMAX       -  NORMALIZATION CONSTANT
C ============================================================================
      COMPLEX CX
      DIMENSION CX(1),RX(1)
999   FORMAT(' *** WARNING  :  YOU ARE NORMALIZING A NULL ARRAY ***')
C ----------------------------------------------------------------------------
C     DETERMINE MAXIMUM REAL OR IMAGINARY COMPONENT MAGNITUDE
C     OF GIVEN ARRAY AND PERFORM NORMALIZATION IF  IOPTN=1
C ----------------------------------------------------------------------------
      XMAX=0
      IF(IND.EQ.2) GOTO 1000
      DO 10 I=1,NX
      ABSX=ABS(RX(I))
10    IF(ABSX.GT.XMAX) XMAX=ABSX
      IF(XMAX.EQ.0) GOTO 9999
      IF(IOPTN.EQ.0) RETURN
      DO 20 I=1,NX
```

```
20    RX(I)=RX(I)/XMAX
      RETURN
1000  DO 30 I=1,NX
      ABSX=ABS(REAL(CX(I)))
      IF(ABSX.GT.XMAX) XMAX=ABSX
      ABSX=ABS(AIMAG(CX(I)))
30    IF(ABSX.GT.XMAX) XMAX=ABSX
      IF(XMAX.EQ.0) GOTO 9999
      IF(IOPTN.EQ.0) RETURN
      DO 40 I=1,NX
40    CX(I)=CX(I)/XMAX
C     -----------------------------------------------------------------
      RETURN
9999  WRITE(108,999)
      STOP
      END
```

```
C ==========================================================================
      SUBROUTINE CMBSGNL(S0,S1,W,SC,          NSX,NS,NPM1)
C ==========================================================================
C                   COMPUTATION OF COMBINED PORT SIGNAL
C                                   AND
C                     ASSOCIATED POWER SUPPRESSION
C
C              PROGRAM        :  CMBSGNL:S
C              ORIGINAL       :  SEPTEMBER  6, 1980
C              REVISION       :  SEPTEMBER 13, 1980
C
C              PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C --------------------------------------------------------------------------
C     INPUT  :     S0          -  MAIN-PORT SIGNAL
C                  S1          -  AUXILIARY SIGNAL VECTOR
C                  W           -  ADAPTIVE WEIGHT VECTOR, CMAX, BMAX
C                  NSX         -  MAXIMUM NUMBER OF SIGNAL SAMPLES
C                  NS          -  ACTUAL NUMBER OF SIGNAL SAMPLES
C                  NPM1        -  NUMBER OF AUXILIARY PORTS
C
C     OUTPUT :     SC          -  COMBINED PORT SIGNAL
C ==========================================================================
      COMPLEX S0,S1,W,SC,CARR
      DIMENSION S0(1),S1(NSX,1),W(1),SC(1),CARR(1),RARR(1),IARR(1)
  100 FORMAT(1H1)
  200 FORMAT(1X,79('-'))
  300 FORMAT(33X,'COMBINED PORT',/,33X,13('-'))
  400 FORMAT(25X,'RECEIVED BASEBAND PORT SIGNAL',/,19X
     -,'NORMALIZATION CONSTANT      :  ',E13.6)
  500 FORMAT(8F10.5)
C --------------------------------------------------------------------------
C     COMPUTE COMBINED PORT SIGNAL AND ASSOCIATED POWER SUPPRESSION
C --------------------------------------------------------------------------
      WMAX=BMAX/CMAX
      P0=PC=0
      DO 10 IS=1,NS
      SC(IS)=S0(IS)
      DO 20 IP=1,NPM1
   20 SC(IS)=SC(IS)+S1(IS,IP)*WMAX*W(IP)
      P0=P0+CABS(S0(IS))**2
```

```
10    PC=PC+CABS(SC(IS))**2
      PCAN=100
      IF(PC.NE.0) PCAN=10*ALOG10(P0/PC)
      RARR(1)=PCAN
      CALL RCNORM(SC,RARR,SMAX,NS,2,1)
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,300)
      WRITE(108,200)
      WRITE(108,400) SMAX
      WRITE(108,200)
      WRITE(108,500) (SC(IS),IS=1,NS)
      WRITE(108,200)
      RARR(1)=PCAN
      CALL FORM('  SUPPRESSION  (DB)         :',CARR,RARR,IARR,1,0,1)
      WRITE(108,200)
C -----------------------------------------------------------------
      RETURN
      END
```

Table 2-15. Functional Description of Modules Comprising BCRS

```
--------------------------------------------------------------------------------
                FUNCTIONAL DESCRIPTION OF BCRS SOURCE MODULES
                ---------------------------------------------

--------------------------------------------------------------------------------
BCRMAIN:S  -  MAIN EXECUTIVE PROGRAM.  IT CALLS OTHER EXECUTIVE SUBPROGRAMS
              WHICH TOGETHER PERFORM THE DIGITAL ARRAY PROCESSING USING BATCH
              COVARIANCE RELAXATION.

BCRSET:S   -  EXECUTIVE SUBPROGRAM.  IT IS DESIGNED TO READ FROM THE INPUT DA-
              TA FILE, BCRS:D, USING THE GENERAL INPUT AND OUTPUT SUBROUTINES
              RW, RWIRC, AND SKIPR IN ACCORDANCE TO PRESET FORMAT.  ALL DATA IS
              MADE AVAILABLE TO APPROPRIATE SUBROUTINES THROUGH COMMON BLOCKS.

RW:S       -  GENERAL SUBPROGRAM.  (DEFINED IN SIGGEN.)

RWIRC:S    -  GENERAL SUBPROGRAM.  (DEFINED IN SIGGEN.)

SKIPR:S    -  GENERAL SUBPROGRAM.  (DEFINED IN SIGGEN.)

BCREXEC:S  -  EXECUTIVE SUBPROGRAM.  IT CALLS DEDICATED SUBPROGRAMS TO PERFORM
              THE BCR SIMULATION AND SIGNAL COMBINATION.  NO ACTUAL CALCULATI-
              ONS TAKE PLACE IN THIS SUBPROGRAM.

CANDB:S    -  DEDICATED SUBPROGRAM.  THIS SUBPROGRAM WILL COMPUTE THE BATCH CO-
              VARIANCE MATRIX OF THE AUXILIARY PORT SIGNALS (COVARIANCE), AND
              THE CROSS CORRELATION VECTOR OF THE MAIN PORT SIGNAL AND AUXILI-
              ARY PORTS SIGNAL VECTORS (FORCING VECTOR).

BCRSIM:S   -  DEDICATED SUBPROGRAM.  THIS SUBPROGRAM WILL APPLY THE BATCH COVA-
              RIANCE RELAXATION TECHNIQUE TO ESTIMATE THE ADAPTIVE WEIGHT VEC-
              TOR USING THE COVARIANCE MATRIX AND FORCING VECTOR.

FORM:S     -  GENERAL SUBPROGRAM.  IT IS DESIGNED TO PRINT A 28-CHARACTER COM-
              MENT FOLLOWED BY AN INTEGER, REAL, OR COMPLEX SCALAR, OR BY REAL
              OR COMPLEX ARRAY PLACED INTO AN 80-COLUMN FIELD.

PWRSUP:S   -  DEDICATED SUBPROGRAM.  IT IS DESIGNED TO COMPUTE THE RELATIVE
              COMBINED POWER SUPPRESSION AT SPECIFIED BCR ITERATIONS.

WITER:S    -  DEDICATED SUBPROGRAM.  IT IS DESIGNED TO FORM AN ARRAY BY CONCA-
```

```
                         TENATION OF THE ADAPTIVE WEIGHT VECTORS.  IT ALSO NORMALIZES THE
                         ARRAY WHEN DIRECTED TO DO SO.

RCNORM:S     -           GENERAL SUBPROGRAM.  (DEFINED IN SIGGEN.)

CMBSGNL:S    -           DEDICATED SUBPROGRAM.  THIS SUBPROGRAM WILL USE THE FINAL VALUE
                         OF THE ADAPTIVE WEIGHTS TO COMBINE THE MAIN AND AUXILIARY PORT
                         SIGNALS.  FROM THIS IT CALCULATES THE POWER SUPPRESSSION RATIO.

--------------------------------------------------------------------------------
```

Table 2-16. JCL Program, JCLBCRS:BL

```
!JOB 1269,DANIEL(8512),7,BLDG90
!LIMIT (TIME,1),(UO,2),(CO,16),(ACCOUNT)
!.....JCLBCRS:BL........................
!PCL
C     BCRMAIN:B.1269         OVER BCRS:B
C       BCRSET:B.1269
C         RWIRC:B.1269
C         RW:B.1269  SKIPR:B.1269
C       BCREXEC:B.1269
C         CANDB:B.1269
C         BCRSIM:B.1269
C           FORM:B.1269
C           PWRSUP:B.1269
C           WITER:B.1269
C             RCNORM:B.1269
C         CMBSGNL:B.1269
!LYNX BCRS:B                 OVER BCRS:L
```

2.3.2.3 Binary Modules

Each source module in the BCRS program has its corresponding binary version. As shown before, the general JCL program, JCL:B, (see Table 2-2) can be used to create any of these binary modules.

2.3.2.4 Composite Binary and Load Module

The creation of the executable load module for the BCRS program can be accomplished through the execution of the JCL program, JCLBCRS:BL, which is shown in Table 2-16. This program works in the same manner as the one described in paragraph 2.2.2.4. Here, all the member-programs of RADAR:LIB pertaining to BCRS are concatenated to form the composite binary module, BCRS:B. This new module then is linked with necessary system functions to form the executable program, BCRS:L.

2.3.3 Program Execution

The program BCRS:L can be executed upon assigning the proper input and output files. The input file for this case is BCRS:D, the one discussed in paragraph 2.3.1. If the output is planned to be used as input to a subsequent program, as in this case, it should be a file other than the normal line printer output.

One of the methods of file assignments and program execution is shown as the JCL:X in Table 2-9. This general program can be executed by entering the interactive command

!BATCH JCL:X 'NAME' = BCRS

When completed, there will be an output data file, BCRS:0, which will contain the results of the BCRS program run.

2.3.4 Output File - BCRS:0

The output data file, BCRS:0, is shown in Table 2-17. It was produced by executing BCRS:L with input data BCRS:D. Up to the port 0 signal list, BCRS:0 is identical to BCRS:D[†]. This is done to insure that the output is interpreted in the proper context and is also a way of passing BCR and system description data to the subsequent program. The result of the BCRS process output begins with the covariance matrix and the forcing vector if IWCB = 1. These are followed by the BCR simulation output. That begins with the initialization stage and continues with a listing of pertinent quantities of each subsequent iteration, two in the present case. A composite weight-vector listing that follows includes a normalized set of all weight-vector iterates.

[†] Note that the sequence numbers preceding each entry is not a part of the output data but is a useful artifice for editing. However, as a consequence of this keying, BCR performance data has exceeded the page size and hence not shown. For this benchtest example, however, it will be crucial to have the sequence numbers, especially in explaining the formation of the BCRP:D input data file that follows.

Table 2-17. Output Data File of the BCR Simulation Program, BCRS:0 Benchtest Example (See Figure 2-2)

```
1
 ------------------------------------------------------------------------------
                         DIGITAL ADAPTIVE ARRAY PROCESSING
                         ---------------------------------
                                       USING
                                       -----
                            BATCH COVARIANCE RELAXATION
                            ---------------------------

 ------------------------------------------------------------------------------
  PRINT OPTIONS
  -------------
     IWS0    -  OPTIONAL OUTPUT OF S0                  :          1
     IWS1    -  OPTIONAL OUTPUT OF  S1                 :          0
     IWCB    -  OPTIONAL OUTPUT OF  C  AND  B          :          1

  BCR PARAMETERS
  --------------
     NSX     -  MAXIMUM NUMBER OF SIGNAL SAMPLES       :        256
     IW0     -  SIZE OF BAND DIAGONAL                  :          0
     NWX     -  MAXIMUM NUMBER OF WEIGHTS              :         20
     ISW     -  WEIGHT SELECTOR ARRAY                  :    1 1 1 1 0 0 0 0 0 0
                                                       :    0 0 0 0 0 0 0 0 0 0
     ITERX   -  MAXIMUM NUMBER OF ITERATIONS           :         20
 ------------------------------------------------------------------------------
     ITER    -  ACTUAL NUMBER OF ITERATIONS            :          X
 ------------------------------------------------------------------------------
1
 ------------------------------------------------------------------------------
                       SPECIFICATION OF SYSTEM PARAMETERS
                       ----------------------------------


 ------------------------------------------------------------------------------
  PRINT OPTIONS
  -------------
     IWANT   -  MAIN ANTENNA ARRAY WEIGHTING           :          0
     IWRAP   -  RECEIVER AMPLITUDE AND PHASE           :          0
     IWRI    -  RECEIVER IMPULSE RESPONSE              :          0
     IWCAP   -  CHANNEL AMPLITUDE AND PHASE            :          0
     IWCI    -  CHANNEL IMPULSE RESPONSE               :          0
     IWSC    -  INDIVIDUAL CHANNEL SIGNALS             :          0
```

```
FILTER PARAMETERS
-----------------
  NPOL     -  NUMBER OF LOWPASS PROTOTYPE POLES        :         2
              POL(1)                                   :   -.70700   -.70700
              POL(2)                                   :   -.70700    .70700
  FBIF     -  FRACTIONAL BANDWIDTH AT FINAL IF         :    .10000
  FBRF     -  FRACTIONAL BANDWIDTH AT RF               :    .00100
  RADRNG   -  NORMALIZED RADIAN FREQUENCY RANGE        :   4.00000
  RADIN    -  NORMALIZED INITIAL RADIAN FREQUENCY      :  -2.00000
  NF       -  NUMBER OF FREQUENCY SAMPLES              :        32
  LPBP     -  LOWPASS/BANDPASS OPTION                  :         1
              0  :   LOWPASS
              1  :   BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS   -  NUMBER OF CHANNELS PER PORT              :        20
  ISC      -  CHANNEL SELECTOR ARRAY                   :   1 1 0 0 0 0 0 0 0 0
                                                       :   0 0 0 0 0 0 0 0 0 0

CHANNEL  1:
  AN       -  AMPLITUDE OF NOISE SOURCE                :   1.00000
  IX0      -  INITIAL RANDU SETTING                    :         1
  N1       -  FIRST-TIME-ON SAMPLE NUMBER              :         1
  NB       -  BLINK DURATION IN SAMPLES                :     10000
  TH       -  AZIMUTH ANGLES OF INCIDENCE (DEG)        :  45.00000
  CDEL     -  CHANNEL DELAY IN SAMPLE-TIME UNITS       :    .00000

CHANNEL  2:
  AN       -  AMPLITUDE OF NOISE SOURCE                :   1.00000
  IX0      -  INITIAL RANDU SETTING                    :        11
  N1       -  FIRST-TIME-ON SAMPLE NUMBER              :         1
  NB       -  BLINK DURATION IN SAMPLES                :     10000
  TH       -  AZIMUTH ANGLES OF INCIDENCE (DEG)        :  10.00000
  CDEL     -  CHANNEL DELAY IN SAMPLE-TIME UNITS       :    .10000

PORT PARAMETERS
---------------
  D0       -  ANTENNA-ELEMENT SEPARATION FACTOR        :   1.00000
  NS       -  NUMBER OF SIGNAL SAMPLES                 :       128
  NPORTS   -  NUMBER OF PORTS                          :        21
  ISP      -  PORT SELECTOR ARRAY                      :   1 1 1 1 0 0 0 0 0 0
```

```
                                                              :     0 0 0 0 0 0 0 0 0 0

PORT      0:
  NEL      -  NUMBER OF ANTENNA ELEMENTS                      :          255
  LOC1     -  LOCATION OF THE FIRST ELEMENT                   :            1
  BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                      :      1.00000
  BWOFF    -  BANDWIDTH OFFSET FACTOR                         :       .00000
  PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY              :       .00000

PORT      1:
  NEL      -  NUMBER OF ANTENNA ELEMENTS                      :            1
  LOC1     -  LOCATION OF THE FIRST ELEMENT                   :            1
  BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                      :      1.00000
  BWOFF    -  BANDWIDTH OFFSET FACTOR                         :       .00000
  PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY              :       .00000

PORT      2:
  NEL      -  NUMBER OF ANTENNA ELEMENTS                      :            1
  LOC1     -  LOCATION OF THE FIRST ELEMENT                   :            1
  BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                      :      1.00000
  BWOFF    -  BANDWIDTH OFFSET FACTOR                         :       .00000
  PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY              :       .10000

PORT      3:
  NEL      -  NUMBER OF ANTENNA ELEMENTS                      :            1
  LOC1     -  LOCATION OF THE FIRST ELEMENT                   :          255
  BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                      :      1.00000
  BWOFF    -  BANDWIDTH OFFSET FACTOR                         :       .00000
  PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY              :       .00000

PORT      4:
  NEL      -  NUMBER OF ANTENNA ELEMENTS                      :            1
  LOC1     -  LOCATION OF THE FIRST ELEMENT                   :          255
  BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                      :      1.00000
  BWOFF    -  BANDWIDTH OFFSET FACTOR                         :       .00000
  PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY              :       .10000
--------------------------------------------------------------------------------
1
--------------------------------------------------------------------------------
                                   PORT  0
                                   -------
--------------------------------------------------------------------------------
                         RECEIVED BASEBAND PORT SIGNAL
```

NORMALIZATION CONSTANT : .359902E+00

.05464	.03818	-.27935	-.18506	-.80850	-.80400	-.96805	-1.00000
-.89117	-.93809	-.86581	-.91809	-.51389	-.71681	-.01191	-.37046
.51795	.07067	.69954	.48863	.33405	.37157	-.72323	-.56527
-.48070	-.23752	-.28359	.13504	-.05666	.10339	.17505	.00267
-.06350	-.11284	-.27573	-.25835	-.11408	.03863	.07952	.27616
-.00055	-.11476	.45160	.31795	-.25093	-.05661	-.53158	-.23006
.10718	-.05535	.29687	.03986	-.22163	-.13246	-.32173	-.22894
-.37581	.00988	-.03880	.09207	.23237	.18048	-.03938	.26499
-.60312	-.16596	-.22783	-.27798	.15059	.04556	.49784	.37388
.05924	.20007	-.67014	-.26716	-.51796	-.17555	.41518	.42498
.46477	.55190	-.24890	.00095	-.22332	-.22121	-.17883	-.20944
-.14506	-.08280	-.48174	-.33351	-.15840	-.23864	.03017	.12233
-.24552	.07159	.07142	.10982	.79052	.48894	.42138	.42662
-.56041	-.25491	-.64554	-.51348	.04592	.16574	.45397	.33689
-.15339	-.20463	-.60183	-.43150	-.23293	.15092	-.05293	.03788
.70177	.26965	.66569	.35406	.76548	.50369	.33144	.11207
.34198	.16070	-.22887	.04206	-.10226	.16876	.08978	.24863
.07344	-.10328	.46115	.36154	.51861	.34287	.17532	.33440
-.01579	.20464	.13438	.29128	-.17331	-.08766	.00878	.12494
-.21701	-.41771	.06459	-.23581	-.57297	-.59389	-.25658	-.38524
-.71101	-.39013	-.17867	-.31976	.16877	.45003	-.15970	.15444
.40744	.45485	.68224	.67182	.68668	.85575	-.08121	.25855
.30583	.30440	.19326	.26502	-.49589	-.22002	-.67978	-.41590
-.30489	-.53854	.35300	.05499	-.29986	.01299	-.08097	-.02180
.53392	.59253	-.08945	.13345	-.56345	-.37763	-.77421	-.36072
-.67038	-.15848	-.09812	.07619	-.07994	.11141	-.19309	-.00092
-.09846	-.05803	-.32363	-.11191	-.70204	-.51644	-.42421	-.15956
.35976	.29671	.55861	.31601	.34769	-.05135	.29570	.17155
-.02046	.16898	-.40331	-.40163	-.33261	-.31393	.02263	.05234
-.29163	-.15834	.20011	.26848	.21790	.33672	.14524	.09034
.12943	.14092	-.11299	.08524	.11257	.27772	.18866	.44657
.08273	.17843	.09462	.15594	-.23630	-.08516	.11599	-.09951

1

COVARIANCE MATRIX C

NORMALIZATION CONSTANT : .152546E+02

.80547	.00000	-.00073	.80509	.58753	-.33441	.33558	.58896
-.00073	-.80509	.80534	.00000	-.33321	-.58546	.58734	-.33458

```
     .58753     .33441    -.33321     .58546    1.00000     .00000     .00052     .99954
     .33558    -.58896     .58734     .33458     .00052    -.99954     .99980     .00000

                                FORCING VECTOR  B
                      NORMALIZATION CONSTANT     :     .720710E+01

     .85241    -.12855    -.12721    -.85123     .99894     .01176     .01417   -1.00000
 ---------------------------------------------------------------------------------
1
 -----------------------------------------------------------------------------------------------------------

                                        PERFORMANCE SUMMARY OF BCR SIMULATION
                                        -------------------------------------
 -----------------------------------------------------------------------------------------------------------
  INITIAL CONDITIONS
  ------------------

  C
                          :    .80547     .00000    -.00073     .80509     .58753    -.33441     .33558

                          :   -.00073    -.80509     .80534     .00000    -.33321    -.58546     .58734

                          :    .58753     .33441    -.33321     .58546    1.00000     .00000     .00052

                          :    .33558    -.58896     .58734     .33458     .00052    -.99954     .99980


  B
  BNORM                   :    .85241    -.12855    -.12721    -.85123     .99894     .01176     .01417
  R                       :   1.86604
  RNORM                   :    .85241    -.12855    -.12721    -.85123     .99894     .01176     .01417
  P                       :   1.86604
  W                       :    .85241    -.12855    -.12721    -.85123     .99894     .01176     .01417
  WNORM                   :    .00000     .00000     .00000     .00000     .00000     .00000     .00000
  POC   (DR)              :    .00000
                          :    .00000

  MAIN PROCESS
  ------------


  ITER
  CP                      :         1
  PCP                     :   2.55656    -.85971    -.85879   -2.55255    3.08305     .44410     .44687
                          :  10.74911
```

```
ALPHA                  :     .32395
W                      :   -.27614    .04164    .04121    .27575   -.32360   -.00381   -.00459
WNORM                  :    .60450
R                      :    .02423    .14995    .15099   -.02434    .00020   -.13210   -.13059
RNORM                  :    .28455
ROBDB                  : -16.33525
BETA                   :    .02325
P                      :    .04405    .14696    .14803   -.04414    .02343   -.13183   -.13026
POC   (DB)             :   8.54392


ITER                   :         2
CP                     :    .01075    .06767    .06812   -.01116    .00017   -.05959   -.05894
PCP                    :    .03654
ALPHA                  :   2.21618
W                      :   -.37375   -.28405   -.28686    .37356   -.37553    .28835    .28409
WNORM                  :    .94265
R                      :    .00040   -.00002    .00002    .00040   -.00017   -.00003    .00003
RNORM                  :    .00061
RORDB                  : -69.65309
BETA                   :    .00000
P                      :    .00040   -.00002    .00002    .00040   -.00017   -.00004    .00003
POC   (DB)             :  25.23053
-----------------------------------------------------------------------------------------------
1
------------------------------------------------------------------------------
                          COMPOSITE WEIGHT-VECTOR ARRAY
                          -----------------------------
                 ADAPTIVE WEIGHT VECTOR AT INDICATED BCR ITERATION
                    NORMALIZATION CONSTANT     :  .177725E+00
------------------------------------------------------------------------------
    .00000    .00000    .00000    .00000    .00000    .00000    .00000    .00000
   -.73406    .11070    .10955    .73304   -.86024   -.01013   -.01220    .86116
   -.99357   -.75511   -.76258    .99306   -.99828    .76653    .75521   1.00000
------------------------------------------------------------------------------
1
------------------------------------------------------------------------------
                                 COMBINED PORT
                                 -------------
------------------------------------------------------------------------------
                          RECEIVED BASEBAND PORT SIGNAL
                    NORMALIZATION CONSTANT     :    .230300E-01
------------------------------------------------------------------------------
```

-.03882	-.01324	.19068	.17606	.06095	.16566	.05320	-.07284
.02983	-.00813	.07463	-.12494	.26855	-.17769	.18889	-.25052
-.11826	-.07935	-.48928	-.04969	.14694	.37927	-.66591	.31812
-.14739	-.27506	-.10647	.01626	.88967	-.26926	.09201	.11673
-.20856	.03510	-.23095	.15381	-.39792	-.22452	.61232	.05542
.04492	-.09820	.08912	-.02986	-.77634	.44226	.26923	-.25755
1.00000	-.23182	-.54976	.16129	-.19127	.13861	-.41197	.07181
-.23734	-.08672	.76398	-.18493	-.26977	.04421	-.59678	.20887
.39799	.03208	.52495	-.19592	-.15485	-.12996	.12723	-.07019
-.60791	.39562	-.33631	.15349	.12069	-.26230	.69098	-.37495
-.45620	.32997	.13521	.06601	.31844	.00246	-.15046	-.04648
-.08588	.07297	-.00853	.05014	.19793	-.18658	-.56760	.10908
-.02747	.00685	.61953	-.29260	.31569	-.12731	-.74138	.40361
-.02039	.26390	-.16356	-.12187	.31915	-.39370	.64571	.03263
-.60272	.43443	-.57501	-.07628	-.00653	-.11521	.98491	-.26487
.31008	-.11340	-.33386	.00741	.14769	.03221	-.08355	.18245
-.38720	.05156	-.66413	.28236	.17833	-.30709	.66246	.11549
-.09248	-.10721	.10212	-.11135	-.03255	.06142	-.73040	.26786
.21263	-.16096	.18605	.13513	-.12474	-.02220	.33997	-.04687
.63003	-.01096	-.36406	.03134	.00490	.15946	-.43003	-.09487
-.06626	.18153	.18173	-.40262	-.72759	.20561	.37521	-.14094
.15502	-.11839	.02080	-.12896	-.38385	.26245	.00175	.11956
.55600	-.24311	-.29779	.32143	-.31922	.13678	.45629	-.03848
.75443	-.31475	-.70349	.06777	-.52023	.21697	.37744	-.30429
-.07888	-.01842	.01804	.33538	-.27066	.12158	-.52435	.08089
.19229	-.25167	.60283	-.14237	-.44529	.09207	.46714	-.07031
-.20124	.04263	.10831	.11555	-.40199	.15664	.09070	-.36940
.79021	-.25598	.24875	.04699	.00929	.11623	-.92485	.05834
.42465	.09246	-.11334	.20699	-.07138	-.23966	.07399	.06591
-.37167	.04002	.17212	-.25058	.10549	.13588	.18000	-.05109
-.35736	.14107	-.22773	.00883	-.04493	-.09309	.05999	.00589
.34461	.01571	-.25240	.07354	.32533	.03121	.12023	-.19239

SUPPRESSION (DB) : 25.23703

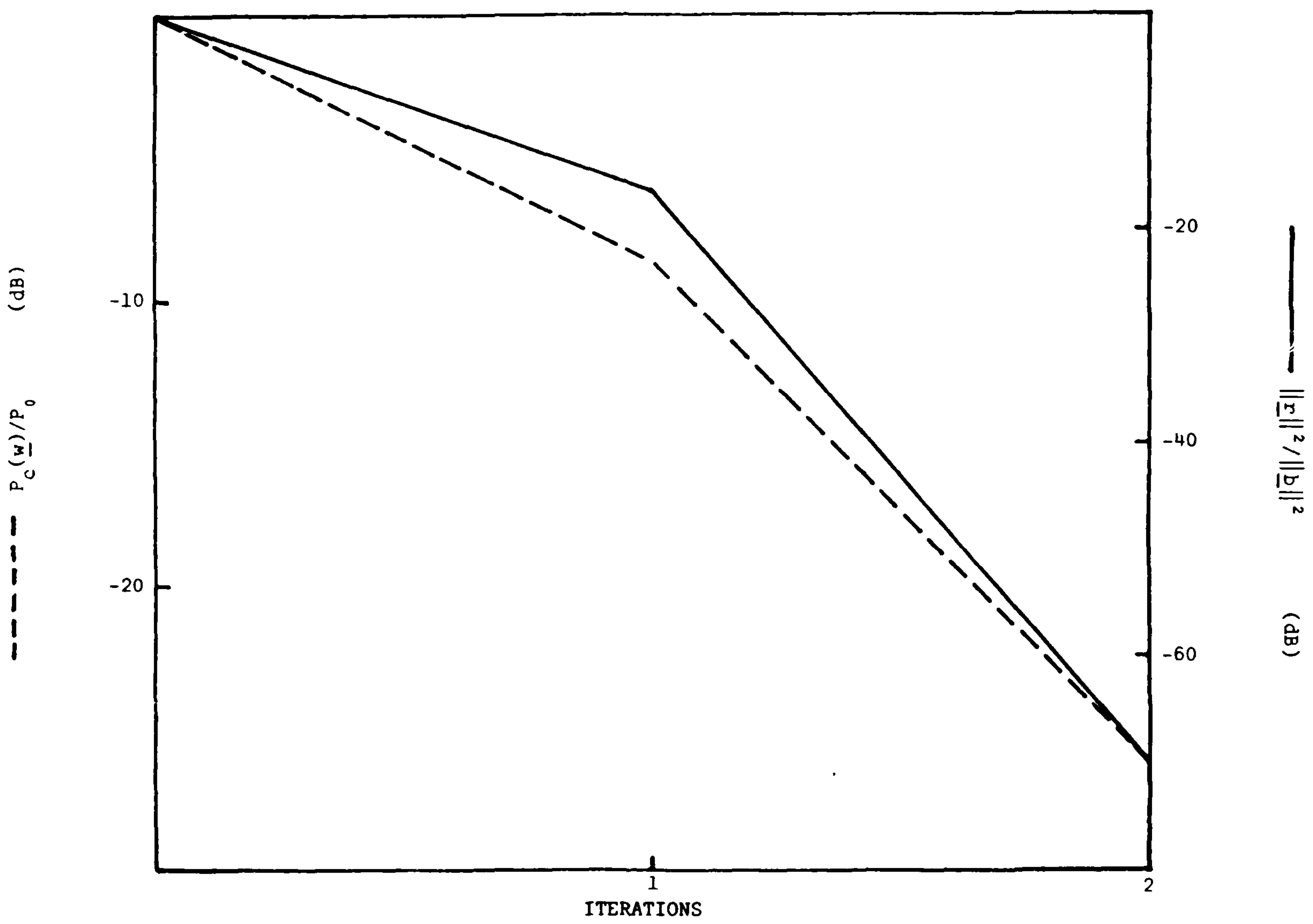


Figure 2-6. BCR Convergence Characteristics
Benchtest Example
(See Figure 2-1)

Valuable insight into the BCR process may be gained by careful examination of the printed output. For example, the relative combined power suppression, $10 \log_{10}(P_c(\underline{w})/P_0)$, and the relative gradient metric, $10 \log (\| \underline{r} \|^2/\| \underline{b} \|^2)$, pertain to the nulling performance and convergence, respectively. In the output listing of Table 2-17, these quantities are -POC and ROBDB given in dB. Figure 2-6 summarizes the behavior of these parameters as a function of iteration for the benchtest example under consideration.

Whereas POC is computed directly from BCR variables at each iteration[‡], it is also computed directly, at the end of the process, using the explicit combined signal listed next in BCRS:O. This value of POC which may differ from its equivalent indirect computation is listed last.

2.4 BCR Performance and Plotting Program - BCRP

The purpose of the BCRP program is to analyze the performance of the BCR process and produce graphical output that characterizes this performance. As such, it uses the data produced by the BCRS program.

2.4.1 Input Data File - BCRP:D

The input data file, BCRP:D, for the BCR performance and plotting program, BCRP, is shown in Table 2-18. It happens to be a subset of BCRS:O except for one character; namely, "X", the unspecified number of actual iterations in the header data file, BCRS:DO.

The BCRP program requires all BCRS:O information except any C and $\underline{b}$ output or the BCR performance summary. To create BCRP:D from BCRS:O, the first change on BCRS:O is the entry of the actual number of iterations (2 in our case) over and right-justified to the letter "X". Subsequently, all C and $\underline{b}$ and BCR performance data ranging from lines 165 ghrough 239 is deleted, which is obvious by noting the missing range of numbers in Table 2-18.

2.4.2 Program Structure

The BCRP program was designed with the same methods as the previous two major programs. It has a similar modular-linear stfucture with the data input restricted to one executive program branch. The "library" approach is used by referencing subprograms, many of which were already introduced in earlier programs. Details of the program structure are given in the following paragraphs.

2.4.2.1 Tree Diagram

The structure and modular order of execution of the BCRP program are clearly demonstrated by the tree diagram of Figure 2-7. The data input is alone in the BCRPSET branch, again using the utility subprograms RW, RWIRC, and SKIPR. The main program, BCRPMAIN, calls a series of dedicated subprograms. CMPCHNL is called to generate composite baseband channel amplitude responses. The resulting quantities are printed and held for plotting. Next, FIELD is

---

† See equation (3-86) in Project Memorandum 8512-03.

Table 2-18. Input Data File of the BCR Plotting Program, BCRP:D Benchtest Example (See Figure 2-2)

```
 1 -     1.000 1
 2 -     2.000  -----------------------------------------------------------------------------
 3 -     3.000                      DIGITAL ADAPTIVE ARRAY PROCESSING
 4 -     4.000                      ---------------------------------
 5 -     5.000                                    USING
 6 -     6.000                                    -----
 7 -     7.000                         BATCH COVARIANCE RELAXATION
 8 -     8.000                         ---------------------------
 9 -     9.000
10 -    10.000  -----------------------------------------------------------------------------
11 -    11.000   PRINT OPTIONS
12 -    12.000   -------------
13 -    13.000     IWS0     -  OPTIONAL OUTPUT OF S0                 :           1
14 -    14.000     IWS1     -  OPTIONAL OUTPUT OF  S1                :           0
15 -    15.000     IWCB     -  OPTIONAL OUTPUT OF  C  AND  B         :           1
16 -    16.000
17 -    17.000   BCR PARAMETERS
18 -    18.000   --------------
19 -    19.000     NSX      -  MAXIMUM NUMBER OF SIGNAL SAMPLES      :         256
20 -    20.000     IW0      -  SIZE OF BAND DIAGONAL                 :           0
21 -    21.000     NWX      -  MAXIMUM NUMBER OF WEIGHTS             :          20
22 -    22.000     ISW      -  WEIGHT SELECTOR ARRAY                 :    1 1 1 1 0 0 0 0 0 0
23 -    23.000                                                       :    0 0 0 0 0 0 0 0 0 0
24 -    24.000     ITERX    -  MAXIMUM NUMBER OF ITERATIONS          :          20
25 -    25.000  -----------------------------------------------------------------------------
26 -    26.000     ITER     -  ACTUAL NUMBER OF ITERATIONS           :           2
27 -    27.000  -----------------------------------------------------------------------------
28 -    28.000 1
29 -    29.000  -----------------------------------------------------------------------------
30 -    30.000                     SPECIFICATION OF SYSTEM PARAMETERS
31 -    31.000                     ----------------------------------
32 -    32.000
33 -    33.000  -----------------------------------------------------------------------------
34 -    34.000   PRINT OPTIONS
35 -    35.000   -------------
36 -    36.000     IWANT    -  MAIN ANTENNA ARRAY WEIGHTING          :           0
37 -    37.000     IWRAP    -  RECEIVER AMPLITUDE AND PHASE          :           0
38 -    38.000     IWRI     -  RECEIVER IMPULSE RESPONSE             :           0
39 -    39.000     IWCAP    -  CHANNEL AMPLITUDE AND PHASE           :           0
40 -    40.000     IWCI     -  CHANNEL IMPULSE RESPONSE              :           0
41 -    41.000     IWSC     -  INDIVIDUAL CHANNEL SIGNALS            :           0
```

```
FILTER PARAMETERS
-----------------
  NPOL     -  NUMBER OF LOWPASS PROTOTYPE POLES       :        2
              POL(1)                                  :   -.70700   -.70700
              POL(2)                                  :   -.70700    .70700
  FBIF     -  FRACTIONAL BANDWIDTH AT FINAL IF        :    .10000
  FBRF     -  FRACTIONAL BANDWIDTH AT RF              :    .00100
  RADRNG   -  NORMALIZED RADIAN FREQUENCY RANGE       :   4.00000
  RADIN    -  NORMALIZED INITIAL RADIAN FREQUENCY     :  -2.00000
  NF       -  NUMBER OF FREQUENCY SAMPLES             :        32
  LPBP     -  LOWPASS/BANDPASS OPTION                 :         1
              0  :   LOWPASS
              1  :   BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS   -  NUMBER OF CHANNELS PER PORT             :        20
  ISC      -  CHANNEL SELECTOR ARRAY                  :   1 1 0 0 0 0 0 0 0 0
                                                      :   0 0 0 0 0 0 0 0 0 0

CHANNEL  1:
  AN       -  AMPLITUDE OF NOISE SOURCE               :   1.00000
  IX0      -  INITIAL RANDU SETTING                   :         1
  N1       -  FIRST-TIME-ON SAMPLE NUMBER             :         1
  NB       -  BLINK DURATION IN SAMPLES               :     10000
  TH       -  AZIMUTH ANGLES OF INCIDENCE (DEG)       :  45.00000
  CDEL     -  CHANNEL DELAY IN SAMPLE-TIME UNITS      :    .00000

CHANNEL  2:
  AN       -  AMPLITUDE OF NOISE SOURCE               :   1.00000
  IX0      -  INITIAL RANDU SETTING                   :        11
  N1       -  FIRST-TIME-ON SAMPLE NUMBER             :         1
  NB       -  BLINK DURATION IN SAMPLES               :     10000
  TH       -  AZIMUTH ANGLES OF INCIDENCE (DEG)       :  10.00000
  CDEL     -  CHANNEL DELAY IN SAMPLE-TIME UNITS      :    .10000

PORT PARAMETERS
---------------
  D0       -  ANTENNA-ELEMENT SEPARATION FACTOR       :   1.00000
  NS       -  NUMBER OF SIGNAL SAMPLES                :       128
  NPORTS   -  NUMBER OF PORTS                         :        21
  ISP      -  PORT SELECTOR ARRAY                     :   1 1 1 1 0 0 0 0 0 0
```

```
 85 -   85.000                                                      :    0 0 0 0 0 0 0 0 0 0
 86 -   86.000
 87 -   87.000   PORT     0:
 88 -   88.000     NEL      -  NUMBER OF ANTENNA ELEMENTS           :        255
 89 -   89.000     LOC1     -  LOCATION OF THE FIRST ELEMENT        :          1
 90 -   90.000     BWFCTR   -  BANDWIDTH TOLERANCE FACTOR           :    1.00000
 91 -   91.000     BWOFF    -  BANDWIDTH OFFSET FACTOR              :     .00000
 92 -   92.000     PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY   :     .00000
 93 -   93.000
 94 -   94.000   PORT     1:
 95 -   95.000     NEL      -  NUMBER OF ANTENNA ELEMENTS           :          1
 96 -   96.000     LOC1     -  LOCATION OF THE FIRST ELEMENT        :          1
 97 -   97.000     BWFCTR   -  BANDWIDTH TOLERANCE FACTOR           :    1.00000
 98 -   98.000     BWOFF    -  BANDWIDTH OFFSET FACTOR              :     .00000
 99 -   99.000     PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY   :     .00000
100 -  100.000
101 -  101.000   PORT     2:
102 -  102.000     NEL      -  NUMBER OF ANTENNA ELEMENTS           :          1
103 -  103.000     LOC1     -  LOCATION OF THE FIRST ELEMENT        :          1
104 -  104.000     BWFCTR   -  BANDWIDTH TOLERANCE FACTOR           :    1.00000
105 -  105.000     BWOFF    -  BANDWIDTH OFFSET FACTOR              :     .00000
106 -  106.000     PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY   :     .10000
107 -  107.000
108 -  108.000   PORT     3:
109 -  109.000     NEL      -  NUMBER OF ANTENNA ELEMENTS           :          1
110 -  110.000     LOC1     -  LOCATION OF THE FIRST ELEMENT        :        255
111 -  111.000     BWFCTR   -  BANDWIDTH TOLERANCE FACTOR           :    1.00000
112 -  112.000     BWOFF    -  BANDWIDTH OFFSET FACTOR              :     .00000
113 -  113.000     PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY   :     .00000
114 -  114.000
115 -  115.000   PORT     4:
116 -  116.000     NEL      -  NUMBER OF ANTENNA ELEMENTS           :          1
117 -  117.000     LOC1     -  LOCATION OF THE FIRST ELEMENT        :        255
118 -  118.000     BWFCTR   -  BANDWIDTH TOLERANCE FACTOR           :    1.00000
119 -  119.000     BWOFF    -  BANDWIDTH OFFSET FACTOR              :     .00000
120 -  120.000     PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY   :     .10000
121 -  121.000  ---------------------------------------------------------------------------------
122 -  122.000 1
123 -  123.000  ---------------------------------------------------------------------------------
124 -  124.000                                     PORT  0
125 -  125.000                                     -------
126 -  126.000  ---------------------------------------------------------------------------------
127 -  127.000                           RECEIVED BASEBAND PORT SIGNAL
```

```
128 -  128.000                     NORMALIZATION CONSTANT    :    .359902E+00
129 -  129.000  ------------------------------------------------------------------------------------------
130 -  130.000     .05464    .03818   -.27935   -.18506   -.80850   -.80400   -.96805  -1.00000
131 -  131.000   -.89117   -.93809   -.86581   -.91809   -.51389   -.71681   -.01191   -.37046
132 -  132.000    .51795    .07067    .69954    .48863    .33405    .37157   -.72323   -.56527
133 -  133.000   -.48070   -.23752   -.28359    .13504   -.05666    .10339    .17505    .00267
134 -  134.000   -.06350   -.11284   -.27573   -.25835   -.11408    .03863    .07952    .27616
135 -  135.000   -.00055   -.11476    .45160    .31795   -.25093   -.05661   -.53158   -.23006
136 -  136.000    .10718   -.05535    .29687    .03986   -.22163   -.13246   -.32173   -.22894
137 -  137.000   -.37581    .00988   -.03880    .09207    .23237    .18048   -.03938    .26499
13A -  13A.000   -.60312   -.16596   -.22783   -.27798    .15059    .04556    .49784    .37388
139 -  139.000    .05924    .20007   -.67014   -.26716   -.51796   -.17555    .41518    .42498
140 -  140.000    .46477    .55190   -.24890    .00095   -.22332   -.22121   -.17883   -.20944
141 -  141.000   -.14506   -.08280   -.48174   -.33351   -.15840   -.23864    .03017    .12233
142 -  142.000   -.24552    .07159    .07142    .10982    .79052    .48894    .42138    .42662
143 -  143.000   -.56041   -.25491   -.64554   -.51348    .04592    .16574    .45397    .33689
144 -  144.000   -.15339   -.20463   -.60183   -.43150   -.23293    .15092   -.05293    .03788
145 -  145.000    .70177    .26965    .66569    .35406    .76548    .50369    .33144    .11207
146 -  146.000    .34198    .16070   -.22887    .04206   -.10226    .16876    .08978    .24863
147 -  147.000    .07344   -.10328    .46115    .36154    .51861    .34287    .17532    .33440
148 -  148.000   -.01579    .20464    .13438    .29128   -.17331   -.08766    .00878    .12494
149 -  149.000   -.21701   -.41771    .06459   -.23581   -.57297   -.59389   -.25658   -.38524
150 -  150.000   -.71101   -.39013   -.17867   -.31976    .16877    .45003   -.15970    .15444
151 -  151.000    .40744    .45485    .68224    .67182    .68668    .85575   -.08121    .25855
152 -  152.000    .30583    .30440    .19326    .26502   -.49589   -.22002   -.67978   -.41590
153 -  153.000   -.30489   -.53854    .35300    .05499   -.29986    .01299   -.08097   -.02180
154 -  154.000    .53392    .59253   -.08945    .13345   -.56345   -.37763   -.77421   -.36072
155 -  155.000   -.67038   -.15848   -.09812    .07619   -.07994    .11141   -.19309   -.00092
156 -  156.000   -.09846   -.05803   -.32363   -.11191   -.70204   -.51644   -.42421   -.15956
157 -  157.000    .35976    .29671    .55861    .31601    .34769   -.05135    .29570    .17155
158 -  15A.000   -.02046    .16898   -.40331   -.40163   -.33261   -.31393    .02263    .05234
159 -  159.000   -.29163   -.15834    .20011    .26848    .21790    .33672    .14524    .09034
160 -  160.000    .12943    .14092   -.11299    .08524    .11257    .27772    .18866    .44657
161 -  161.000    .08273    .17843    .09462    .15594   -.23630   -.08516    .11599   -.09951
162 -  162.000  ------------------------------------------------------------------------------------------
163 -  163.000 1
164 -  164.000  ------------------------------------------------------------------------------------------
165 -  240.000                        COMPOSITE WEIGHT-VECTOR ARRAY
166 -  241.000                        -----------------------------
167 -  242.000                ADAPTIVE WEIGHT VECTOR AT INDICATED BCR ITERATION
16A -  243.000                     NORMALIZATION CONSTANT    :  .177725E+00
169 -  244.000  ------------------------------------------------------------------------------------------
170 -  245.000    .00000    .00000    .00000    .00000    .00000    .00000    .00000    .00000
```

```
171 -  246.000   -.73406   .11070   .10955   .73304  -.86024  -.01013  -.01220   .86116
172 -  247.000   -.99357  -.75511  -.76258   .99306  -.99828   .76653   .75521  1.00000
173 -  248.000  ------------------------------------------------------------------------------
174 -  249.000 1
175 -  250.000  ------------------------------------------------------------------------------
176 -  251.000                               COMBINED PORT
177 -  252.000                               -------------
178 -  253.000  ------------------------------------------------------------------------------
179 -  254.000                       RECEIVED BASEBAND PORT SIGNAL
180 -  255.000                 NORMALIZATION CONSTANT    :    .230300E-01
181 -  256.000  ------------------------------------------------------------------------------
182 -  257.000   -.03882  -.01324   .19068   .17606   .06095   .16566   .05320  -.07284
183 -  258.000    .02983  -.00813   .07463  -.12494   .26855  -.17769   .18889  -.25052
184 -  259.000   -.11826  -.07935  -.48928  -.04969   .14694   .37927  -.66591   .31812
185 -  260.000   -.14739  -.27506  -.10647   .01626   .88967  -.26926   .09201   .11673
186 -  261.000   -.20856   .03510  -.23095   .15381  -.39792  -.22452   .61232   .05542
187 -  262.000    .04492  -.09820   .08912  -.02986  -.77634   .44226   .26923  -.25755
188 -  263.000   1.00000  -.23182  -.54976   .16129  -.19127   .13861  -.41197   .07181
189 -  264.000   -.23734  -.08672   .76398  -.18493  -.26977   .04421  -.59678   .20887
190 -  265.000    .39799   .03208   .52495  -.19592  -.15485  -.12996   .12723  -.07019
191 -  266.000   -.60791   .39562  -.33631   .15349   .12069  -.26230   .69098  -.37495
192 -  267.000   -.45620   .32997   .13521   .06601   .31844   .00246  -.15046  -.04648
193 -  268.000   -.08588   .07297  -.00853   .05014   .19793  -.18658  -.56760   .10908
194 -  269.000   -.02747   .00685   .61953  -.29260   .31569  -.12731  -.74138   .40361
195 -  270.000   -.02039   .26390  -.16356  -.12187   .31915  -.39370   .64571   .03263
196 -  271.000   -.60272   .43443  -.57501  -.07628  -.00653  -.11521   .98491  -.26487
197 -  272.000    .31008  -.11340  -.33386   .00741   .14769   .03221  -.08355   .18245
198 -  273.000   -.38720   .05156  -.66413   .28236   .17833  -.30709   .66246   .11549
199 -  274.000   -.09248  -.10721   .10212  -.11135  -.03255   .06142  -.73040   .26786
200 -  275.000    .21263  -.16096   .18605   .13513  -.12474  -.02220   .33997  -.04687
201 -  276.000    .63003  -.01096  -.36406   .03134   .00490   .15946  -.43003  -.09487
202 -  277.000   -.06626   .18153   .18173  -.40262  -.72759   .20561   .37521  -.14094
203 -  278.000    .15502  -.11839   .02080  -.12896  -.38385   .26245   .00175   .11956
204 -  279.000    .55600  -.24311  -.29779   .32143  -.31922   .13678   .45629  -.03848
205 -  280.000    .75443  -.31475  -.70349   .06777  -.52023   .21697   .37744  -.30429
206 -  281.000   -.07888  -.01842   .01804   .33538  -.27066   .12158  -.52435   .08089
207 -  282.000    .19229  -.25167   .60283  -.14237  -.44529   .09207   .46714  -.07031
208 -  283.000   -.20124   .04263   .10831   .11555  -.40199   .15664   .09070  -.36940
209 -  284.000    .79021  -.25598   .24875   .04699   .00929   .11623  -.92485   .05834
210 -  285.000    .42465   .09246  -.11334   .20699  -.07138  -.23966   .07399   .06591
211 -  286.000   -.37167   .04002   .17212  -.25058   .10549   .13588   .18000  -.05109
212 -  287.000   -.35736   .14107  -.22773   .00883  -.04493  -.09309   .05999   .00589
213 -  288.000    .34461   .01571  -.25240   .07354   .32533   .03121   .12023  -.19239
214 -  289.000  ------------------------------------------------------------------------------
215 -  290.000   SUPPRESSION  (DB)        :  25.23703
216 -  291.000  ------------------------------------------------------------------------------
```

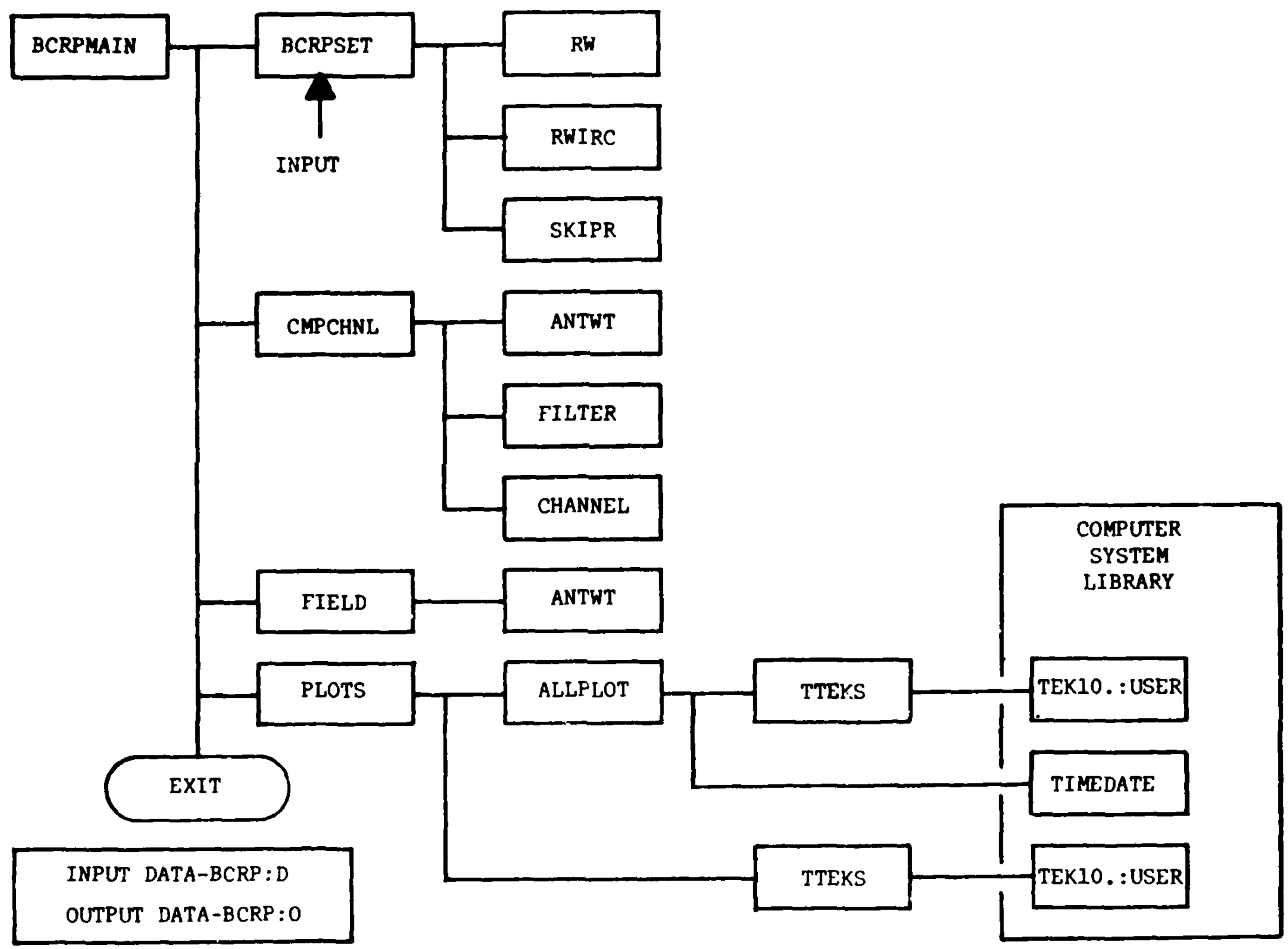


Figure 2-7. Tree Diagram of the BCR Plotting Program, BCRP

called to compute the electric field in the angular neighborhoods of the noise sources. The amplitudes of the fields over a 0.25° window are printed and held for plotting. The last branch of the structure is the plotting branch, PLOTS. It directs an auxiliary plotting program to output on a CRT several pictures containing the relevant plots in a desired format. The module called TEK.:USER is a collection of TEKTRONIX functions and subroutines which are part of the system library; therefore they are not transportable with BCRP. Also, it makes BCRP:L dependent on the availability of certain TEKTRONIX terminals.

All the data which are to be plotted are available prior to calling the PLOTS branch. Since PLOTS is the only branch which needs TEK.:USER, it is possible to remove or replace the PLOTS branch. Removal is accomplished by simply eliminating the PLOTS branch and the call to it. Replacement is done by substituting subprograms which interface with BCRP as PLOTS does, and uses local system-supported plotting software and hardware.

2.4.2.2 Source Modules

The source program list of BCRP is given in Table 2-19. To each subprogram in this list there is a corresponding named module in the BCRP tree diagram. Since all of these subprograms are members of the RADAR:LIB, those with the familiar names, such as RW, RWIRC, etc., refer to the specific member-program already described for earlier programs. A functional description of the newly introduced source programs is given in Table 2-20.

2.4.2.3 Binary Modules

Each source module in the BCRP program has its corresponding binary version. As shown before, the general JCL program, JCL:B, (see Table 2-2) can be used to create any of these binary modules.

2.4.2.4 Composite Binary and Load Modules

The creation of the executable load module for the BCRP program can be accomplished through the execution of the JCL program, JCLBCRP:BL, which is shown in Table 2-21. This program works in the same manner as the one described in paragraph 2.1.2.4. Here, all the member-programs of RADAR:LIB pertaining to BCRP are concatenated to form the composite binary module, BCRP:B. This new module is then linked with the TEK.:USER library and other necessary system supplied functions to form the executable load module, BCRP:L.

2.4.3 Program Execution

The program BCRP:L can be executed after assigning the proper input and output files. The input file for this case is BCRP:D, discussed in paragraph 2.4.1. The printed output can go to a similar file, or to a line printer. This program was designed to be executed in an interactive mode. Using a TEKTRONIX terminal, it is necessary to assign the input and output files. One of the ways this can be done is by entering the two successive device assignments

```
!SET F:105/BCRP:D
!SET F:108/BCRP:O
```

Table 2-19. FORTRAN Listing of Source Modules Comprising the BCR Performance and Plotting Program, BCRP:S

```
C ======================================================================
C                 BCR GRAPHICAL OUTPUT PROGRAM :  BCRP:S
C                 --------------------------------------
C
C                    DIGITAL ADAPTIVE ARRAY PROCESSING
C                                 USING
C                       BATCH COVARIANCE RELAXATION
C
C             PROGRAM       :  BCRPMAIN:S
C             ORIGINAL      :  JULY       23, 1980
C             REVISION      :  FEBRUARY    3, 1981
C
C             PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                              RADAR SYSTEMS ANALYSIS GROUP
C                              MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                              TEMPE, ARIZONA  85282
C ======================================================================
      CALL BCRPSET
      CALL CMPCHNL
      CALL FIELD
      CALL PLOTS
      CALL EXIT
      END
```

```
C ================================================================
      SUBROUTINE BCRPSET
C ================================================================
C                SPECIFICATION OF SYSTEM PARAMETERS
C                MAIN-PORT AND COMBINED-PORT SIGNALS
C                               AND
C                     ADAPTIVE-WEIGHT ITERATES
C
C          PROGRAM       :  BCRPSET:S
C          ORIGINAL      :  JULY      23, 1980
C          REVISION      :  FEBRUARY 13, 1981
C
C          PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                           RADAR SYSTEMS ANALYSIS GROUP
C                           MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                           TEMPE, ARIZONA  85282
C ----------------------------------------------------------------
C     INPUT  :    DATA FILE BCRP:D
C ================================================================
      DATA PI/3.14159265/
      COMPLEX POL,S0,W,WIT,SC,CDUM
      COMMON /BCRP1/ POL(6),FBIF,FBRF,RADRNG,RADIN,DRAD,NPOL,LPBP,NF
      COMMON /BCRP2/ AH0(64,20),AHC(64,20),TH(20),CTAU(20),ISC(20)
     -,IDC(20),NCHNLS
      COMMON /BCRP3/ BWFCTR(21),BWOFF(21),PTAU(21),D0,NEL(21),LOC1(21)
     -,ISP(20),IDP(21),NPM1
      COMMON /BCRP4/ S0(256),W(20),WIT(21,20),SC(256),AS0(256),ASC(256)
     -,X(256),AL(1001),ISW(20),NS,NWX,ITER,ITER1
      DIMENSION AR14(14),AR12(12)
  100 FORMAT(14A4,4X,10I2)
  200 FORMAT(12A4,E13.6)
  300 FORMAT(8F10.5)
  999 FORMAT('*****  SELECTED WEIGHTS ARE NOT COMPATIBLE WITH DESIGNATED
     -PORTS  *****')
C ----------------------------------------------------------------
      CALL RW(20)
      CALL RWIRC(CDUM,RDUM,NWX,0)
C ----------------------------------------------------------------
C     READ IN WEIGHT SELECTOR ARRAY OVER DESIGNATED PORTS
C ----------------------------------------------------------------
      NW1=1
      NW2=10
 1000 READ (105,100) AR14,(ISW(IW),IW=NW1,NW2)
```

```
      WRITE(108,100) AR14,(ISW(IW),IW=NW1,NW2)
      NW1=NW1+10
      NW2=NW2+10
      IF(NW1.GT.NWX) GOTO 2000
      IF(NW2.GT.NWX) NW2=NWX
      GOTO 1000
2000  CALL RW(2)
      CALL RWIRC(CDUM,RDUM,ITER,0)
      ITER1=ITER+1
      CALL RW(18)
      CALL RWIRC(CDUM,RDUM,NPOL,0)
      DO 10 IP=1,NPOL
10    CALL RWIRC(POL(IP),RDUM,IDUM,2)
      CALL RWIRC(CDUM,FBIF,IDUM,1)
      CALL RWIRC(CDUM,FBRF,IDUM,1)
      CALL RWIRC(CDUM,RADRNG,IDUM,1)
      CALL RWIRC(CDUM,RADIN,IDUM,1)
      CALL RWIRC(CDUM,RDUM,NF,0)
      DRAD=RADRNG/NF
      CALL RWIRC(CDUM,RDUM,LPBP,0)
      CALL RW(5)
C -----------------------------------------------------------------
C     READ IN CHANNEL SELECTOR ARRAY AND ASSOCIATED CHANNEL DESCRIPTIONS
C -----------------------------------------------------------------
      CALL RWIRC(CDUM,RDUM,NCHNLS,0)
      NC1=1
      NC2=10
3000  READ (105,100) AR14,(ISC(IC),IC=NC1,NC2)
      WRITE(108,100) AR14,(ISC(IC),IC=NC1,NC2)
      NC1=NC1+10
      NC2=NC2+10
      IF(NC1.GT.NCHNLS) GOTO 4000
      IF(NC2.GT.NCHNLS) NC2=NCHNLS
      GOTO 3000
4000  IIC=0
      DO 20 IC=1,NCHNLS
      IF(ISC(IC).EQ.0) GOTO 20
      CALL RW(6)
      IIC=IIC+1
      IDC(IIC)=IC
      CALL RWIRC(CDUM,TH(IIC),IDUM,1)
      CALL RW(1)
20    CONTINUE
```

```
      NCHNLS=IIC
      CALL RW(3)
C     ------------------------------------------------------------------
C     READ IN PORT SELECTOR ARRAY AND ASSOCIATED PORT DESCRIPTIONS
C     AUGMENT PORT SELECTOR ARRAY VIA WEIGHT SELECTOR ARRAY
C     ------------------------------------------------------------------
      CALL RWIRC(CDUM,D0,IDUM,1)
      CALL RWIRC(CDUM,RDUM,NS,0)
      CALL RWIRC(CDUM,RDUM,NPORTS,0)
      NPM1=NPORTS-1
      NP1=1
      NP2=10
5000  READ (105,100) AR14,(ISP(IP),IP=NP1,NP2)
      WRITE(108,100) AR14,(ISP(IP),IP=NP1,NP2)
      NP1=NP1+10
      NP2=NP2+10
      IF(NP1.GT.NPM1) GOTO 6000
      IF(NP2.GT.NPM1) NP2=NPM1
      GOTO 5000
6000  DO 30 IP=1,NPM1
      ISP(IP)=ISP(IP)*ISW(IP)
30    IF(ISP(IP).NE.ISW(IP)) GOTO 9999
      CALL RW(2)
      IIP=1
      DO 40 IP=1,NPORTS
      IF(IP.EQ.1) GOTO 7000
      IP1=IP-1
      IF(ISP(IP1).EQ.0) GOTO 40
      CALL RW(2)
      IIP=IIP+1
      IDP(IIP)=IP1
7000  CALL RWIRC(CDUM,RDUM,NEL(IIP),0)
      CALL RWIRC(CDUM,RDUM,LOC1(IIP),0)
      CALL RWIRC(CDUM,BWFCTR(IIP),IDUM,1)
      CALL RWIRC(CDUM,BWOFF(IIP),IDUM,1)
      CALL RWIRC(CDUM,PDEL,IDUM,1)
      IF(IP.EQ.1) PTAUX=PI*D0*NEL(1)
      PTAU(IIP)=PDEL*PTAUX
40    CONTINUE
      NPM1=IIP-1
      IDP(1)=0
C     ------------------------------------------------------------------
C     READ IN, DENORMALIZE AND COMPUTE AMPLITUDE OF S0
```

```
C ---------------------------------------------------------------------
      CALL RW(7)
      READ (105,200) AR12,SOSCL
      WRITE(108,200) AR12,SOSCL
      CALL RW(1)
      READ (105,300) (SO(IS),IS=1,NS)
      WRITE(108,300) (SO(IS),IS=1,NS)
      DO 50 IS=1,NS
      SO(IS)=SO(IS)*SOSCL
50    ASO(IS)=CABS(SO(IS))
C ---------------------------------------------------------------------
C     READ IN AND DENORMALIZE ADAPTIVE WE GHT ITERATES
C ---------------------------------------------------------------------
      CALL RW(6)
      READ (105,200) AR12,WITSCL
      WRITE(108,200) AR12,WITSCL
      CALL RW(1)
      DO 60 IT=1,ITER1
      READ (105,300) (WIT(IT,IP),IP=1,NPM1)
60    WRITE(108,300) (WIT(IT,IP),IP=1,NPM1)
      DO 70 IT=1,ITER1
      DO 70 IP=1,NPM1
70    WIT(IT,IP)=WIT(IT,IP)*WITSCL
      DO 80 IP=1,NPM1
80    W(IP)=WIT(ITER1,IP)
C ---------------------------------------------------------------------
C     READ IN, DENORMALIZE AND COMPUTE AMPLITUDE OF SC
C ---------------------------------------------------------------------
      CALL RW(7)
      READ (105,200) AR12,SCSCL
      WRITE(108,200) AR12,SCSCL
      CALL RW(1)
      READ (105,300) (SC(IS),IS=1,NS)
      WRITE(108,300) (SC(IS),IS=1,NS)
      DO 90 IS=1,NS
      SC(IS)=SC(IS)*SCSCL
90    ASC(IS)=CABS(SC(IS))
      CALL RW(3)
C ---------------------------------------------------------------------
      RETURN
9999  WRITE(108,999)
      STOP
      END
```

```
C =====================================================================
      SUBROUTINE RW(N)
C =====================================================================
C                READING AND WRITING 80-CHARACTER DATA COMMENTS
C
C                PROGRAM       :  RW:S
C                ORIGINAL      :  APRIL 19, 1978
C                REVISION      :  JUNE  15, 1980
C
C                PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                                 RADAR SYSTEMS ANALYSIS GROUP
C                                 MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                 TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C     INPUT  :    N          -    NUMBER OF LINES TO BE READ AND WRITTEN
C =====================================================================
      DIMENSION AR20(20)
  100 FORMAT(20A4)
      DO 10 I=1,N
      READ (105,100) AR20
   10 WRITE(108,100) AR20
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE RWIRC(CX,RX,IX,IND)
C ======================================================================
C                READING AND WRITING AN 80-CHARACTER LINE
C                               CONSISTING OF
C                          A 56-CHARACTER COMMENT
C                                    AND
C            A COMPLEX, REAL OR INTEGER SCALAR VARIABLE VALUE
C
C              PROGRAM        :  RWIRC:S
C              ORIGINAL       :  APRIL  19, 1980
C              REVISION       :  AUGUST 13, 1980
C
C              PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     INPUT  :     IND        -  OPTION INDICATOR
C                                 0  : READ & WRITE INTEGER
C                                 1  : READ & WRITE REAL
C                                 2  : READ & WRITE COMPLEX
C
C     OUTPUT :     CX         -  COMPLEX SCALAR EXCLUSIVE OF RX & IX
C                  RX         -  REAL SCALAR EXCLUSIVE OF CX & IX
C                  IX         -  INTEGER SCALAR EXCLUSIVE OF CX & RX
C ======================================================================
      COMPLEX CX
      DIMENSION AR14(14)
  100 FORMAT(14A4,I12)
  200 FORMAT(14A4,F12.5)
  300 FORMAT(14A4,2X,2(F10.5))
      GOTO (1,2),IND
      READ (105,100) AR14,IX
      WRITE(108,100) AR14,IX
      RETURN
    1 READ (105,200) AR14,RX
      WRITE(108,200) AR14,RX
      RETURN
    2 READ (105,300) AR14,CX
      WRITE(108,300) AR14,CX
      RETURN
      END
```

```
C ====================================================================
      SUBROUTINE SKIPR(NU,NS)
C ====================================================================
C SKIP NS LINES ON UNIT NU.
C
C              PROGRAM        :  SKIPR:S
C              ORIGINAL       :  FEBRUARY 23, 1981
C              REVISION       :  FEBRUARY 23, 1981
C
C              PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C --------------------------------------------------------------------
C     INPUT  :    NU          -  NUMBER OF UNIT OR DEVICE
C                 NS          -  NUMBER OF LINES TO SKIP
C
C     OUTPUT  :               -  NONE
C --------------------------------------------------------------------
C     ENTRY POINTS            :  SKIPR
C     SUBROUTINES CALLED      :  NONE
C ====================================================================
100   FORMAT(1X)
C --------------------------------------------------------------------
C     SKIP NS LINES ON UNIT NU
C --------------------------------------------------------------------
      DO 10 I=1,NS
10    READ(NU,100)
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE CMPCHNL
C ======================================================================
C     COMPUTATION OF COMPOSITE BASEBAND CHANNEL AMPLITUDE RESPONSE
C                   BEFORE AND AFTER BCR ADAPTATION
C
C               PROGRAM       :  CMPCHNL:S
C               ORIGINAL      :  AUGUST   4, 1980
C               REVISION      :  JANUARY 30, 1981
C
C               PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     INPUT  :    SYSTEM PARAMETERS IN COMMON
C
C     OUTPUT :    AH0        -  MAIN BASEBAND CHANNEL AMPLITUDE
C                               RESPONSES
C                 AHC        -  COMPOSITE BASEBAND CHANNEL AMPLITUDE
C                               RESPONSES
C ======================================================================
      COMPLEX POL,S0,W,WIT,SC,HR,H0,H
      COMMON /BCRP1/ POL(6),FBIF,FBRF,RADRNG,RADIN,DRAD,NPOL,LPBP,NF
      COMMON /BCRP2/ AH0(64,20),AHC(64,20),TH(20),CTAU(20),ISC(20)
     -,IDC(20),NCHNLS
      COMMON /BCRP3/ BWFCTR(21),BWOFF(21),PTAU(21),D0,NEL(21),LOC1(21)
     -,ISP(20),IDP(21),NPM1
      COMMON /BCRP4/ S0(256),W(20),WIT(21,20),SC(256),AS0(256),ASC(256)
     -,X(256),AL(1001),ISW(20),NS,NWX,ITER,ITER1
      DIMENSION H0(64),H(64),HR(64)
  100 FORMAT(1H1)
  200 FORMAT(1X,79('-'))
  300 FORMAT(22X,'BASEBAND CHANNEL AMPLITUDE RESPONSE',I4,/
     -,22X,40('-'))
  400 FORMAT(18X,'MAIN-PORT BASEBAND CHANNEL AMPLITUDE RESPONSE',/
     -,18X,45('-'),/,(8F10.5))
  500 FORMAT(18X,'COMPOSITE BASEBAND CHANNEL AMPLITUDE RESPONSE',/
     -,18X,45('-'),/,(8F10.5))
C ----------------------------------------------------------------------
C     PROVIDE MAIN ANTENNA ARRAY ELEMENT WEIGHTING
C ----------------------------------------------------------------------
      CALL ANTWT(NEL(1),LOC1(1),AL,1)
```

```
C ----------------------------------------------------------------
C     DERIVE COMPOSITE BASEBAND AMPLITUDE RESPONSES
C     FOR SELECTED CHANNELS
C ----------------------------------------------------------------
      DO 10 IC=1,NCHNLS
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,300) IDC(IC)
      WRITE(108,200)
C ----------------------------------------------------------------
C     DERIVE MAIN-PORT BASEBAND CHANNEL TRANSFER FUNCTION
C ----------------------------------------------------------------
      CALL FILTER(NF,DRAD,RADIN,FBIF,LPBP,BWFCTR(1),BWOFF(1),NPOL,POL
     -,HR)
      CTAU(IC)=0
      CALL CHANNEL(NEL(1),LOC1(1),AL,D0,TH(IC),CTAU(IC),PTAU(1),NF
     -,RADIN,DRAD,BWFCTR(1),BWOFF(1),FBRF,HR,H0)
C ----------------------------------------------------------------
C     DERIVE MAIN-PORT BASEBAND CHANNEL AMPLITUDE RESPONSE
C ----------------------------------------------------------------
      DO 20 IF=1,NF
20    AH0(IF,IC)=20*ALOG10(CABS(H0(IF)))
      WRITE(108,400) (AH0(IF,IC),IF=1,NF)
      WRITE(108,200)
C ----------------------------------------------------------------
C     DERIVE BASEBAND CHANNEL TRANSFER FUNCTIONS
C     FOR SELECTED AUXILIARY PORTS
C ----------------------------------------------------------------
      AL1=AL(1)
      AL(1)=1
      DO 30 IP=1,NPM1
      IP1=IP+1
      CALL FILTER(NF,DRAD,RADIN,FBIF,LPBP,BWFCTR(IP1),BWOFF(IP1),NPOL
     -,POL,HR)
      CALL CHANNEL(NEL(IP1),LOC1(IP1),AL,D0,TH(IC),CTAU(IC),PTAU(IP1)
     -,NF,RADIN,DRAD,BWFCTR(IP1),BWOFF(IP1),FBRF,HR,H)
C ----------------------------------------------------------------
C     ACCUMULATE COMPOSITE BASEBAND CHANNEL TRANSFER FUNCTION
C ----------------------------------------------------------------
      DO 30 IF=1,NF
30    H0(IF)=H0(IF)+W(IP)*H(IF)
C ----------------------------------------------------------------
C   DERIVE COMPOSITE CHANNEL AMPLITUDE RESPONSE
```

```
C -----------------------------------------------------------------------
      DO 40 IF=1,NF
40    AHC(IF,IC)=20*ALOG10(CABS(H0(IF)))
      WRITE(108,500) (AHC(IF,IC),IF=1,NF)
      WRITE(108,200)
      AL(1)=AL1
10    CONTINUE
C -----------------------------------------------------------------------
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE ANTWT(NEL,LOC1,AL,IW)
C ======================================================================
C         COMPUTATION OF LINEAR ANTENNA ARRAY TAYLOR WEIGHTING
C
C              PROGRAM       :  ANTWT:S
C              ORIGINAL      :  APRIL     19, 1980
C              REVISION      :  SEPTEMBER  1, 1980
C
C              PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                               RADAR SYSTEMS ANALYSIS GROUP
C                               MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                               TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     INPUT  :    NEL        -  NUMBER OF ELEMENTS IN LINEAR ARRAY
C                 LOC1       -  LOCATION (NUMBER) OF FIRST ELEMENT
C
C     OUTPUT :    AL         -  ARRAY ELEMENT WEIGHTING
C ======================================================================
      DIMENSION AL(1)
  100 FORMAT(1H1)
  200 FORMAT(1X,79('-'))
  300 FORMAT(20X,'TAYLOR WEIGHTING FOR MAIN ANTENNA ARRAY',/,20X
     -,39('-'))
  400 FORMAT(8F10.5)
      DATA PI/3.1415965/
      PI2=2*PI
      LOC=LOC1-1
      CON2=0.5*(NEL+1)
      CON1=PI2/NEL
      DO 10 I=1,NEL
      LOC=LOC+1
      ARG=CON1*(LOC-CON2)
   10 AL(I)=1+0.5*COS(ARG)
      IF(IW.EQ.0) RETURN
      WRITE(108,100)
      WRITE(108,200)
      WRITE(108,300)
      WRITE(108,400) (AL(I),I=1,NEL)
      WRITE(108,200)
      RETURN
      END
```

```
C =====================================================================
      SUBROUTINE FILTER(NF,DRAD,RADIN,FBIF,LPBP,BWFCTR,BWOFF,NPOL,POL
     -,H)
C =====================================================================
C EVALUATION OF NORMALIZED RECEIVER BASEBAND SAMPLED TRANSFER FUNCTION
C                         AT DISCRETE RADIAN FREQUENCIES
C                                     USING
C            LOWPASS PROTOTYPE FILTER OR ITS BANDPASS EQUIVALENT
C
C              PROGRAM        :  FILTER:5
C              ORIGINAL       :  APRIL 19, 1980
C              REVISION       :  JUNE  23, 1980
C
C              PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C     INPUT  :     NF         -  NUMBER OF FREQUENCY SAMPLES
C                  DRAD       -  NORMALIZED RADIAN FREQUENCY INCREMENT
C                  RADIN      -  INITIAL RADIAN FREQUENCY
C                  FBIF       -  FRACTIONAL BANDWIDTH AT FINAL IF
C                  LPBP       -  LOWPASS / BANDPASS OPTION
C                                0  :  LOWPASS
C                                1  :  BANDPASS
C                  BWFCTR     -  BANDWIDTH FACTOR
C                  BWOFF      -  BANDWIDTH OFFSET FACTOR
C                  NPOL       -  NUMBER OF POLES
C                  POL        -  POLE LOCATIONS
C
C     OUTPUT :     H          -  SAMPLED TRANSFER FUNCTION
C =====================================================================
      COMPLEX POL,S,DEN,H
      DIMENSION POL(1),H(1)
C ---------------------------------------------------------------------
C     ADJUST INCREMENT AND INITIAL RADIAN FREQUENCY
C     COMPUTE SAMPLED TRANSFER FUNCTION
C ---------------------------------------------------------------------
      DRADL=0.5*DRAD/BWFCTR
      RAD1=(RADIN/2-BWOFF)/BWFCTR-DRADL
      DO 10 I=1,NF
      RAD=RAD1+I*DRADL
      H(I)=1
```

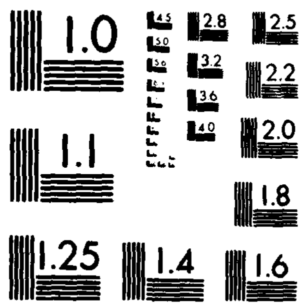
```
      S=CMPLX(0,RAD)
      IF(LPBP.EQ.1) S=S*(1+1/(1+RAD*FBIF))
      DO 10 J=1,NPOL
      DEN=S-POL(J)
   10 H(I)=H(I)*POL(J)/DEN
C     ----------------------------------------------------------------
      RETURN
      END
```

```
      SUBROUTINE CHANNEL(NEL,LOC1,AL,D0,TH,CTAU,PTAU,NF,RADIN,DRAD
     -,BWFCTR,BWOFF,FBRF,HR,H)
C  ====================================================================
C  EVALUATION OF NORMALIZED CHANNEL BASEBAND SAMPLED TRANSFER FUNCTION
C                     AT DISCRETE RADIAN FREQUENCIES
C                                INCLUDING
C            LINEAR ANTENNA ARRAY AND RECEIVER CHARACTERISTICS
C                  WITH SPEIFIED CHANNEL AND PORT DELAYS
C
C            PROGRAM      :  CHANNEL:S
C            ORIGINAL     :  MAY      4, 1980
C            REVISION     :  JANUARY 25, 1981
C
C            PREPARED  BY :  S. M. DANIEL  &  I. KERTESZ
C                            RADAR SYSTEMS ANALYSIS GROUP
C                            MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                            TEMPE, ARIZONA  85282
C  --------------------------------------------------------------------
C     INPUT  :    NEL        -  NUMBER OF ELEMENTS IN LINEAR ARRAY
C                 LOC1       -  LOCATION (NUMBER) OF FIRST ELEMENT
C                 AL         -  ARRAY ELEMENT WEGHTING
C                 D0         -  ELEMENT SEPARATION FACTOR
C                 TH         -  AZIMUTH ANGLE (DEG) OF INCIDENCE
C                 CTAU       -  NORMALIZED CHANNEL DELAY
C                 PTAU       -  NORMALIZED PORT DELAY
C                 NF         -  NUMBER OF FREQUENCY SAMPLES
C                 RADIN      -  INITIAL RADIAN FREQUENCY
C                 DRAD       -  NORMALIZED RADIAN FREQUENCY INCREMENT
C                 BWFCTR     -  BANDWIDTH FACTOR
C                 BWOFF      -  BANDWIDTH OFFSET FACTOR
C                 FBRF       -  FRACTIONAL BANDWIDTH AT RF
C                 HR         -  RECEIVER SAMPLED TRANSFER FUNCTION
C
C     OUTPUT :    H          -  CHANNEL SAMPLED TRANSFER FUNCTION
C  ====================================================================
      DOUBLE COMPLEX  CH
      COMPLEX HR,H
      DIMENSION HR(1),H(1),AL(1)
      DATA PI/3.14159265/
      RATIO=FBRF/BWFCTR
      DRADL=RATIO*DRAD/2
      RAD1=1+RATIO*(RADIN/2-BWOFF)-DRADL
```

```
      PDSTH=PI*D0*SIN(TH*PI/180)
C --------------------------------------------------------------------
C     EVALUATION OF BASEBAND CHANNEL SAMPLED TRANSFER FUNCTION
C --------------------------------------------------------------------
      DO 10 I=1,NF
      RAD=RAD1+I*DRADL
      ARG0=PDSTH*RAD
      CH=0
      LOC=LOC1-1
      DO 20 J=1,NEL
      LOC=LOC+1
      ARG=ARG0*LOC
      CH=CH+AL(J)*DCMPLX(DCOS(ARG),DSIN(ARG))
20    CONTINUE
      ARG=-RAD*(CTAU+PTAU)
      CH=CH*DCMPLX(DCOS(ARG),DSIN(ARG))
10    H(I)=CH*HR(I)
C --------------------------------------------------------------------
      RETURN
      END
```

AD-A109 927 MOTOROLA INC TEMPE AZ GOVERNMENT ELECTRONICS DIV F/G 20/14
BATCH COVARIANCE RELAXATION (BCR) ADAPTIVE PROCESSING.(U)
AUG 81 S M DANIEL, I KERTESZ F30602-80-C-0031
UNCLASSIFIED 8512-F RADC-TR-81-212 NL

5 of 7
AD A109927


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

```
C ======================================================================
      SUBROUTINE FIELD
C ======================================================================
C              COMPUTATION OF ELECTRIC FIELD AMPLITUDE PATTERNS
C                    ABOUT INTERFERENCE ANGLES OF ARRIVAL
C                       BEFORE AND AFTER BCR ADAPTATION
C
C              PROGRAM       :  FIELD:S
C              ORIGINAL      :  AUGUST   4, 1980
C              REVISION      :  JANUARY 31, 1981
C
C              PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                               RADAR SYSTEMS ANALYSIS GROUP
C                               MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                               TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     INPUT  :   SYSTEM PARAMETERS IN COMMON
C
C     OUTPUT :    AEB        -  ELECTRIC FIELD AMPLITUDE PATTERN
C                               BEFORE ADAPTATION
C                 AEA        -  ELECTRIC FIELD AMPLITUDE PATTERN
C                               AFTER ADAPTATION
C ======================================================================
      DOUBLE COMPLEX E
      COMPLEX S0,W,WIT,SC,POL
      COMMON /BCRP1/ POL(6),FBIF,FBRF,RADRNG,RADIN,DRAD,NPOL,LPBP,NF
      COMMON /BCRP2/ AH0(64,20),AHC(64,20),TH(20),CTAU(20),ISC(20)
     -,IDC(20),NCHNLS
      COMMON /BCRP3/ BWFCTR(21),BWOFF(21),PTAU(21),D0,NEL(21),LOC1(21)
     -,ISP(20),IDP(21),NPM1
      COMMON /BCRP4/ S0(256),W(20),WIT(21,20),SC(256),AS0(256),ASC(256)
     -,X(256),AL(1001),ISW(20),NS,NWX,ITER,ITER1
      COMMON /BCRP5/ NTH,THETA(64,21),AEB(64,21),AEA(64,21)
  100 FORMAT(1H1)
  200 FORMAT(1X,79('-'))
  300 FORMAT(6X,'FIELD PATTERN BEFORE AND AFTER BCR ADAPTATION ABOUT INT
     -ERFERENCE',I4,/,6X,68('-'))
  400 FORMAT(15X,'MAIN BEAM PATTERN BEFORE AND AFTER BCR ADAPTATION',/
     -,15X,49('-'))
  500 FORMAT(/,31X,'MAIN FIELD PATTERN',/,31X,18('-'))
  600 FORMAT(/,29X,'COMPOSITE FIELD PATTERN',/,29X,23('-'))
  700 FORMAT(8F10.5)
      DATA PI/3.14159265/
```

```
      CONV=PI/180
      RTH=CONV/4
      NTH=64
      DTH=RTH/NTH
      THO=-RTH/2-DTH
      NCP1=NCHNLS+1
      LOC0=LOC1(1)-1
C -----------------------------------------------------------------
C     COMPUTE ELECTRIC FIELD PATTERNS BEFORE AND AFTER ADAPTATION
C     OVER A 0.25 DEG RANGE ABOUT EACH INTERFERENCE ANGLE OF ARRIVAL
C     INCLUDING MAIN BEAM EFFECT
C -----------------------------------------------------------------
      CALL ANTWT(NEL(1),LOC1(1),AL,0)
      DO 10 IC=1,NCP1
      IF(IC.LT.NCP1) THE=TH(IC)
      IF(IC.EQ.NCP1) THE=0
      THE0=CONV*THE+THO
      DO 20 IA=1,NTH
      THE=THE0+IA*DTH
      ARG1=PI*SIN(THE)
      E=0
      DO 30 IL=1,NEL(1)
      ARG=(LOC0+IL)*ARG1
30    E=E+AL(IL)*DCMPLX(DCOS(ARG),DSIN(ARG,)
      AEB(IA,IC)=20*ALOG10(CABS(E))
      THETA(IA,IC)=THE/CONV
      DO 40 IP=1,NPM1
      IPP1=IP+1
      ARG=LOC1(IPP1)*ARG1-PTAU(IPP1)
40    E=E+W(IP)*DCMPLX(DCOS(ARG),DSIN(ARG))
20    AEA(IA,IC)=20*ALOG10(CABS(E))
      WRITE(108,100)
      WRITE(108,200)
      IF(IC.LT.NCP1) WRITE(108,300) IDC(IC)
      IF(IC.EQ.NCP1) WRITE(108,400)
      WRITE(108,500)
      WRITE(108,700) (THETA(IA,IC),AEB(IA,IC),IA=1,NTH)
      WRITE(108,600)
      WRITE(108,700) (THETA(IA,IC),AEA(IA,IC),IA=1,NTH)
10    WRITE(108,200)
C -----------------------------------------------------------------
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE PLOTS
C ======================================================================
C                               GRAPHICAL OUTPUT
C                       MAIN AND ADAPTED SIGNAL AMPLITUDE
C                         EVOLUTION OF ADAPTIVE WEIGHTS
C                  MAIN AND COMPOSITE CHANNEL AMPLITUDE RESPONSES
C     MAIN AND COMPOSITE ELECTIC FIELD AMPLITUDES ABOUT INTERFERENCES
C
C              PROGRAM        :  PLOTS:S
C              ORIGINAL       :  SEPTEMBER  6, 1980
C              REVISION       :  FEBRUARY   2, 1981
C
C              PREPARED  BY   :  I. KERTESZ  &  S. M. DANIEL
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     INPUT  :     S0         -  MAIN-PORT SIGNAL
C                  SC         -  BCR-ADAPTED COMBINED SIGNAL
C                  NS         -  ACTUAL NUMBER OF SIGNAL SAMPLES
C                  WIT        -  COMPOSITE ARRAY OF ADAPTIVE WEIGHT
C                                ITERATES
C
C     OUTPUT :     INDICATED PLOTS
C ======================================================================
      COMPLEX S0,SC,W,WIT,WW,POL
      COMMON /BCRP1/ POL(6),FBIF,FBRF,RADRNG,RADIN,DRAD,NPOL,LPBP,NF
      COMMON /BCRP2/ AH0(64,20),AHC(64,20),TH(20),CTAU(20),ISC(20)
     -,IDC(20),NCHNLS
      COMMON /BCRP3/ BWFCTR(21),BWOFF(21),PTAU(21),D0,NEL(21),LOC1(21)
     -,ISP(20),IDP(21),NPM1
      COMMON /BCRP4/ S0(256),W(20),WIT(21,20),SC(256),AS0(256),ASC(256)
     -,X(256),AL(1001),ISW(20),NS,NWX,ITER,ITER1
      COMMON /BCRP5/ NTH,THETA(64,21),AEB(64,21),AEA(64,21)
      DIMENSION WW(420),WR(21),WI(21)
      DATA  DMAX /1.0E10/
C ----------------------------------------------------------------------
C     PLOT AMPLITUDES OF S0 AND SC
C ----------------------------------------------------------------------
      DO 10 IS=1,NS
   10 X(IS)=IS
      R1=1
```

```
      R2=NS
      D1=DMAX
      D2=-D1
      CALL TRANGE(NS,1,D1,D2,ASO)
      CALL TRANGE(NS,1,D1,D2,ASC)
      CALL ALLPLOT(1,NS,0,R1,R2,D1,D2,X,ASO,ASC)
C ----------------------------------------------------------------------
C     PLOT ADAPTIVE WEIGHT EVOLUTION
C ----------------------------------------------------------------------
      D1=DMAX
      D2=-D1
      K=0
      DO 20 J=1,NPM1
      DO 20 I=1,ITER1
      K=K+1
   20 WW(K)=WIT(I,J)
      K=2*K
      II=0
      CALL TRANGE(K,1,D1,D2,WW)
      D=AMAX1(ABS(D1),ABS(D2))
      DO 40 J=1,NPM1
      DO 30 I=1,ITER1
      WR(I)=REAL(WIT(I,J))
   30 WI(I)=AIMAG(WIT(I,J))
      IF(J.EQ.NPM1) II=-1
   40 CALL ALLPLOT(2,ITER1,II,-D,D,-D,D,WR,WI)
C ----------------------------------------------------------------------
C     PLOT MAIN AND COMPOSITE CHANNEL AMPLITUDE RESPONSES
C ----------------------------------------------------------------------
      RAD=RADIN-DRAD
      DO 50 IF=1,NF
   50 X(IF)=RAD+IF*DRAD
      R1=X(1)
      R2=X(NF)
      DO 60 IC=1,NCHNLS
      D1=DMAX
      D2=-D1
      CALL TRANGE(NF,1,D1,D2,AMO(1,IC))
      CALL TRANGE(NF,1,D1,D2,AMC(1,IC))
      II=IOC(IC)
      IF(IC.EQ.NCHNLS) II=-II
      CALL ALLPLOT(3,NF,II,R1,R2,D1,D2,X,AMO(1,IC),AMC(1,IC))
   60 CONTINUE
```

```
C ---------------------------------------------------------------------
C     PLOT MAIN AND COMPOSITE ELECTRIC FIELD ABOUT INTERFERENCES
C ---------------------------------------------------------------------
      NCHP1=NCHNLS+1
      IPLACE=4
      DO 70 IC=1,NCHP1
      R1=THETA(1,IC)
      R2=THETA(NTH,IC)
      D1=DMAX
      D2=-D1
      CALL TRANGE(NTH,1,D1,D2,AEB(1,IC))
      CALL TRANGE(NTH,1,D1,D2,AEA(1,IC))
      II=IDC(IC)
      IF(IC.EQ.NCHNLS) II=-II
      IF(IC.EQ.NCHP1) IPLACE=5
      CALL ALLPLOT(IPLACE,NTH,II,R1,R2,D1,D2,THETA(1,IC),AEB(1,IC)
     -,AEA(1,IC))
   70 CONTINUE
C ---------------------------------------------------------------------
      RETURN
      END
```

```
C =====================================================================
      SUBROUTINE ALLPLOT(IPLACE,   II,R1,R2,D1,D2,X,Y1,Y2)
C =====================================================================
C                    AUXILIARY PLOTTING ROUTINE FOR PLOTS:S
C
C              PROGRAM       :  ALLPLOT:S
C              ORIGINAL      :  AUGUST   20, 1980
C              REVISION      :  FEBRUARY  3, 1981
C
C              PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                               RADAR SYSTEMS ANALYSIS GROUP
C                               MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                               TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C     INPUT  :     IPLACE     -  PLOT SELECTOR
C                                 1  : TITLE PAGE AND SIGNAL AMPLITUDES
C                                 2  : EVOLUTION OF WEIGHTS
C                                 3  : MAIN AND COMPOSITE CHANNEL
C                                      AMPLITUDE RESPONSES
C                                 4  : NULL FIELD PATTERNS, NX
C                                 5  : MAIN BEAM FIELD PATTERN
C                  II         -  SPARE INTEGER ARGUMENT
C                  R1,R2      -  SPARE REAL ARGUMENT
C                  D1,D2      -  SPARE REAL ARGUMENT
C                  X          -  ABSCISSA
C                  Y1         -  ORDINATE 1
C                  Y2         -  ORDINATE 2
C
C =====================================================================
      DIMENSION HV(5)
      DATA HV/630.,610.,310.,280.,250./
      DIMENSION X(1),Y1(1),Y2(1),LABH(60),LAB20(20),LAB12(12)
      DIMENSION LABEL(15,8)
      DATA LABEL/
     1 '                  BCR  ADAPTIVE  PROCESSING                  '
     2,'                  -------------------------                  '
     3,'        SIMULATION BY :  S. M. DANIEL & I. KERTESZ           '
     4,'         ADVANCED TECHNOLOGY AND SYSTEMS ANALYSIS            '
     5,'                   MOTOROLA, INC. - GED                      '
     6,'      MAIN AND COMPOSITE CHANNEL AMPLITUDE RESPONSES         '
     7,'                BEFORE AND AFTER ADAPTATION                  '
     8,'      FIELD PATTERN ABOUT INTERFERENCE NEIGHBORHOODS         '
     9/
```

```
      INTEGER ITDATE(4)
      INTEGER IGXS(4)/100,600,100,600/,IGXF(4)/450,950,450,950/
     -        ,IGYS(4)/460,460,100,100/,IGYF(4)/700,700,340,340/
     -        ,IGP(4) /175,675,175,675/,IGQ(4) /712,712,360,360/
     -        ,NGP(4) /135,635,135,635/,NGQ(4) /395,395, 20, 20/
100   FORMAT(8HCHANNEL ,I4)
200   FORMAT(F6.1,5H  TO ,F6.1,3H DB)
C ------------------------------------------------------------------
C     GO TO APPROPRIATE PLOT
C ------------------------------------------------------------------
      GOTO (1000,2000,3000,3000,4000), IPLACE
C ------------------------------------------------------------------
C     INITIALIZE PLOTTING
C ------------------------------------------------------------------
1000  I2=0
      I3=0
      I5=0
      CALL TIMEDATE(ITDATE)
      CALL TINIT
C ------------------------------------------------------------------
C     PLOT TITLE PAGE AND SIGNALS AMPLITUDES
C ------------------------------------------------------------------
      DO 10 I=1,5
10    CALL TTYPEW(60,LABEL(1,I),100.,HV(I))
      CALL TTYPEW(16,ITDATE,380.,200.)
      CALL TRING
      CALL NEWPAG
      CALL TTYPEA(16,ITDATE,700,0)
      CALL TTYPEA(40,40H             ORIGINAL SIGNAL
     -200,390)
      CALL TPLOT1(0,-11,NX,R1,R2,D1,D2,128,896,470,720,X,Y1)
      CALL TTYPEA(40,40H             COMBINED SIGNAL
     -200,20)
      CALL TPLOT1(0,-11,NX,R1,R2,D1,D2,128,896,100,350,X,Y2)
      CALL TRING
      RETURN
C ------------------------------------------------------------------
C     PLOT EVOLUTION OF ADAPTIVE WEIGHTS
C ------------------------------------------------------------------
2000  ISC=-1
      I2=I2+1
      IF(I2.NE.1) GOTO 2100
      CALL NEWPAG
```

```
      CALL TTYPEA(16,ITDATE,700,0)
      ISC=0
      CALL TTYPEA(30,30H EVOLUTION OF ADAPTIVE WEIGHTS,280,60)
2100  CALL TPLOT1(ISC,0,NX,R1,R2,D1,D2,200,800,160,760,X,Y1)
      IF(II.LT.0) CALL TRING
      RETURN
C ------------------------------------------------------------------
C     PLOT MAIN AND COMPOSITE CHANNEL AMPLITUDE RESPONSES
C     PLOT ELECTRIC FIELD PATTERN ABOUT INTERFERENCE NEIGHBORHOODS
C ------------------------------------------------------------------
3000  NXO2P1=NX/2+1
      ILAB1=6
      ILAB2=7
      IIA=IABS(II)
      IF(IPLACE.EQ.4) ILAB1=8
3100  I3=I3+1
      IF(I3.NE.1) GOTO 3200
      CALL NEWPAG
      CALL TTYPEA(16,ITDATE,700,0)
      CALL TTYPEA(60,LABEL(1,ILAB1),100,750)
      CALL TTYPEA(60,LABEL(1,ILAB2),100,730)
3200  RL=Y1(NXO2P1)
      RH=Y2(NXO2P1)
      ENCODE (12,100,LAB12) IIA
      CALL TTYPEA(12,LAB12,IGP(I3),IGQ(I3))
      CALL TPLOT1(0,0,NX,R1,R2,D1,D2,IGXS(I3),IGXF(I3),IGYS(I3)
     -            ,IGYF(I3),X,Y1)
      ENCODE(20,200,LAB20) RL,RH
      CALL TTYPEA(20,LAB20,NGP(I3),NGQ(I3))
      CALL TPLOT1(-1,1,NX,R1,R2,D1,D2,IGXS(I3),IGXF(I3),IGYS(I3)
     -            ,IGYF(I3),X,Y2)
      IF(II.LT.0.OR.I3.EQ.4) GOTO 3300
      CALL BELL
      RETURN
3300  I3=0
      CALL TRING
      RETURN
C ------------------------------------------------------------------
C     PLOT MAIN BEAM ELECTRIC FIELD PATTERN BEFORE AND AFTER ADAPTATION
C ------------------------------------------------------------------
4000  CALL NEWPAG
      CALL TTYPEA(16,ITDATE,700,0)
      CALL TTYPEA(37,37HMAIN BEAM BEFORE AND AFTER ADAPTATION,260,40)
```

```
      CALL TPLOT1(0,0,NX,R1,R2,D1,D2,120,970,120,780,X,Y1)
      CALL TPLOT1(-1,4,NX,R1,R2,D1,D2,120,970,120,7R0,X,Y2)
      CALL TRING
C -----------------------------------------------------------------
      RETURN
      END
```

```
C =================================================================
C
C    COMPOSITE SUBPROGRAM TTEKS
C =================================================================
C    PROGRAM TTEKS IS A COLLECTION OF SIMPLE SUBROUTINES, ALL OF WHICH
C    ARE RELATED TO TEKTRONIX-USER PROGRAMS.  THEY ARE COMPILED UNDER
C    THE NAME TTEKS, TO PRODUCE THE BINARY LIBRARY TTEKS:B
C    THE CONTENTS ARE LISTED BELOW.
C
C
C            SUBPROGRAM    :  TTEKS:S
C            ORIGINAL      :  AUGUST,       1980
C            REVISION      :  FEBRUARY 23, 1981
C
C            PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                             RADAR SYSTEMS ANALYSIS GROUP
C                             MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                             TEMPE, ARIZONA  85282
C -----------------------------------------------------------------
C
C    MEMBER      DESCRIPTION
C    ------      -----------
C    TCLEAR      CLEARS THE SCREEN AND GETS INTO THE WRITE-MODE
C    TDASHTO     DRAWS A DASHED LINE BETWEEN TWO GIVEN WINDOW
C                  COORDINATES
C    TDRAWTO     DRAWS A SOLID LINE BETWEEN TWO GIVEN WINDOW COORDINA-
C                TES
C    TINIT       INITIALIZES OR RESETS THE PLOTTER
C    TPLOT1      PLOTS 1 OR MORE CURVES ON A GRID. X-Y PLOTTER.
C    TRANGI      GIVES MINIMUM AND MAXIMUM VALUES OF AN ARRAY
C    TRANGE      SAME AS TRANGI, BUT THE MIN  AND MAX VALUES MUST BE
C                  INITIALIZED PRIOR TO CALL
C    TRING       GIVES THREE BELLS AND ASKS FOR "RETURN" COMMAND
C    TTICKER     PLACES HORIZONTAL AND VERTICAL TICK-MARKS (CROSSES)
C                  AT THE GIVEN WINDOW COORDINATES
C    TTICKH      PLACES HORIZONTAL TICK-MARK AT THE GIVEN WINDOW
C                  COORDINATES
C    TTICKV      PLACES VERTICAL TICK-MARK AT THE GIVEN WINDOW COORDI-
C                  NATES
C    TTYPEA      TYPES A 1 TO 80 CHARACTER MESSAGE AT THE GIVEN
C                  ABSOLUTE (SCREEN) COORDINATES
C    TTYPEW      TYPES A 1 TO 80-CHARACTER MESSAGE AT THE GIVEN
C                  WINDOW COORDINATES
```

```
C
C
C
C ====================================================================
      SUBROUTINE TTEK(NC,MESSAGE,Y,H1,V1,H2,V2,N,M,L)
C ====================================================================
C     ALL OF THE   ENTRY...RETURN   GROUPS IN SUBROUTINE TTEK ARE IN-
C       DEPENDENT, AND CAN BE REPLACED OR MODIFIED WITHOUT AFFECTING
C       THE OTHERS.  THEY WERE PLACED IN ONE SUBROUTINE ONLY TO REDUCE
C       THE OVERHEAD ASSOCIATED WITH MANY SUBROUTINES.
C
C             ORIGINAL       :  AUGUST,       1980
C             REVISION       :  FEBRUARY 16, 1981
C
C             PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                               RADAR SYSTEMS ANALYSIS GROUP
C                               MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                               TEMPE, ARIZONA  85282
C --------------------------------------------------------------------
C     INPUT  :                -  DESCRIBED FOR EACH ENTRY SEPARATELY
C --------------------------------------------------------------------
C     ENTRY POINTS           :  TTYPEA
C                               TTYPEW
C                               TRANGI
C                               TRANGE
C                               TDASHTO
C                               TDRAWTO
C                               TTICKER
C                               TTICKV
C                               TTICKH
C                               TCLEAR
C                               TINIT
C                               TRING
C
C     SUBROUTINES CALLED     :  :TEK10.:USER LIBRARY
C
C ====================================================================
      DIMENSION MESSAGE(NC),LABEL(80)
      REAL Y(1)
      ENTRY TTYPEA(NC,MESSAGE,IH,IV)
C --------------------------------------------------------------------
C
C     THIS ENTRY WILL WRITE A MESSAGE OF NC CHARACTERS ON THE SCREEN
```

```
C       STARTING AT THE GIVEN COORDINATES.
C       NOTE:  USE THIS ENTRY WITH THE ACTUAL INTEGER SCREEN COORDINA-
C              TES (IH,IV) CORRESPONDING TO THE 1024 HORIZONTAL AND
C              780 VERTICAL ADDRESSABLE SCREEN ELEMENTS.
C
C              USE ENTRY TTYPEW IF TWINDO HAS BEEN CALLED SINCE INITI-
C              ALIZATION, OR IF NO WINDOW HAS BEEN SET.  THE REAL NUM-
C              BER COORDINATES (RH,RV) GIVEN WILL BE MAPPED INTO THE
C              1024 BY 780 REDUCED SYSTEM BASED ON A WINDOW PROVIDED
C              BY TWINDO.
C
C     INPUT  :  NC          -  NUMBER OF CHARACTERS IN THE MESSAGE
C               MESSAGE     -  MESSAGE
C               RH          -  HORIZONTAL RELATIVE COORDINATE
C               RV          -  VERTICAL RELATIVE COORDINATE
C               IH          -  HORIZONTAL ABSOLUTE COORDINATE
C               IV          -  VERTICAL ABSOLUTE COORDINATE
C
C ------------------------------------------------------------------
      CALL KAM2AS(NC,MESSAGE,LABEL)
      CALL MOVABS(IH,IV)
      CALL HLABEL(NC,LABEL)
      RETURN
C ------------------------------------------------------------------
      ENTRY TTYPEW(NC,MESSAGE,RH,RV)
C ------------------------------------------------------------------
      CALL KAM2AS(NC,MESSAGE,LABEL)
      CALL MOVEA(RH,RV)
      CALL HLABEL(NC,LABEL)
      RETURN
C ==================================================================
      ENTRY TRANGI(I,J,YMIN,YMAX,Y)
C ------------------------------------------------------------------
C     THIS ENTRY WILL FIND THE MINIMUM AND MAXIMUM VALUES OF AN ARRAY.
C       USE ENTRY TRANGE WHEN YMIN AND YMAX NEED TO BE INITIALIZED
C       EXTERNALLY PRIOR TO CALL.
C
C     INPUT  :  I           -  NUMBER OF ROWS IN Y
C               J           -  NUMBER OF COLUMNS IN Y
C               YMIN        -  MINIMUM VALUE OF Y
C               YMAX        -  MAXIMUM VALUE OF Y
C               Y           -  REAL OR COMPLEX ARRAY
C
```

```
C-----------------------------------------------------------------------
      YMIN=1.E40
      YMAX=-1.E40
      ENTRY TRANGE(I,J,YMIN,YMAX,Y)
      NY = J*I
      DO 100 K=1,NY
      IF(Y(K).LT.YMIN) YMIN = Y(K)
  100 IF(Y(K).GT.YMAX) YMAX = Y(K)
      RETURN
C =====================================================================
      ENTRY TDASHTO(H1,V1,H2,V2,LINE)
C ---------------------------------------------------------------------
C
C     THIS ENTRY WILL DRAW DASH OR DOT LINE FROM (H1,V1) TO (H2,V2).
C       USE ENTRY TDRAWTO FOR A SOLID LINE.
C
C     INPUT  :  H1          -  'FROM' HORIZONTAL COORDINATE
C               V1          -  'FROM' VERTICAL COORDINATE
C               H2          -  'TO' HORIZONTAL COORDINATE
C               V2          -  'TO' VERTICAL COORDINATE
C               LINE        -  LINE TYPE :
C                                1  DOTTED
C                                2  DASH-DOT
C                                3  SHORT-DASHED
C                                4  LONG-DASHED
C ---------------------------------------------------------------------
      CALL MOVEA(H1,V1)
      CALL DASHA(H2,V2,LINE)
      RETURN
C ---------------------------------------------------------------------
      ENTRY TDRAWTO(H1,V1,H2,V2)
C ---------------------------------------------------------------------
      CALL MOVEA(H1,V1)
      CALL DRAWA(H2,V2)
      RETURN
C =====================================================================
      ENTRY TTICKER(H,V)
C ---------------------------------------------------------------------
C
C     THIS ENTRY WILL PUT HORIZONTAL AND VERTICAL TICK-MARKS (CROSSES)
C       AT THE GIVEN WINDOW OR RELATIVE COORDINATES.
C     USE ENTRY TTICKV OR TTICKH FOR VERTICAL OR HORIZONTAL TICKS ONLY.
C     INPUT  :  H           -  HORIZONTAL COORDINATE
```

```
C                   V                   - VERTICAL COORDINATE
C------------------------------------------------------------------------
      DH = H - 7.
      RH = H + 7.
      CALL MOVEA(DH,V)
      CALL DRAWA(RH,V)
C ----------------------------------------------------------------------
      ENTRY TTICKV(H,V)
C ----------------------------------------------------------------------
      DV = V - 7.
      RV = V + 7.
      CALL MOVEA(H,RV)
      CALL DRAWA(H,DV)
      RETURN
C ----------------------------------------------------------------------
      ENTRY TTICKH(H,V)
C ----------------------------------------------------------------------
      DH = H - 7.
      RH = H + 7.
      CALL MOVEA(DH,V)
      CALL DRAWA(RH,V)
      RETURN
C ======================================================================
      ENTRY TCLEAR
C ----------------------------------------------------------------------
C
C     TCLEAR IS USED TO CLEAR THE SCREEN, HOME THE CURSOR, AND ENTER
C       WRITE-MODE.  NO ARGUMENTS ARE NEEDED.
C
C------------------------------------------------------------------------
      CALL ERASE
      CALL HOME
      CALL ANMODE
      RETURN
C ======================================================================
      ENTRY TINIT
C ----------------------------------------------------------------------
C
C     INITIALIZE OR RESET THE PLOTTER.  IT RESETS ALL WINDOW FUNCTIONS.
C       NO ARGUMENTS ARE NEEDED.
C
C ----------------------------------------------------------------------
      CALL INITT(120)
```

```
      CALL TERM(1,1024)
      CALL BINITT
      RETURN
C =====================================================================
      ENTRY TRING
C ---------------------------------------------------------------------
C
C     THREE BELLS AND A POKE SOUNDS THREE BELLS THEN PAUSES UNTIL A
C        'RETURN' OR 'ENTER' RESPONSE FROM THE OPERATOR IS RECEIVED.
C       NO ARGUMENTS ARE NEEDED.
C
C ---------------------------------------------------------------------
      CALL BELL
      CALL BELL
      CALL BELL
      CALL TINPUT(K)
      RETURN
      END
```

```
C =================================================================
      SUBROUTINE TPLOT1(IPLOT,ILINE,NP,XS,XB,YS,YB,IXS,IXF,IYS,IYF,X,Y)
C =================================================================
C
C     THIS IS A SUBROUTINE TO PLOT ONE OR MORE CURVES ON THE SAME GRID.
C       SUBROUTINE ULINE, FOLLOWING BELOW WITHOUT A HEADER, IS A REPLA-
C       CEMENT FOR THE TEKTRONIX SUBROUTINE OF SAME NAME TO PRODUCE
C       THE SAMPLE-AND-HOLD EFFECT WHEN SPECIFIED.
C
C             ORIGINAL      :  AUGUST,      1980
C             REVISION      :  FEBRUARY 23, 1981
C
C             PREPARED BY   :  S. M. DANIEL, G. C. WANG, & I. KERTESZ
C                              RADAR  SYSTEMS ANALYSIS GROUP
C                              MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                              TEMPE, ARIZONA  85282
C -----------------------------------------------------------------
C     INPUT  :    IPLOT      -  PLOT OPTION:
C                               -1  SAME GRID, ADD ANOTHER CURVE
C                                0  USE THE INPUT X AND Y LIMITS
C                                1  COMPUTE THE X AND Y LIMITS
C                 ILINE      -  LINE TYPE:
C                               0  SOLID
C                               1  BROKEN
C                               11 SAMPLE AND HOLD STYLE
C                 NP         -  NUMBER OF POINTS TO PLOT
C                 XS,XB      -  X-GRID LIMITS
C                 YS,YB      -  Y-GRID LIMITS
C                 IXS, IXF   -  X (HORIZONTAL)  GRAPH LIMITS IN SCREEN
C                                 COORDINATES
C                 IYS, IYF   -  Y (VERTICAL)  GRAPH LIMITS IN SCREEN
C                                 COORDINATES
C                 X, Y          DATA TO  BE PLOTTED
C -----------------------------------------------------------------
      DIMENSION X(1),Y(1)
      CALL BINITT
      CALL NPTS(NP)
      IF(IPLOT.EQ.-1) GO TO 3000
      CALL SLIMX(IXS,IXF)
      CALL SLIMY(IYS,IYF)
 1000 IF (IPLOT.EQ.0) GO TO 2000
      CALL TRANGI(NP,1,XS,XB,X)
      CALL TRANGI(NP,1,YS,YB,Y)
```

```
 2000 CALL DLIMX(XS,XB)
      CALL DLIMY(YS,YB)
      CALL CHECK(X,Y)
      CALL LINE(ILINE)
      CALL DSPLAY(X,Y)
      RETURN
 3000 CALL LINE(ILINE)
      CALL NPTS(NP)
      CALL CPLOT(X,Y)
      RETURN
      END
```

```
C ----------------------------------------------------------------------
      SUBROUTINE ULINE(X,Y,I)
C ----------------------------------------------------------------------
      IF(I.NE.1) CALL DRAWA(X,YOLD) $ CALL DRAWA(X,Y)
      YOLD=Y
      RETURN
      END
```

Table 2-20. Functional Description of the Modules Comprising BCRP

```
-------------------------------------------------------------------------------
                   FUNCTIONAL DESCRIPTION OF BCRP SOURCE MODULES
                   ---------------------------------------------

-------------------------------------------------------------------------------
BCRPMAIN:S - MAIN EXECUTIVE PROGRAM.  IT CALLS OTHER EXECUTIVE AND DEDICATED
             SUBPROGRAMS TO COMPUTE DATA INTENDED FOR PLOTTING. THESE DATA
             CHARACTERIZE THE ANTENNA-RECEIVER SYSTEM, SCENARIO, AND THE APP-
             LIED BCR TECHNIQUE.

BCRPSET:S  - EXECUTIVE SUBPROGRAM.  IT IS DESIGNED TO READ FROM THE INPUT DA-
             TA FILE, BCRP:D, USING THE GENERAL INPUT AND OUTPUT SUBROUTINES
             RW, RWIRC, AND SKIPR IN ACCORDANCE TO PRESET FORMAT.  ALL DATA IS
             MADE AVAILABLE TO APPROPRIATE SUBROUTINES THROUGH COMMON BLOCKS.

RW:S       - GENERAL SUBPROGRAM.  (DEFINED IN SIGGEN.)

RWIRC:S    - GENERAL SUBPROGRAM.  (DEFINED IN SIGGEN.)

SKIPR:S    - GENERAL SUBPROGRAM.  (DEFINED IN SIGGEN.)

CMPCHNL:S  - DEDICATED SUBPROGRAM.  THIS SUBPROGRAM WILL COMPUTE THE COMPOSI-
             TE BASEBAND CHANNEL AMPLITUDE RESPONSE BEFORE AND AFTER ADAPTATI-
             ON USING THE ADAPTIVE WEIGHTS SUPPLIED BY THE BCR PROCESS.  IT
             WILL MAKE AVAILABLE THE MAGNITUDE OF THESE AMPLITUDE RESPONSES
             THROUGH THE COMMON BLOCKS.

ANTWT:S    - DEDICATED SUBPROGRAM.  (DEFINED IN SIGGEN.)

FILTER:S   - DEDICATED SUBPROGRAM.  (DEFINED IN SIGGEN.)

CHANNEL:S  - DEDICATED SUBPROGRAM.  (DEFINED IN SIGGEN.)

FIELD:S    - DEDICATED SUBPROGRAM.  THIS SUBPROGRAM WILL COMPUTE THE ELECTRIC
             FIELD PATTERN IN THE NEIGHBORHOOD OF EACH INTERFERENCE BEFORE AND
             AFTER ADAPTATION, AND MAKE AVAILABLE THE MAGNITUDE OF THESE ELEC-
             TRIC FIELDS AND THAT OF THE MAIN BEAM THROUGH COMMON BLOCKS.

PLOTS:S    - EXECUTIVE SUBPROGRAM.  THIS SUBPROGRAM EXTRACTS THE DATA FROM THE
             COMMON BLOCKS, AND COORDINATES THE PLOTTING ACTIVITY THROUGH THE
             DEDICATED PLOTTING ROUTINES.
```

```
ALLPLOT:S  -  DEDICATED SUBPROGRAM.  THIS SUBPROGRAM IS DESIGNED TO BE CALLED
              BY AN EXECUTIVE SUBPROGRAM SUCH AS PLOTS:S TO PLOT ACCORDING TO
              A BUILT-IN FORMAT THE FOLLOWING BCR SIMULATION RESULTS: TITLE-PA-
              GE, EVOLUTION OF THE WEIGHTS, MAIN AND COMPOSITE CHANNEL TRANSFER
              FUNCTION AMPLITUDES BEFORE AND AFTER ADAPTATION, ELECTRIC FIELD
              AMPLITUDES IN THE NEGIHBORHOOD  OF THE INTERFERENCES, AND THE MAIN
              BEAM FIELD MAGNITUDE.  THIS SUBPROGRAM MAKES REFERENCES TO THE
              COMPUTER SYSTEM LIBRARY FOR THE FUNCTION  TIMEDATE, AND THE TEK-
              TRONIX USER LIBRARY.

TTEKS:S    -  GENERAL SUBPROGRAM.  THIS SUBPROGRAM IS A COLLECTION OF UTILITY
              ROUTINES WHICH ARE REFERENCED BY TEKTRONIX PLOTTING-RELATED SUB-
              PROGRAMS.  THIS SUBPROGRAM MAKES REFERENCES TO THE COMPUTER SYS-
              TEM'S  TEK10.:USER LIBRARY.

-------------------------------------------------------------------------------
```

Table 2-21. JCL Program, JCLBCRP:BL

```
!JOB 1269,DANIEL(8512),7,BLDG90
!LIMIT (TIME,2),(UO,40),(CO,35),(ACCOUNT)
!.....JCLBCRP:BL........................
!PCL
C     BCRPMAIN:B.1269          OVER BCRP:B
C       BCRPSET:B.1269
C         RW:B.1269
C         RWIRC:B.1269 SKIPR
C       CMPCHNL:B.1269
C         ANTWT:B.1269
C         FILTER:B.1269
C         CHANNEL:B.1269
C       FIELD:B.1269
*         ANTWT:B.1269
C       PLOTS:B.1269
C         ALLPLOT:B.1269
C           TTEKS:B.1269
*         TTEKS:B.1269
!LYNX BCRP:B                   OVER BCRP:L ; :TEK10.:USER
```

After this, the command

!BCRP:L.

carries out the execution. Only the graphical activity will be apparent on the terminal. As each page or picture is completed, a series of 3 closely spaced but distinct beeps will be heard, followed by a pause. If a permanent (hard-copy) is desired, it should be made at this time, since the picture will remain on the screen until the proper response is given by the operator. The required response is actuating the "RETURN" or "ENTER" key. The 3 beeps constitute a warning to copy the screen or lose it.

After the proper response, the screen is blanked and a new picture begins to form, again followed by 3 beeps when completed. When all pictures have been drqwn, the word "EXIT" appears, meaning the execution is properly terminated. The complete graphical output obtained this way for the bench-test example is shown in Figure 2-8.

2.4.4 Output File - BCRP:O

The output data file, BCRP:O is shown in Table 2-22. It was produced by executing the program BCRP:L with the input data BCRP:D. Note that the front part of the BCRP:O output is simply a repetition of the input data file BCRPLD. This is followed by data to be plotted, as shown in Figure 2-8.

The title page of the CRT-plots identifies the program and the date and time of execution. This ties the pictures to the printed data, since that, too, is dated by the operating system. This is important because the system, scenario, and BCR description are not always discernible from these pictures.

The next frame contains the original and combined signal plots. These are actually the amplitudes of the Port 0 signal, and the combined port signal, respectively. The signals were read as part of the input data and the amplitudes were found in BCRPSET via an appropriately simple operation.

The next frame presents the two-step iterative evolution of the adaptive weights in a common complex plane. Initial value and weight iterates, $\underline{w}^0 = 0$, $\underline{w}^1$ and $\underline{w}^2$ are read from the "composite weight-vector array" input. Since $\underline{w}^0 = \underline{0}$, all weight-vector components begin at the origin, progress in various directions from there according to w^1 and terminate at w^2. The breaks in the weight-component loci indicate their values at the first iteration. The end points constitute their values at the second iteration.

The channel and field graphs are plotted in groups of four per frame. As such, in general, more than one frame may be needed for these graphs. The main and composite channel transfer function amplitudes are plotted for each interference. In these and subsequent plots, the solid line indicates the original response while the dotted one stands for the adapted one. The abscissa is the normalized radian frequency range, and the values in dB below each plot indicate the amplitude levels before and after adaptation at the center frequency. The data for these plots are produced by CMPCHNL.

Title Page

BCR ADAPTIVE PROCESSING

BENCHTEST EXAMPLE

(See Figure 2-2)

SOURCES	:	2 (10°, 45°)
RF BANDWIDTH	:	0.1%
AUXILIARY PORTS	:	4
TAP WEIGHTS	:	1/PORT

SIMULATION BY : S. M. DANIEL & I. KERTESZ
ADVANCED TECHNOLOGY AND SYSTEMS ANALYSIS
MOTOROLA, INC. - GED

13:03 FEB 14, '81

Figure 2-8

Relative Signal Amplitude Before and After BCR Adaptation

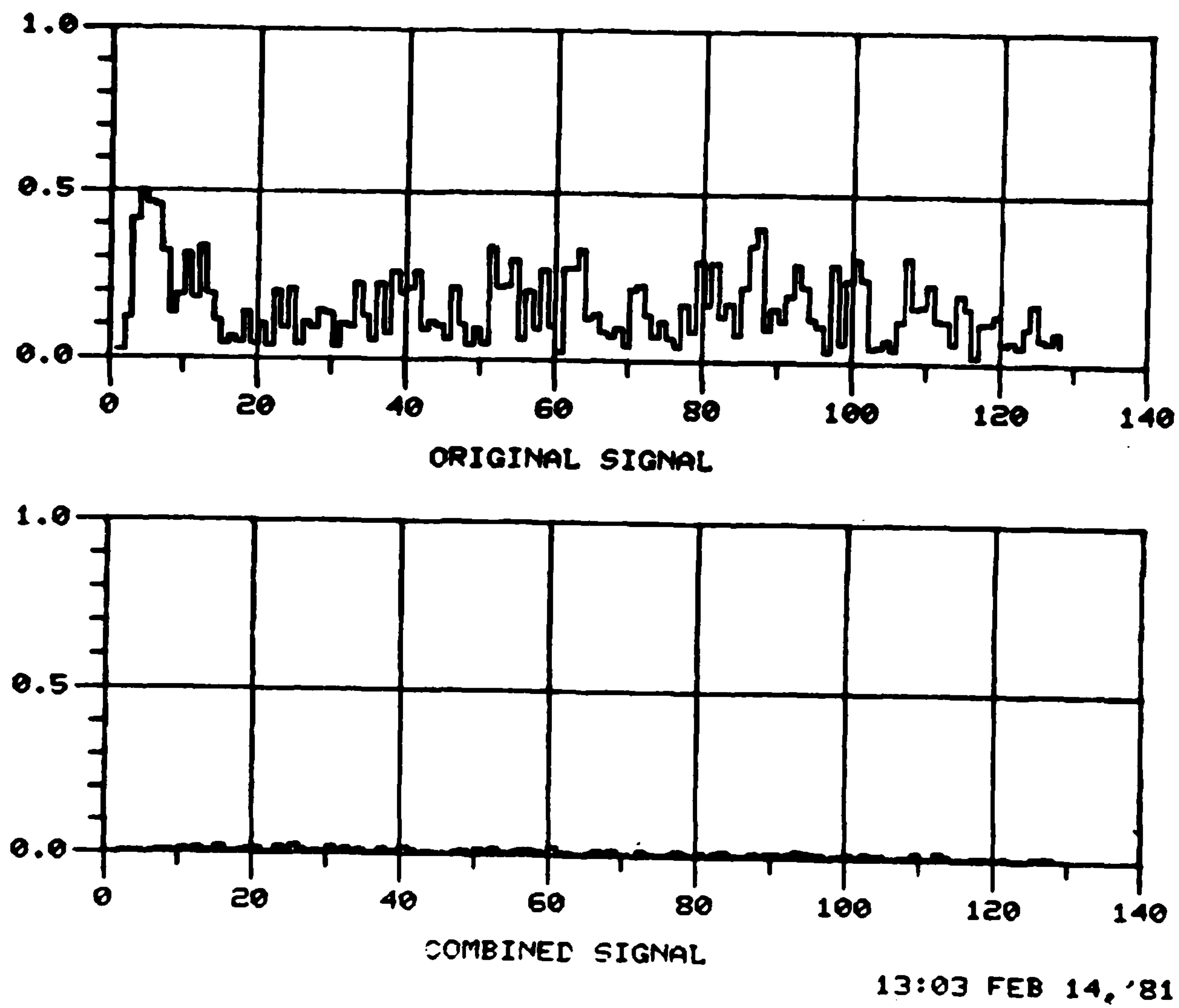


Figure 2-8 (Cont'd)

BCR Adaptive Weight Evolution in the Complex Plane

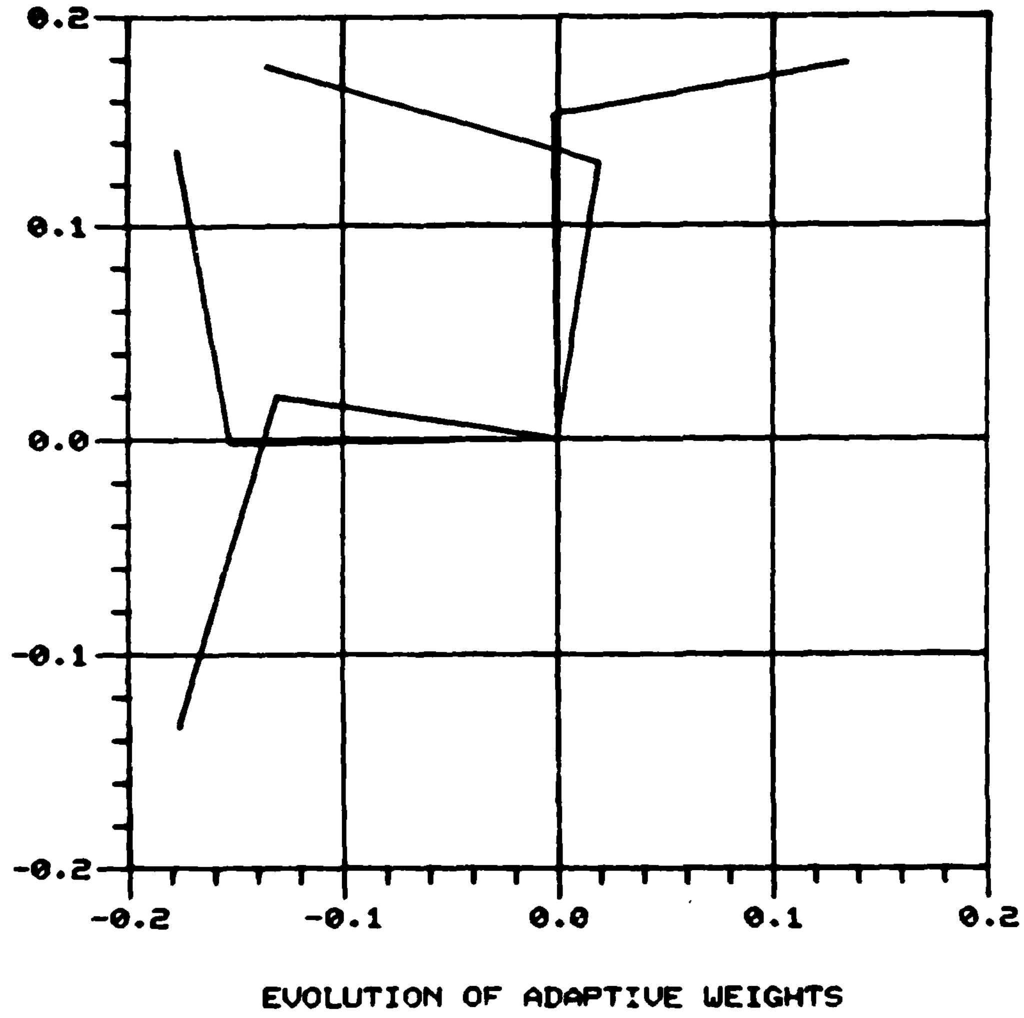


Figure 2-8 (Cont'd) 13:03 FEB 14,'81

Source Transfer Function Amplitude Responses Before and After BCR Adaptation

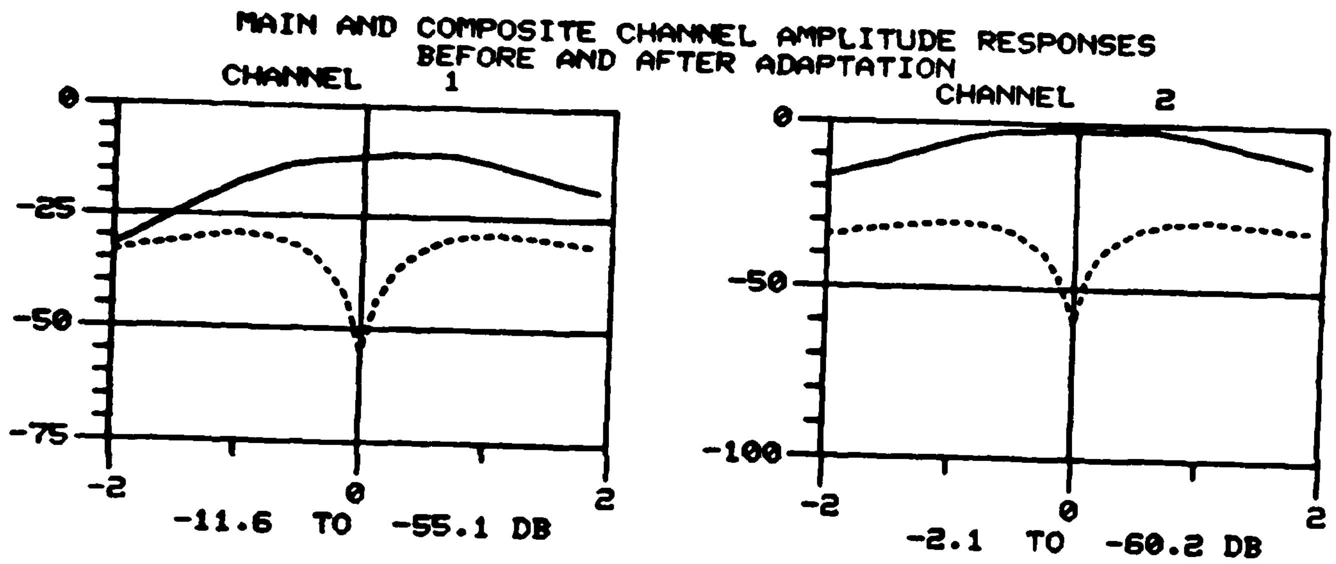
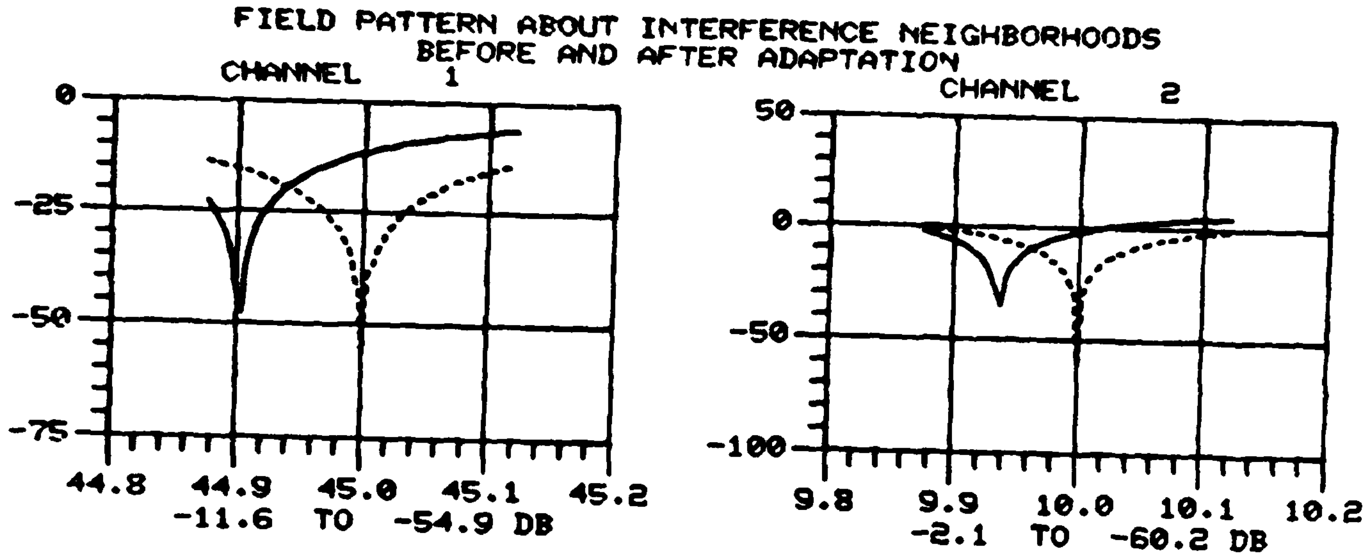


Figure 2-8 (Cont'd)

13:03 FEB 14, '81

Field Pattern Amplitudes Over 0.25 Neighborhoods About Source Angles of Arrival at RF Center



13:03 FEB 14, '81

Main Beam Field Pattern Amplitude at Center RF Frequency, Before and After BCR Adaptation

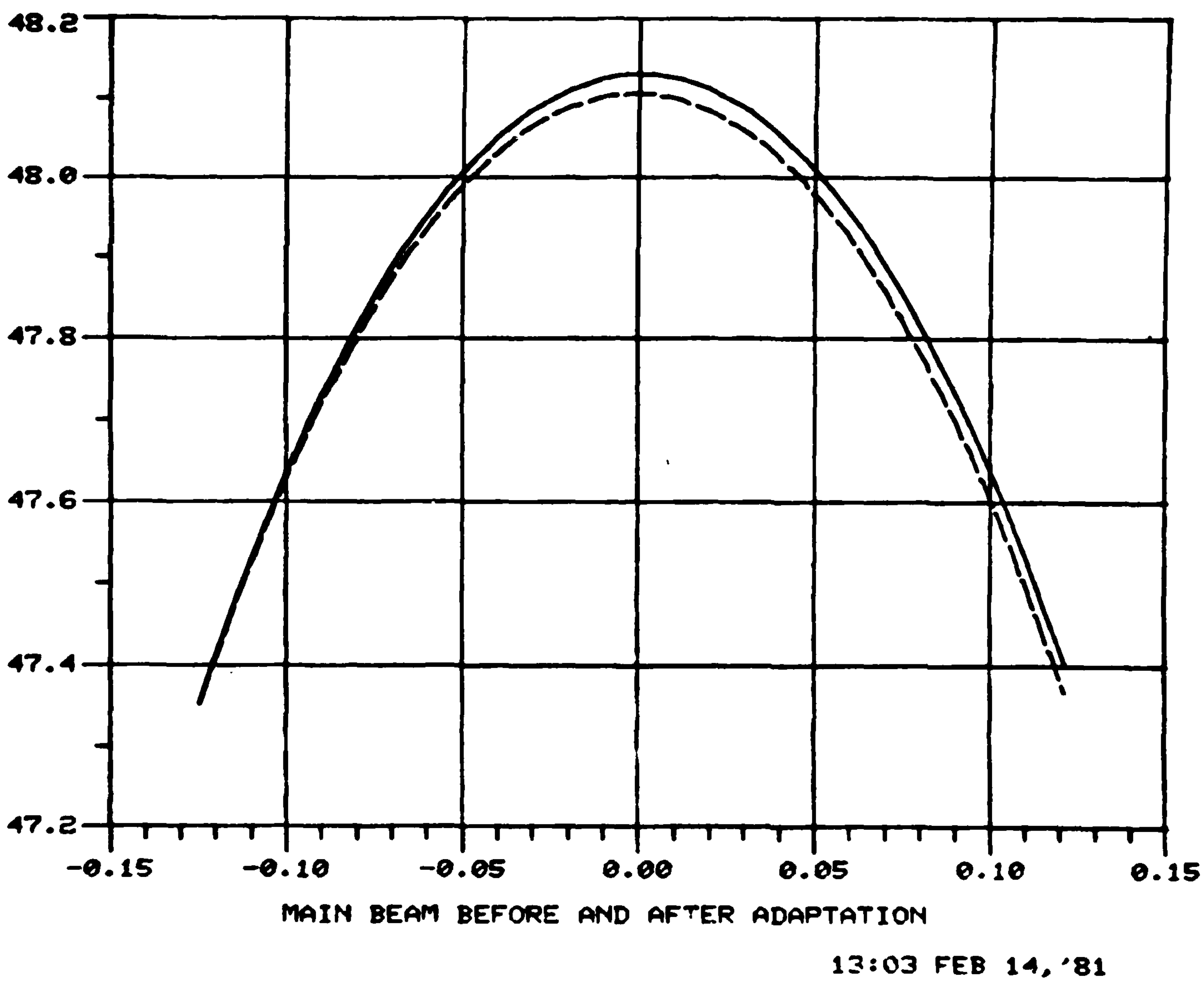


Table 2-22. Output Data File of the BCR Plotting Program, BCRP:0 Benchtest Example (See Figure 2-2)

```
------------------------------------------------------------------------------
                    DIGITAL ADAPTIVE ARRAY PROCESSING
                    ---------------------------------
                                 USING
                                 -----
                       BATCH COVARIANCE RELAXATION
                       ---------------------------
------------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWS0   -  OPTIONAL OUTPUT OF  S0                 :          1
  IWS1   -  OPTIONAL OUTPUT OF  S1                 :          0
  IWCR   -  OPTIONAL OUTPUT OF  C   AND  B         :          1
BCR PARAMETERS
--------------
  NSX    -  MAXIMUM NUMBER OF SIGNAL SAMPLES       :        256
  IWO    -  INITIAL WEIGHT-VALUE OPTION            :          0
  NWX    -  MAXIMUM NUMBER OF WEIGHTS              :         20
  ISW    -  WEIGHT SELECTOR ARRAY                  :    1 1 1 1 0 0 0 0 0 0
                                                   :    0 0 0 0 0 0 0 0 0 0
  ITERX  -  MAXIMUM NUMBER OF ITERATIONS           :         20
------------------------------------------------------------------------------
  ITER   -  ACTUAL NUMBER OF ITERATIONS            :          2
------------------------------------------------------------------------------
```

```
---------------------------------------------------------------------------
                    SPECIFICATION OF SYSTEM PARAMETERS
                    ----------------------------------

---------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWANT   -  MAIN ANTENNA ARRAY WEIGHTING                :        0
  IWRAP   -  RECEIVER AMPLITUDE AND PHASE                :        0
  IWRI    -  RECEIVER IMPULSE RESPONSE                   :        0
  IWCAP   -  CHANNEL AMPLITUDE AND PHASE                 :        0
  IWCI    -  CHANNEL IMPULSE RESPONSE                    :        0
  IWSC    -  INDIVIDUAL CHANNEL SIGNALS                  :        0

FILTER PARAMETERS
-----------------
  NPOL    -  NUMBER OF LOWPASS PROTOTYPE POLES           :        2
             POL(1)                                      :  -.70700   -.70700
             POL(2)                                      :  -.70700    .70700
  FBIF    -  FRACTIONAL BANDWIDTH AT FINAL IF            :   .10000
  FBRF    -  FRACTIONAL BANDWIDTH AT RF                  :   .00100
  RADRNG  -  NORMALIZED RADIAN FREQUENCY RANGE           :  4.00000
  RADIN   -  NORMALIZED INITIAL RADIAN FREQUENCY         : -2.00000
  NF      -  NUMBER OF FREQUENCY SAMPLES                 :       32
  LPBP    -  LOWPASS/BANDPASS OPTION                     :        1
             0  :   LOWPASS
             1  :   BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS  -  NUMBER OF CHANNELS PER PORT                 :       20
  ISC     -  CHANNEL SELECTOR ARRAY                      :  1 1 0 0 0 0 0 0 0 0
                                                         :  0 0 0 0 0 0 0 0 0 0

CHANNEL  1:
  AN      -  AMPLITUDE OF NOISE SOURCE                   :  1.00000
  IX0     -  INITIAL RANDU SETTING                       :        1
  N1      -  FIRST-TIME-ON SAMPLE NUMBER                 :        1
  NB      -  BLINK DURATION IN SAMPLES                   :    10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)           : 45.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS          :   .00000
```

```
CHANNEL   2:
   AN      -  AMPLITUDE OF NOISE SOURCE                    :     1.00000
   IX0     -  INITIAL RANDU SETTING                        :          11
   N1      -  FIRST-TIME-ON SAMPLE NUMBER                  :           1
   NB      -  BLINK DURATION IN SAMPLES                    :       10000
   TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)            :    10.00000
   CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS           :      .10000

PORT PARAMETERS
---------------
   D0      -  ANTENNA-ELEMENT SEPARATION FACTOR            :     1.00000
   NS      -  NUMBER OF SIGNAL SAMPLES                     :         128
   NPORTS  -  NUMBER OF PORTS                              :          21
   ISP     -  PORT SELECTOR ARRAY                          :     1 1 1 1 0 0 0 0 0 0
                                                           :     0 0 0 0 0 0 0 0 0 0

PORT      0:
   NEL     -  NUMBER OF ANTENNA ELEMENTS                   :         255
   LOC1    -  LOCATION OF THE FIRST ELEMENT                :           1
   BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                   :     1.00000
   BWOFF   -  BANDWIDTH OFFSET FACTOR                      :      .00000
   PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY           :      .00000

PORT      1:
   NEL     -  NUMBER OF ANTENNA ELEMENTS                   :           1
   LOC1    -  LOCATION OF THE FIRST ELEMENT                :           1
   BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                   :     1.00000
   BWOFF   -  BANDWIDTH OFFSET FACTOR                      :      .00000
   PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY           :      .00000

PORT      2:
   NEL     -  NUMBER OF ANTENNA ELEMENTS                   :           1
   LOC1    -  LOCATION OF THE FIRST ELEMENT                :           1
   BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                   :     1.00000
   BWOFF   -  BANDWIDTH OFFSET FACTOR                      :      .00000
   PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY           :      .10000

PORT      3:
   NEL     -  NUMBER OF ANTENNA ELEMENTS                   :           1
   LOC1    -  LOCATION OF THE FIRST ELEMENT                :         255
   BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                   :     1.00000
   BWOFF   -  BANDWIDTH OFFSET FACTOR                      :      .00000
   PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY           :      .00000
```

```
PORT      4:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :        255
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :     .10000
------------------------------------------------------------------------------
```

PORT 0

RECEIVED BASEBAND PORT SIGNAL

NORMALIZATION CONSTANT : .359881E 00

.05464	.03818	-.27937	-.18505	-.80856	.80400	-.96813	-1.00000
-.89125	-.93809	-.86588	-.91810	-.51393	-.71682	-.01192	-.37048
.51798	.07065	.69959	.48862	.33408	.37157	-.72328	-.56526
-.48074	-.23750	-.28361	.13506	-.05666	.10339	.17505	.00266
-.06351	-.11283	-.27575	-.25835	-.11409	.03865	.07953	.27617
-.00055	-.11477	.45163	.31794	-.25094	-.05660	-.53162	-.23005
.10719	-.05537	.29689	.03985	-.22164	-.13245	-.32176	-.22894
-.37584	.00990	-.03879	.09208	.23239	.18048	-.03938	.26500
-.60316	-.16595	-.22785	-.27799	.15061	.04556	.49787	.37387
.05925	.20008	-.67018	-.26714	-.51800	-.17553	.41521	.42498
.46481	.55190	-.24892	.00096	-.22334	-.22122	-.17884	-.20945
-.14507	-.08280	-.48178	-.33350	-.15841	-.23864	.03017	.12234
-.24554	.07161	.07142	.10982	.79057	.48893	.42141	.42662
-.56044	-.25490	-.64559	-.51348	.04592	.16575	.45400	.33688
-.15341	-.20463	-.60188	-.43149	-.23294	.15095	-.05292	.03788
.70182	.26963	.66574	.35405	.76554	.50367	.33146	.11205
.34201	.16069	-.22889	.04207	-.10227	.16877	.08978	.24863
.07344	-.10329	.46118	.36153	.51865	.34286	.17534	.33441
-.01579	.20465	.13439	.29129	-.17332	-.08765	.00878	.12494
-.21703	-.41773	.06459	-.23583	-.57301	-.59389	-.25661	-.38524
-.71106	-.39012	-.17869	-.31977	.16879	.45005	-.15971	.15446
.40747	.45486	.68229	.67181	.68674	.85576	-.08121	.25857
.30585	.30440	.19328	.26502	-.49591	-.22000	-.67983	-.41589
-.30492	-.53856	.35302	.05498	-.29988	.01301	-.08097	-.02180
.53397	.59253	-.08945	.13345	-.56349	-.37762	-.77427	-.36070
-.67043	-.15846	-.09813	.07619	-.07995	.11142	-.19310	-.00091
-.09846	-.05802	-.32365	-.11190	-.70209	-.51642	-.42425	-.15954
.35979	.29670	.55865	.31600	.34771	-.05137	.29572	.17155
-.02046	.16899	-.40334	-.40163	-.33264	-.31392	.02263	.05234
-.29165	-.15834	.20012	.26848	.21792	.33672	.14525	.09035
.12944	.14092	-.11299	.08525	.11258	.27773	.18868	.44658
.08274	.17843	.09463	.15595	-.23632	-.08516	.11600	-.09952

COMPOSITE WEIGHT-VECTOR ARRAY

ADAPTIVE WEIGHT VECTOR AT INDICATED BCH ITERATION
NORMALIZATION CONSTANT : .177713E 00

.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
-.73410	.11073	.10958	.73308	-.86029	-.01010	-.01218	.86120
-.99367	-.75509	-.76257	.99317	-.99829	.76661	.75528	1.00000

COMBINED PORT

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .230309E-01

-.03888	-.01323	.19071	.17610	.06099	.16573	.05320	-.07275
.02968	-.00799	.07461	-.12495	.26860	-.17760	.18883	-.25049
-.11842	-.07928	-.48937	-.04960	.14693	.37919	-.66575	.31826
-.14744	-.27502	-.10648	.01617	.88963	-.26945	.09181	.11671
-.20862	.03532	-.23092	.15384	-.39795	-.22437	.61231	.05531
.04496	-.09826	.08906	-.02994	-.77621	.44229	.26915	-.25769
1.00000	-.23204	-.54974	.16137	-.19118	.13871	-.41202	.07177
-.23740	-.08672	.76412	-.18493	-.26971	.04424	-.59677	.20878
.39803	.03190	.52495	-.19608	-.15468	-.12987	.12703	-.07027
-.60785	.39567	-.33618	.15351	.12069	-.26231	.69092	-.37503
-.45617	.32986	.13514	.06595	.31843	.00235	-.15037	-.04658
-.08588	.07293	-.00856	.05025	.19804	-.18646	-.56763	.10918
-.02754	.00689	.61948	-.29271	.31554	-.12729	-.74146	.40364
-.02025	.26382	-.16347	-.12197	.31903	-.39370	.64548	.03260
-.60283	.43451	-.57496	-.07616	-.00654	-.11513	.98510	-.26494
.30998	-.11339	-.33380	.00750	.14768	.03202	-.08365	.18227
-.38708	.05156	-.66419	.28229	.17821	-.30714	.66224	.11530
-.09250	-.10721	.10204	-.11146	-.03253	.06137	-.73031	.26788
.21265	-.16106	.18602	.13517	-.12473	-.02217	.33990	-.04699
.62998	-.01110	-.36409	.03137	.00500	.15949	-.43009	-.09470
-.06632	.18151	.18172	-.40263	-.72752	.20569	.37521	-.14097
.15484	-.11836	.02065	-.12918	-.38379	.26239	.00183	.11941
.55595	-.24317	-.29769	.32131	-.31891	.13693	.45635	-.03862
.75436	-.31485	-.70351	.06789	-.52017	.21701	.37759	-.30430
-.07877	-.01852	.01811	.33523	-.27060	.12152	-.52439	.08092
.19224	-.25177	.60273	-.14255	-.44537	.09207	.46715	-.07034
-.20111	.04277	.10832	.11553	-.40186	.15682	.09062	-.36923
.79020	-.25614	.24858	.04701	.00931	.11621	-.92485	.05844
.42459	.09243	-.11328	.20698	-.07141	-.23948	.07394	.06588
-.37167	.03991	.17203	-.25067	.10553	.13580	.17998	-.05091
-.35735	.14106	-.22756	.00886	-.04484	-.09309	.06001	.00578
.34456	.01564	-.25236	.07364	.32527	.03103	.12031	-.19236

SUPPRESSION (DB) : 25.23689

TAYLOR WEIGHTING FOR MAIN ANTENNA ARRAY

.50004	.50034	.50095	.50186	.50307	.50458	.50640	.50851
.51093	.51364	.51664	.51994	.52353	.52741	.53157	.53602
.54076	.54577	.55105	.55661	.56244	.56853	.57489	.58150
.58837	.59549	.60285	.61046	.61830	.62637	.63467	.64319
.65193	.66088	.67004	.67939	.68894	.69868	.70860	.71870
.72897	.73941	.75000	.76074	.77163	.78266	.79382	.80511
.81651	.82802	.83964	.85136	.86317	.87506	.88702	.89906
.91115	.92330	.93550	.94773	.96000	.97229	.98460	.99692
1.00924	1.02155	1.03385	1.04613	1.05838	1.07060	1.08278	1.09490
1.10697	1.11897	1.13090	1.14275	1.15451	1.16618	1.17774	1.18920
1.20055	1.21177	1.22287	1.23383	1.24465	1.25531	1.26583	1.27618
1.28637	1.29638	1.30621	1.31585	1.32531	1.33456	1.34362	1.35246
1.36109	1.36950	1.37769	1.38565	1.39337	1.40086	1.40810	1.41509
1.42183	1.42832	1.43454	1.44051	1.44620	1.45162	1.45677	1.46164
1.46624	1.47054	1.47457	1.47830	1.48175	1.48490	1.48776	1.49032
1.49258	1.49454	1.49621	1.49757	1.49863	1.49939	1.49985	1.50000
1.49985	1.49939	1.49863	1.49757	1.49621	1.49454	1.49258	1.49032
1.48776	1.48490	1.48175	1.47830	1.47457	1.47054	1.46624	1.46164
1.45677	1.45162	1.44620	1.44051	1.43454	1.42832	1.42183	1.41509
1.40810	1.40086	1.39337	1.38565	1.37769	1.36950	1.36109	1.35246
1.34362	1.33456	1.32531	1.31585	1.30621	1.29638	1.28637	1.27618
1.26583	1.25531	1.24465	1.23383	1.22287	1.21177	1.20055	1.18920
1.17774	1.16618	1.15451	1.14275	1.13090	1.11897	1.10697	1.09490
1.08278	1.07060	1.05838	1.04613	1.03385	1.02155	1.00924	.99692
.98460	.97229	.96000	.94773	.93550	.92330	.91115	.89906
.88702	.87506	.86317	.85136	.83964	.82802	.81651	.80511
.79382	.78266	.77163	.76074	.75000	.73941	.72897	.71870
.70860	.69868	.68894	.67939	.67004	.66088	.65193	.64319
.63467	.62637	.61830	.61046	.60285	.59549	.58837	.58150
.57489	.56853	.56244	.55661	.55105	.54577	.54076	.53602
.53157	.52741	.52353	.51994	.51664	.51364	.51093	.50851
.50640	.50458	.50307	.50186	.50095	.50034	.50004	

BASEBAND CHANNEL AMPLITUDE RESPONSE 1

MAIN-PORT BASEBAND CHANNEL AMPLITUDE RESPONSE

-32.00455	-30.16212	-28.35980	-26.52783	-24.70956	-22.88368	-21.04797	-19.30652
-17.64091	-16.16315	-14.87927	-13.88285	-13.13818	-12.61402	-12.23330	-11.88884
-11.59961	-11.33837	-11.07288	-10.89269	-10.80316	-10.89853	-11.19811	-11.72890
-12.44590	-13.30921	-14.25012	-15.21103	-16.18156	-17.13153	-18.03403	-18.89853

COMPOSITE BASEBAND CHANNEL AMPLITUDE RESPONSE

-33.21780	-32.61180	-32.03041	-31.42552	-30.83665	-30.28641	-29.71341	-29.37326
-29.15300	-29.29034	-29.68282	-30.61176	-32.03050	-34.27242	-37.79625	-42.71439
-55.10971	-43.46461	-38.01500	-34.38458	-32.18727	-30.56454	-29.54636	-28.97539
-28.76286	-28.80223	-28.97453	-29.37656	-29.73537	-30.14111	-30.61868	-31.08455

BASEBAND CHANNEL AMPLITUDE RESPONSE 2

MAIN-PORT BASEBAND CHANNEL AMPLITUDE RESPONSE

-16.65526	-15.43969	-14.17362	-12.85739	-11.49469	-10.09601	-8.68230	-7.28719
-5.96633	-4.79063	-3.82824	-3.12086	-2.65897	-2.38732	-2.23139	-2.13645
-2.05608	-1.97848	-1.91513	-1.90377	-1.99607	-2.25224	-2.71445	-3.39295
-4.25410	-5.24691	-6.31034	-7.40009	-8.48114	-9.53406	-10.54722	-11.51543

COMPOSITE BASEBAND CHANNEL AMPLITUDE RESPONSE

-34.11560	-33.55356	-32.97908	-32.39885	-31.82541	-31.27599	-30.77669	-30.40169
-30.21181	-30.29446	-30.78613	-31.78796	-33.41043	-35.85718	-39.78143	-46.53316
-60.15071	-43.43925	-38.03241	-34.81860	-32.58838	-31.04456	-30.03014	-29.48505
-29.26477	-29.33008	-29.54668	-29.88869	-30.28293	-30.71077	-31.14575	-31.58029

FIELD PATTERN BEFORE AND AFTER BCR ADAPTATION ABOUT INTERFERENCE 1

MAIN FIELD PATTERN

44.87498	-22.91161	44.87889	-24.32169	44.88280	-26.02716	44.88670	-28.07201
44.89061	-30.58412	44.89452	-35.12675	44.89842	-43.23663	44.90233	-47.90111
44.90623	-37.76054	44.91014	-32.07393	44.91405	-28.60013	44.91795	-26.51216
44.92186	-24.73946	44.92577	-23.33789	44.92967	-22.00999	44.93358	-20.88618
44.93748	-19.96809	44.94139	-19.09592	44.94530	-18.28661	44.94920	-17.57083
44.95311	-16.91776	44.95702	-16.31866	44.96092	-15.72820	44.96483	-15.22130
44.96873	-14.71785	44.97264	-14.34896	44.97655	-13.82789	44.98045	-13.37713
44.98436	-13.01956	44.98827	-12.62630	44.99217	-12.26140	44.99608	-11.94249
44.99998	-11.59961	45.00389	-11.44618	45.00780	-11.01364	45.01170	-10.73277
45.01561	-10.45701	45.01952	-10.23641	45.02342	-9.96329	45.02733	-9.72684
45.03123	-9.48957	45.03514	-9.26974	45.03905	-9.06605	45.04295	-8.86177
45.04686	-8.66108	45.05077	-8.49413	45.05467	-8.29583	45.05858	-8.09089
45.06248	-7.96554	45.06639	-7.80245	45.07030	-7.65774	45.07420	-7.50201
45.07811	-7.36855	45.08202	-7.23990	45.08592	-7.07059	45.08983	-6.96996
45.09373	-6.84608	45.09764	-6.73148	45.10155	-6.61768	45.10545	-6.51336
45.10936	-6.40608	45.11327	-6.31509	45.11717	-6.21940	45.12108	-6.12637

COMPOSITE FIELD PATTERN

44.87498	-13.69902	44.87889	-13.93719	44.88280	-14.18113	44.88670	-14.45752
44.89061	-14.76876	44.89452	-14.99607	44.89842	-15.30442	44.90233	-15.61294
44.90623	-16.03488	44.91014	-16.31573	44.91405	-16.59506	44.91795	-17.00180
44.92186	-17.41490	44.92577	-17.86496	44.92967	-18.28619	44.93358	-18.72659
44.93748	-19.26300	44.94139	-19.82639	44.94530	-20.36674	44.94920	-21.01154
44.95311	-21.68538	44.95702	-22.43054	44.96092	-23.21063	44.96483	-24.16283
44.96873	-25.14122	44.97264	-26.69908	44.97655	-27.66263	44.98045	-29.03015
44.98436	-31.18027	44.98827	-33.46918	44.99217	-36.64508	44.99608	-42.34927
44.99998	-54.94867	45.00389	-38.65898	45.00780	-37.40787	45.01170	-33.83502
45.01561	-31.38101	45.01952	-29.09464	45.02342	-27.75264	45.02733	-26.40709
45.03123	-25.30199	45.03514	-24.28693	45.03905	-23.35655	45.04295	-22.53894
45.04686	-21.83128	45.05077	-21.06075	45.05467	-20.49840	45.05858	-20.04282
45.06248	-19.33177	45.06639	-18.83263	45.07030	-18.32764	45.07420	-17.89638
45.07811	-17.44212	45.08202	-17.01955	45.08592	-16.74706	45.08983	-16.31133
45.09373	-15.96855	45.09764	-15.63376	45.10155	-15.32909	45.10545	-15.01738
45.10936	-14.74584	45.11327	-14.45187	45.11717	-14.19274	45.12108	-13.95106

FIELD PATTERN BEFORE AND AFTER HCR ADAPTATION ABOUT INTERFERENCE 2

MAIN FIELD PATTERN

9.87499	-2.32133	9.87890	-2.86640	9.88281	-3.45642	9.88671	-4.09233
9.89062	-4.78596	9.89452	-5.54609	9.89843	-6.38737	9.90234	-7.32136
9.90625	-8.37942	9.91015	-9.58602	9.91406	-10.99620	9.91796	-12.68318
9.92187	-14.79176	9.92578	-17.58418	9.92968	-21.75583	9.93359	-29.92409
9.93749	-34.78946	9.94140	-23.30136	9.94531	-18.53484	9.94921	-15.47561
9.95312	-13.23025	9.95703	-11.45302	9.96093	-9.98432	9.96484	-8.73459
9.96875	-7.65038	9.97265	-6.68963	9.97656	-5.83354	9.98046	-5.05833
9.98437	-4.35577	9.98828	-3.70984	9.99218	-3.11591	9.99609	-2.56558
10.00000	-2.05608	10.00390	-1.57937	10.00781	-1.13508	10.01171	-.71762
10.01562	-.32580	10.01953	.03987	10.02343	.38935	10.02734	.71681
10.03124	1.02605	10.03515	1.31871	10.03906	1.59518	10.04296	1.85667
10.04687	2.10461	10.05078	2.33906	10.05468	2.56164	10.05859	2.77231
10.06250	2.97135	10.06640	3.15682	10.07031	3.33700	10.07421	3.50537
10.07812	3.66414	10.08203	3.81384	10.08593	3.95449	10.08984	4.08673
10.09374	4.21071	10.09765	4.32641	10.10156	4.43475	10.10546	4.53541
10.10937	4.62833	10.11328	4.71412	10.11718	4.79284	10.12109	4.86454

COMPOSITE FIELD PATTERN

9.87499	-.28364	9.87890	-.49616	9.88281	-.72164	9.88671	-.95683
9.89062	-1.20567	9.89452	-1.46780	9.89843	-1.74505	9.90234	-2.03609
9.90625	-2.34495	9.91015	-2.66968	9.91406	-3.01394	9.91796	-3.37766
9.92187	-3.76444	9.92578	-4.17463	9.92968	-4.61470	9.93359	-5.07532
9.93749	-5.57296	9.94140	-6.10863	9.94531	-6.68376	9.94921	-7.30707
9.95312	-7.98248	9.95703	-8.72061	9.96093	-9.53418	9.96484	-10.43733
9.96875	-11.44837	9.97265	-12.60512	9.97656	-13.93977	9.98046	-15.52864
9.98437	-17.46994	9.98828	-19.99118	9.99218	-23.55244	9.99609	-29.71161
10.00000	-60.15176	10.00390	-29.21854	10.00781	-23.33549	10.01171	-19.85855
10.01562	-17.39069	10.01953	-15.49476	10.02343	-13.92153	10.02734	-12.60655
10.03124	-11.47183	10.03515	-10.47363	10.03906	-9.58627	10.04296	-8.78813
10.04687	-8.06252	10.05078	-7.40029	10.05468	-6.78979	10.05859	-6.22598
10.06250	-5.70394	10.06640	-5.22526	10.07031	-4.76557	10.07421	-4.33998
10.07812	-3.94089	10.08203	-3.56553	10.08593	-3.21259	10.08984	-2.87947
10.09374	-2.56502	10.09765	-2.26845	10.10156	-1.98689	10.10546	-1.72078
10.10937	-1.46932	10.11328	-1.23112	10.11718	-1.00533	10.12109	-.79169

MAIN BEAM PATTERN BEFORE AND AFTER BCR ADAPTATION

MAIN FIELD PATTERN

-.12500	47.35248	-.12109	47.40089	-.11719	47.44769	-.11328	47.49290
-.10937	47.53650	-.10547	47.57854	-.10156	47.61899	-.09766	47.65788
-.09375	47.69519	-.08984	47.73094	-.08594	47.76514	-.08203	47.79778
-.07812	47.82887	-.07422	47.85843	-.07031	47.88644	-.06641	47.91292
-.06250	47.93787	-.05859	47.96129	-.05469	47.98318	-.05078	48.00356
-.04687	48.02240	-.04297	48.03973	-.03906	48.05554	-.03516	48.06985
-.03125	48.08266	-.02734	48.09395	-.02344	48.10371	-.01953	48.11198
-.01562	48.11876	-.01172	48.12401	-.00781	48.12778	-.00391	48.13004
.00000	48.13077	.00391	48.13004	.00781	48.12778	.01172	48.12402
.01562	48.11876	.01953	48.11198	.02344	48.10371	.02734	48.09395
.03125	48.08266	.03516	48.06985	.03906	48.05554	.04297	48.03973
.04687	48.02240	.05078	48.00356	.05469	47.98318	.05859	47.96129
.06250	47.93787	.06641	47.91292	.07031	47.88644	.07422	47.85843
.07812	47.82887	.08203	47.79778	.08594	47.76514	.08984	47.73094
.09375	47.69519	.09766	47.65788	.10156	47.61899	.10547	47.57854
.10937	47.53650	.11328	47.49290	.11719	47.44769	.12109	47.40089

COMPOSITE FIELD PATTERN

-.12500	47.35086	-.12109	47.39839	-.11719	47.44431	-.11328	47.48865
-.10937	47.53140	-.10547	47.57260	-.10156	47.61223	-.09766	47.65027
-.09375	47.68678	-.08984	47.72174	-.08594	47.75516	-.08203	47.78703
-.07812	47.81735	-.07422	47.84616	-.07031	47.87343	-.06641	47.89919
-.06250	47.92343	-.05859	47.94614	-.05469	47.96733	-.05078	47.98703
-.04687	48.00522	-.04297	48.02190	-.03906	48.03708	-.03516	48.05077
-.03125	48.06294	-.02734	48.07362	-.02344	48.08281	-.01953	48.09052
-.01562	48.09671	-.01172	48.10143	-.00781	48.10464	-.00391	48.10638
.00000	48.10663	.00391	48.10536	.00781	48.10265	.01172	48.09840
.01562	48.09270	.01953	48.08549	.02344	48.07680	.02734	48.06660
.03125	48.05492	.03516	48.04176	.03906	48.02707	.04297	48.01091
.04687	47.99324	.05078	47.97406	.05469	47.95338	.05859	47.93121
.06250	47.90752	.06641	47.88232	.07031	47.85561	.07422	47.82735
.07812	47.79759	.08203	47.76630	.08594	47.73349	.08984	47.69913
.09375	47.66322	.09766	47.62578	.10156	47.58679	.10547	47.54626
.10937	47.50415	.11328	47.46049	.11719	47.41525	.12109	47.36842

The main and composite field amplitudes over a 0.25° sector about the incident angle of each interference are then plotted in dB. The numerical quantities below each individual snapshot gives the field amplitude in dB before and after adaptation. The data in these plots are produced by FIELD.

The last graph is also produced by FIELD. It is a plot of the main beam amplitude before and after adaptation.

3.0 COMPUTER SIMULATION RESULTS

The three structured programs, SIGGEN, BCRS and BCRP, described in the previous section provide the basis for investigating the BCR adaptive processing performance in a variety of interference environments and system configurations. The interference environment may consist of wideband continuous noise sources, periodic blinking sources with adjustable period, multipath sources or combinations thereof. The system configuration that may be accommodated includes an RF bandwidth capability of up to 10%, a Taylor-weighted main antenna array of up to 1001 omnidirectional elements, up to 20 omnidirectional auxiliary antenna elements nonuniformity emplaced over the main aperture, and up to 20 complex adaptive weights in single or multitap arrangements. The limit of 20 is due to the dimensionality in the programs, an intentional measure taken to keep the program requirement relatively small. If desired, this number may be increased by appropriately altering certain array dimensionalities as has been done when a case of 100 adaptive weights was exercised.

Presented below is a number of specific examples demonstrating detailed BCR adaptive performance under certain environmental and system conditions. General results derived from a larger set of examples are summarized next. This section concludes with some comments on BCR variations.

3.1 Specific Examples

Of the numerical examples included in this section, only the first is presented in exhaustive detail, both numerical and graphical. To economize on space, all subsequent examples are presented graphically conveying nearly as much information.

3.1.1 Blinking Source

Figure 3-1 gives the scenario and system description that involves one continuous and one blinking source, of equal peak power, incident upon a 255-element Taylor-weighted main linear antenna array in combination with four adaptively-weighted omnidirectional auxiliaries. The specific parameters indicated in Figure 3-1 were appropriately entered in the input data file, SIGGEN:D. Subsequently, the signal generation load module, SIGGEN:L, was executed, producing an output data file, SIGGEN:O, listed in Table 3-1 that follows.

Concatenating SIGGEN:O to the BCR header data file, BCRS:DO, defines the input data file, BCRS:D, for the BCR simulation load module BCRS:L. The BCR simulation performance included in BCRS:O is summarized in Figure 3-2. Shown here is the relative combined power suppression, $P_C(\underline{w})/P_0$, and relative gradient metric, $||\underline{r}||^2/||\underline{b}||^2$, as a function of iteration. To be noted here is the fact that the combined power suppression reaches its limit at the end of the third iteration. However, by choosing the stopping criterion $||\underline{r}||^2/||\underline{b}||^2 < 10^{-6}$, the BCR process goes through an additional iteration at which the relative gradient metric reduces below its threshold of -60 dB.

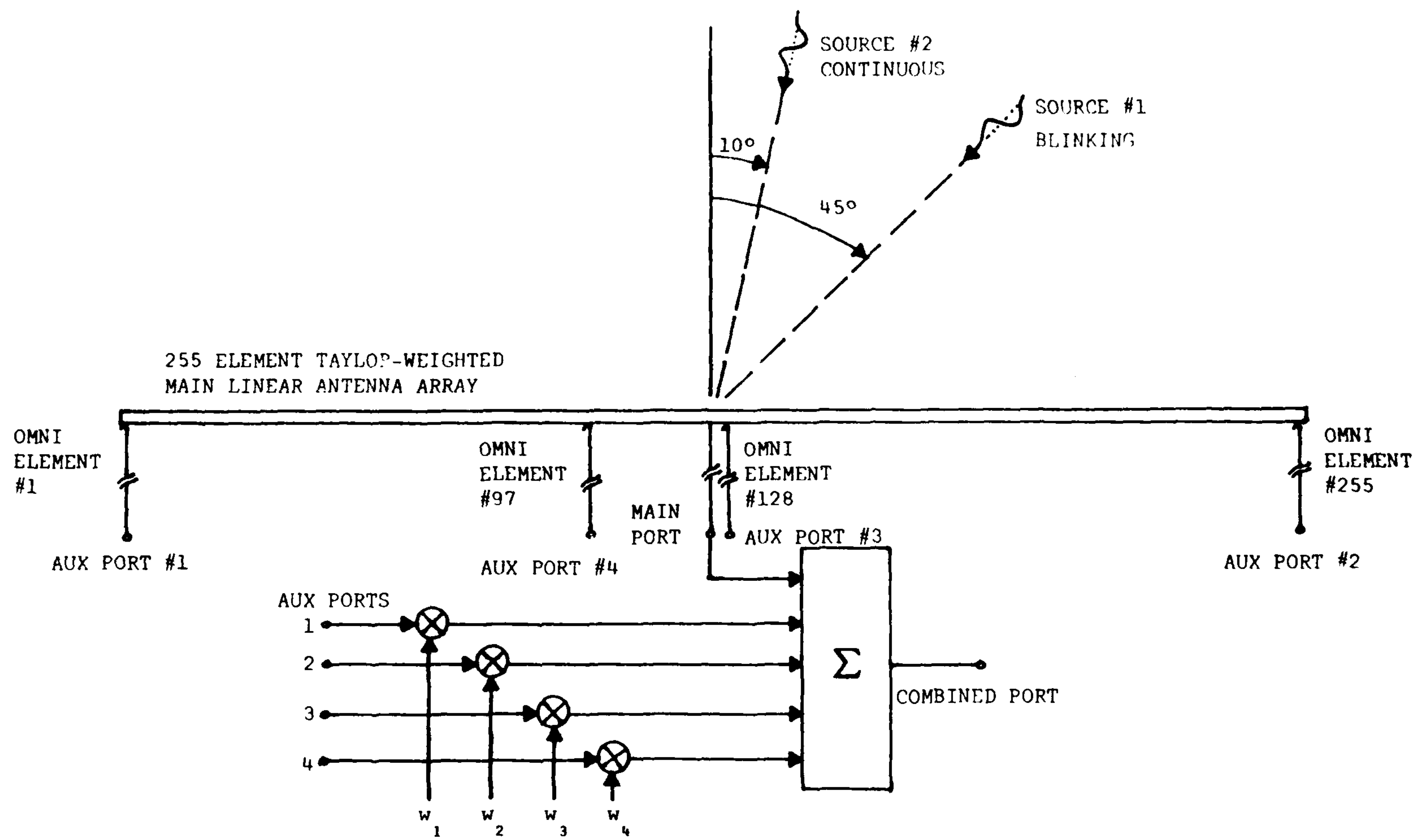


Figure 3-1. Scenario and System Description
Blinking Source Example
0.1% RF Bandwidth

Table 3-1. Signal Generation Program Output File, SIGGEN:0
Blinking Source Example (See Figure 3-1)

```
------------------------------------------------------------------------
                 SPECIFICATION OF SYSTEM PARAMETERS
                 ----------------------------------

------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWANT   -  MAIN ANTENNA ARRAY WEIGHTING              :        0
  IWRAP   -  RECEIVER AMPLITUDE AND PHASE              :        0
  IWRI    -  RECEIVER IMPULSE RESPONSE                 :        0
  IWCAP   -  CHANNEL AMPLITUDE AND PHASE               :        0
  IWCI    -  CHANNEL IMPULSE RESPONSE                  :        0
  IWSC    -  INDIVIDUAL CHANNEL SIGNALS                :        0

FILTER PARAMETERS
-----------------
  NPOL    -  NUMBER OF LOWPASS PROTOTYPE POLES         :        2
             POL(1)                                    :  -.70700   -.70700
             POL(2)                                    :  -.70700    .70700
  FBIF    -  FRACTIONAL BANDWIDTH AT FINAL IF          :   .10000
  FBRF    -  FRACTIONAL BANDWIDTH AT RF                :   .00100
  RADRNG  -  NORMALIZED RADIAN FREQUENCY RANGE         :  4.00000
  RADIN   -  NORMALIZED INITIAL RADIAN FREQUENCY       : -2.00000
  NF      -  NUMBER OF FREQUENCY SAMPLES               :       32
  LPBP    -  LOWPASS/BANDPASS OPTION                   :        1
             0  :   LOWPASS
             1  :   BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS  -  NUMBER OF CHANNELS PER PORT               :       20
  ISC     -  CHANNEL SELECTOR ARRAY                    :  1 1 0 0 0 0 0 0 0 0
                                                       :  0 0 0 0 0 0 0 0 0 0

CHANNEL  1:
  AN      -  AMPLITUDE OF NOISE SOURCE                 :  1.00000
  IX0     -  INITIAL RANDU SETTING                     :        1
  N1      -  FIRST-TIME-ON SAMPLE NUMBER               :        1
  NB      -  BLINK DURATION IN SAMPLES                 :    10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)         : 45.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :   .00000
```

```
CHANNEL   2:
   AN       -  AMPLITUDE OF NOISE SOURCE                     :     1.00000
   IXO      -  INITIAL RANDU SETTING                         :          11
   N1       -  FIRST-TIME-ON SAMPLE NUMBER                   :          64
   NB       -  BLINK DURATION IN SAMPLES                     :          64
   TH       -  AZIMUTH ANGLES OF INCIDENCE (DEG)             :    10.00000
   CDEL     -  CHANNEL DELAY IN SAMPLE-TIME UNITS            :      .00000

PORT PARAMETERS
---------------
   DO       -  ANTENNA-ELEMENT SEPARATION FACTOR             :     1.00000
   NS       -  NUMBER OF SIGNAL SAMPLES                      :         256
   NPORTS   -  NUMBER OF PORTS                               :          21
   ISP      -  PORT SELECTOR ARRAY                           :   1 1 1 1 0 0 0 0 0 0
                                                             :   0 0 0 0 0 0 0 0 0 0

PORT      0:
   NEL      -  NUMBER OF ANTENNA ELEMENTS                    :         255
   LOC1     -  LOCATION OF THE FIRST ELEMENT                 :           1
   BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                    :     1.00000
   BWOFF    -  BANDWIDTH OFFSET FACTOR                       :      .00000
   PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY            :      .00000

PORT      1:
   NEL      -  NUMBER OF ANTENNA ELEMENTS                    :           1
   LOC1     -  LOCATION OF THE FIRST ELEMENT                 :           1
   BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                    :     1.00000
   BWOFF    -  BANDWIDTH OFFSET FACTOR                       :      .00000
   PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY            :      .00000

PORT      2:
   NEL      -  NUMBER OF ANTENNA ELEMENTS                    :           1
   LOC1     -  LOCATION OF THE FIRST ELEMENT                 :         255
   BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                    :     1.00000
   BWOFF    -  BANDWIDTH OFFSET FACTOR                       :      .00000
   PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY            :      .00000

PORT      3:
   NEL      -  NUMBER OF ANTENNA ELEMENTS                    :           1
   LOC1     -  LOCATION OF THE FIRST ELEMENT                 :         128
   BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                    :     1.00000
   BWOFF    -  BANDWIDTH OFFSET FACTOR                       :      .00000
   PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY            :      .00000
```

```
PORT      4:
  NEL     -  NUMRER OF ANTENNA ELEMENTS                :        1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :       97
  RWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :  1.00000
  RWOFF   -  BANDWIDTH OFFSET FACTOR                   :   .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :   .00000
-----------------------------------------------------------------
```

PORT 0

RECEIVED BASEBAND PORT SIGNAL

NORMALIZATION CONSTANT : .344914E 00

.00573	.00859	-.04709	-.09054	-.01194	-.35142	.01546	-.38613
.01198	-.40043	.01519	-.36700	.00996	-.37752	.02541	-.31855
.04065	-.22620	.04685	.09405	-.06996	.03809	.03776	-.13504
.04276	.17080	-.01136	.29592	-.06901	.14884	-.00516	-.14954
.01320	-.07478	.01380	-.10447	.05337	.20487	-.08377	.15101
.01996	-.07612	-.00393	.03382	.02139	.02428	-.01601	.16832
-.05785	-.12442	.05173	-.16691	-.00815	-.02534	.04238	.00130
.00947	.27782	-.05680	.12022	.03213	.04278	.01682	.26613
-.06523	.17723	-.01004	-.09962	.03341	-.01376	-.00494	.07284
.00753	.08830	.00358	.14884	.00954	.22813	-.02532	.22395
.00052	.17930	-.03955	.13508	-.01979	-.08910	.02608	-.07763
-.00211	-.01634	-.01043	-.05144	.01648	-.08650	.04063	.10060
-.01661	.22515	-.03288	.11055	.00527	.02165	.00812	.07103
-.02686	.05376	.02656	-.05850	.02078	.23930	-.09187	-.04907
.05253	-.15603	.02687	-.01275	.03839	.37230	-.12970	.01342
.07369	-.08983	-.04702	-.12781	.07513	.09918	-.13046	-.24395
.24740	.13446	-.19906	.03104	-.06533	.17537	.06554	.28412
.07762	-.10419	.42961	.45015	.48333	.44110	.15388	.36659
-.00762	.21394	.11306	.32985	-.15617	-.12084	.00465	.14188
-.20557	-.46274	.06536	-.24025	-.53974	-.70610	-.22471	-.45208
-.67290	-.51290	-.14201	-.36826	.14857	.49093	-.14552	.14242
.38237	.53824	.63881	.80607	.62690	1.00000	-.08110	.26225
.29137	.36947	.16512	.30688	-.46263	-.30750	-.63580	-.53182
-.27415	-.60615	.33277	.09949	-.28668	-.03339	-.06053	-.03474
.49376	.70079	-.10112	.13201	-.52192	-.48443	-.71622	-.50218
-.61593	-.26686	-.09291	.07050	-.07621	.09941	-.18222	-.02348
-.08995	-.07929	-.30987	-.16201	-.65158	-.65299	-.37928	-.23394
.33474	.37284	.51056	.42029	.32195	-.00121	.28417	.21228
-.02959	.18669	-.37983	-.48275	-.29315	-.38017	.00971	.05183
-.26371	-.20948	.19034	.30418	.19698	.39682	.12739	.10579
.13262	.18191	-.11733	.05332	.12822	.32677	.15025	.47797
.097 7	.230 7	.05577	.14271	-.189 9	-.08525	.0 917	-.13408
.46172	.57038	.04482	.22257	.01098	-.14106	.04026	.03297
-.00780	.27753	-.05780	.12229	.00912	-.05465	.01959	.04948
-.01560	.06675	-.04312	-.08669	-.01674	-.36462	.09917	-.19575

-.00401	.18495	-.03807	-.00944	.03242	.01053	.01083	.12884
.01124	.24960	-.03385	.23288	-.02981	.07963	-.02360	-.11030
.03007	-.13534	-.00096	-.02858	-.03292	-.15392	.00073	-.32400
.04371	-.23959	.01453	-.09561	.02498	.02000	-.01083	.10257
-.01767	.01175	-.01590	-.09401	.02566	-.12352	-.00990	-.03406
-.01343	-.18941	.02940	-.13987	.00399	-.08386	.008[illegible]5	-.01580
-.03166	-.10493	.01248	-.19856	-.00788	-.19656	.01765	-.24069
.00507	-.16559	-.01261	-.25602	.06632	-.18670	.05328	.25159
-.08166	.24028	-.03650	-.15976	.02407	-.20896	.02550	-.12491
.01298	.02310	-.03782	-.04007	.01171	-.17186	.05393	.02871
-.00275	.26110	-.04911	.09570	.02136	.03048	-.03746	.01572
.01396	-.09923	-.00699	-.05579	.00717	-.09491	.03879	-.01844
-.02385	.16789	-.04315	-.17960	.07955	.01614	-.05392	.03416
-.14536	-.23619	-.73208	-.89993	-.69023	-.97140	-.68303	-.75570
-.22940	-.12453	.38024	.50644	.50302	.32913	.10274	-.06014
-.11849	-.02372	.13491	.43974	-.35689	-.30247	.35906	-.02126
.35205	.23795	-.16448	-.13844	-.03571	-.25998	.54954	.27175
.61324	.50681	-.11482	.04576	-.63714	-.41098	-.29919	-.13432
-.29261	.03964	-.37494	-.13848	.33201	.59174	.08889	.14726
.12184	-.04950	.06007	.09520	-.15404	-.28557	-.47699	-.73488
-.47237	-.61200	-.69652	-.33832	-.34242	-.12826	.17787	.29562
-.02565	-.15289	-.05875	-.21879	.20660	.11016	.30049	.41482
.08415	.21406	.16405	.14420	-.06372	-.21187	.36797	.19835
.10707	.37959	-.17839	-.03502	.39013	.34279	.38586	.18749
-.13078	-.36211	.08341	-.03722	.28159	.33073	.46880	.39598
.42048	.23824	.34860	.11143	.36457	.41940	.39107	.31512
.30864	.33590	-.54464	-.30195	-.36961	-.34344	.20714	.29473
-.22027	.08410	-.33393	-.03709	-.22947	.02295	-.51560	-.50495
-.11638	-.03126	.39826	.59778	.26749	.50261	.35425	.56253

PORT 1

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .602822E 00

-.00564	.01082	.07653	-.11958	.44603	-.57805	.51724	-.66812
.52697	-.69309	.48831	-.64155	.49553	-.65320	.43521	-.56642
.32384	-.41799	-.08057	.11081	-.11566	.11091	.20935	-.24315
-.18220	.24940	-.39778	.50434	-.25236	.30659	.19213	-.23310
.11509	-.14066	.14133	-.18590	-.21560	.29718	-.27458	.31315
.11974	-.12907	-.04411	.05568	-.01613	.02783	-.22907	.28828
.10711	-.15931	.26803	-.31479	.02406	-.04681	.03130	-.02691
-.34994	.45255	-.20951	.25009	-.02420	.05999	-.32840	.43247
-.28860	.34755	.12216	-.14660	.05168	-.04811	-.10138	.12130
-.10869	.14445	-.19048	.24981	-.28665	.37955	-.31344	.40017
-.23055	.31016	-.20722	.25904	.09856	-.12637	.12689	-.14898
.01781	-.03079	.05713	-.08085	.12575	-.15578	-.09432	.13573
-.30737	.38656	-.18032	.22022	-.00601	.03379	-.10400	.12332
-.06479	.09641	.06874	-.09719	-.25321	.36461	-.06257	.00520
.30349	-.31152	-.02079	-.02297	-.38250	.56310	-.21179	.15122
.27312	-.21976	.02588	-.15352	.04615	.07518	.06587	-.27987
.22553	.01996	-.32561	.16076	-.51585	.46264	-.06020	.34176
.17775	-.17731	.22684	.35627	.37658	.27024	-.09524	.40104
-.37645	.43263	-.08610	.39069	-.18893	-.01892	-.11509	.21152
.24137	-.57707	.41728	-.47881	-.04419	-.73202	.00093	-.48987
-.49945	-.31018	-.08624	-.31369	-.24371	.59052	-.51441	.43175
-.00611	.58506	.13222	.79185	.05454	1.00000	-.42620	.50173
-.00929	.42991	.09952	.25300	-.46141	-.09739	-.50248	-.29008
.13222	-.66059	.47451	-.16812	-.41045	.13426	-.25887	.12420
.07288	.69066	-.15525	.20909	-.38601	-.31213	-.76341	-.16111
-.94136	.20534	-.27539	.23724	-.23974	.21471	-.30261	.14584
-.08187	-.04225	-.28514	-.03296	-.41313	-.48610	-.61091	.07460
.12853	.35360	.54355	.17479	.64495	-.32099	.19177	.09261
-.17186	.27693	-.04345	-.48254	-.22417	-.27703	.00425	.03969
-.26103	-.09666	-.15403	.40830	-.00409	.41427	.07780	.09130
.06517	.14206	-.32526	.21511	-.23468	.46381	-.28220	.64641
-.05945	.29140	-.06572	.18125	-.18357	.01176	.16443	-.20255
.14882	.50064	-.06361	.26617	.17510	-.21738	-.00981	.01779
-.39245	.47910	-.19566	.24343	.06876	-.07747	-.03391	.06109
-.11014	.12732	.08290	-.11754	.45071	-.59125	.34907	-.41004

-.25287	.29731	-.02013	.01349	.01407	.00120	-.15982	.20797
-.31428	.41240	-.33329	.42194	-.12643	.16490	.12425	-.16086
.20364	-.24801	.03575	-.05533	.16960	-.23712	.42174	-.54437
.35060	-.44263	.13274	-.18246	-.00710	.01080	-.14378	.17746
-.03068	.03668	.10937	-.14325	.18293	-.22471	.03644	-.05823
.23054	-.30695	.20991	-.26019	.10867	-.14897	.02816	-.03755
.10664	-.15457	.27074	-.34230	.24825	-.33153	.32601	-.42117
.21998	-.29215	.31843	-.42610	.29829	-.36397	-.28065	.37060
-.38777	.46065	.17754	-.22549	.29869	-.36731	.18203	-.23535
-.02602	.02362	.01554	-.04112	.23024	-.29095	.00826	.00600
-.33377	.42818	-.17655	.20674	-.00624	.03985	-.06865	.05943
.16451	-.18293	.03720	-.08035	.16226	-.17957	.02573	-.04744
-.20760	.27896	.15805	-.24793	.09257	-.04545	-.12033	.09913
.14035	-.31825	-.18294	-.85804	-.15958	-.93778	-.40299	-.62178
-.48810	.12503	.11589	.48722	.61641	.05720	.37217	-.28499
-.38748	.16517	-.18626	.54429	-.24887	-.16385	.57574	-.27010
.47547	-.03433	-.10759	-.09452	.19698	-.35249	.59046	.00249
.64157	.17481	-.18214	.10297	-.69696	-.08338	-.53524	.14592
-.48672	.26685	-.70094	.23168	-.11891	.69917	.13116	.10189
.22093	-.15376	.04869	.06562	.15654	-.37300	.05943	-.80687
-.22152	-.55966	-.91073	.06994	-.63214	.21969	.03420	.30839
.22953	-.26522	.10673	-.28755	.16394	.03500	.08654	.36625
-.10547	.29158	.18822	.06411	.15633	-.28942	.31414	.04427
-.14566	.43420	-.35745	.16432	.25731	.26759	.67267	-.14795
.20510	-.49710	.07252	-.07854	.11819	.28166	.39454	.23426
.58579	-.04363	.45272	-.11686	.16322	.34282	.34509	.17164
.25252	.19635	-.55540	-.05710	-.45356	-.09696	.03305	.28446
-.43721	.28187	-.67238	.31389	-.36401	.22073	-.32284	-.36488
-.39580	.18134	.03091	.60807	-.11306	.58310	-.04970	.62714

PORT 2

RECEIVED BASEBAND PORT SIGNAL

NORMALIZATION CONSTANT : .715786E 00

.01179	.01666	-.12294	-.16594	-.36424	-.55664	-.39183	-.59137
-.40797	-.61000	-.37388	-.55619	-.38501	-.57573	-.31749	-.47694
-.20870	-.32308	.11580	.17890	-.00324	.02630	-.10759	-.19656
.19496	.29196	.29614	.45891	.11354	.19085	-.14684	-.24484
-.07456	-.11116	-.09487	-.15107	.23485	.35126	.10545	.19432
-.05791	-.11715	.02777	.05035	.04112	.05038	.15596	.25451
-.15524	-.22737	-.13932	-.23798	-.03684	-.03816	.03060	.02707
.28373	.44095	.09486	.15526	.06935	.07633	.27878	.42423
.14336	.24026	-.09960	-.16913	.00365	-.00590	.06976	.11478
.09632	.14127	.15619	.23326	.24054	.35875	.21687	.33296
.18829	.27437	.11446	.18573	-.09761	-.15497	-.06512	-.10939
-.02049	-.02287	-.05968	-.08481	-.07651	-.12655	.12475	.18051
.21758	.34350	.10319	.15840	.01768	.01587	.09290	.13440
.01495	.04030	-.01370	-.05010	.22032	.35018	-.06596	-.08328
-.16632	-.27519	.05148	.06107	.33850	.53478	.00916	.03225
-.11773	-.20581	-.07214	-.11093	.04085	.07203	-.19789	-.29216
.10016	.10935	.05849	.13250	.29448	.43840	.13017	.21688
-.04577	-.12747	.30092	.40822	.26786	.32953	.28599	.41610
.26856	.38959	.21166	.31975	-.02149	-.04066	.08562	.15507
-.42485	-.62800	-.24706	-.37930	-.57629	-.77662	-.27545	-.39548
-.37236	-.42376	-.13945	-.23977	.38252	.60195	.22042	.34002
.43792	.59167	.62663	.83839	.71083	1.00000	.26454	.39310
.31864	.41432	.15288	.22303	-.15054	-.16971	-.34813	-.42371
-.45322	-.66918	.01949	-.01090	.01027	.08949	.09765	.12189
.50951	.71817	.04375	.09621	-.27826	-.37236	-.21671	-.22757
-.00163	.09275	.09030	.15427	.12326	.19511	.01608	.05464
-.02835	-.04356	-.12600	-.12124	-.40406	-.53647	.00100	.05227
.24930	.34103	.17263	.19925	-.11841	-.24821	.18749	.23317
.09454	.18701	-.37423	-.53072	-.18394	-.25327	.00304	.03106
-.08685	-.10537	.30764	.43305	.24598	.36363	.08481	.08749
.12188	.16501	.12168	.19449	.32354	.46231	.39678	.58887
.16636	.22718	.13063	.17889	-.07349	-.07296	-.06405	-.14353
.41220	.57836	.11400	.18496	-.12120	-.21310	.06774	.08640
.28745	.45174	.08696	.14627	-.03714	-.07901	.05425	.07904
.06229	.10373	-.11852	-.16522	-.37430	-.57429	-.14783	-.25527

.17866	.30390	-.03061	-.04105	.03525	.03387	.13871	.20833
.26216	.39510	.22124	.34330	.06758	.10163	-.12341	-.18972
-.12038	-.19617	-.03508	-.04020	-.17802	-.25584	-.32986	-.50580
-.22382	-.34666	-.09410	-.13298	.03377	.04923	.09667	.15635
.00355	.00641	-.10483	-.15791	-.10993	-.17914	-.04690	-.05472
-.19873	-.30113	-.12869	-.20147	-.08484	-.12302	-.01461	-.01755
-.12633	-.17912	-.19580	-.30467	-.21033	-.30751	-.23672	-.36245
-.17276	-.25067	-.27055	-.40207	-.15019	-.25248	.28056	.42941
.19621	.33465	-.17522	-.27891	-.20091	-.31784	-.11537	-.17582
.03107	.05447	-.06276	-.07936	-.16177	-.26060	.06082	.07803
.25362	.40258	.07918	.12686	.03982	.04215	.00425	.02047
-.10843	-.17303	-.04117	-.05851	-.12002	-.17874	.03146	.02762
.12367	.22001	-.17126	-.27047	.02811	.02474	.02015	.05693
-.26219	-.36464	-.67817	-.92126	-.68789	-.94446	-.46790	-.60140
.06440	.13406	.32622	.46476	.10132	.07564	-.12027	-.20964
.14289	.21420	.30034	.47432	-.20650	-.27568	-.07463	-.19321
.06917	.07489	-.11044	-.13076	-.20902	-.33078	.13839	.12477
.23118	.28073	.04708	.09554	-.17909	-.18869	.04784	.10200
.07001	.17196	.09844	.17110	.45073	.65338	.02484	.02686
-.02988	-.09404	.03112	.06062	-.29396	-.41419	-.57966	-.81481
-.36822	-.49710	-.09048	-.00603	.05594	.12482	.17444	.25629
-.18997	-.29245	-.15050	-.23291	.08621	.09861	.29201	.41349
.17279	.24766	.05039	.05130	-.19101	-.29313	.16301	.17614
.26176	.41666	.06523	.10335	.21484	.26619	-.05193	-.12149
-.29029	-.43869	.01198	.00016	.22159	.31146	.21855	.26283
.04182	.00170	.05195	.00276	.28604	.39676	.17248	.19631
.19203	.25089	-.16071	-.15872	-.10907	-.14153	.21214	.30433
.12200	.22648	.12957	.22947	.02207	.08595	-.31341	-.42453
.13590	.20655	.42691	.60865	.41123	.57566	.44616	.61260

PORT 3

RECEIVED BASEBAND PORT SIGNAL

NORMALIZATION CONSTANT : .756774E 00

.00071	.01502	-.00483	-.15227	.02375	-.60738	.02470	-.67117
.02101	-.69426	.01872	-.63703	.01948	-.65466	.01885	-.55483
.01675	-.39470	-.00506	.15861	-.02070	.07339	.02466	-.23728
-.00672	.29121	-.02436	.51502	-.01983	.26457	.01946	-.25886
.00647	-.13316	.00786	-.18116	-.00714	.34950	-.02982	.27052
.01934	-.13422	-.00490	.05757	.00342	.04086	-.01770	.29210
.00055	-.20931	.02617	-.29553	-.00709	-.04371	.00858	-.00045
-.02088	.47983	-.01630	.21439	.01203	.07072	-.01575	.45885
-.02421	.31323	.01409	-.17185	.00870	-.02834	-.00961	.12666
-.00345	.15239	-.00796	.25762	-.01102	.39409	-.01716	.39047
-.00573	.31075	-.01442	.23706	.00785	-.15270	.01175	-.13784
-.00304	-.02782	.00012	-.08821	.01015	-.15149	-.00070	.17018
-.02089	.39061	-.01191	.20148	.00965	.02355	-.00813	.13968
-.00160	.07043	.00579	-.07814	-.00577	.38175	-.02614	-.04037
.04368	-.31924	-.01572	.02614	-.00257	.58387	-.04863	.10211
.06048	-.23715	-.03814	-.13171	.04447	.07015	-.06046	-.29433
.12173	.05078	-.12022	.17432	-.07821	.49607	.07565	.27628
.04091	-.16635	.23338	.36754	.27866	.27034	.07843	.42175
-.02399	.44271	.08193	.35931	-.09211	-.01534	.00993	.19227
-.07383	-.63125	.03673	-.45861	-.28153	-.75580	-.15996	-.43798
-.37012	-.33092	-.12040	-.26933	.05965	.62127	-.09386	.42513
.20496	.58990	.34445	.80784	.35555	1.00000	-.03668	.48072
.15960	.42253	.11755	.22837	-.26331	-.09617	-.34725	-.32204
-.14390	-.68277	.16839	-.12240	-.17220	.14768	-.06116	.14353
.26617	.70235	-.02088	.16040	-.29423	-.31235	-.43381	-.13270
-.38606	.22379	-.05041	.21120	-.05175	.22571	-.09610	.12073
-.05402	-.03651	-.16237	-.05682	-.37085	-.48135	-.26188	.11567
-.19122	.33258	.31985	.14246	.21039	-.33797	.12566	.15846
.00035	.24351	-.18922	-.50797	-.19353	-.24444	.00716	.03307
-.16150	-.07933	.07579	.43649	.12876	.38702	.07802	.08253
.07434	.15019	-.08417	.23309	.04920	.48373	.07892	.64122
.07273	.26009	.02855	.18737	-.08974	-.01955	.02817	-.18674
.23772	.53414	.05117	.22750	.02038	-.23142	-.00040	.05769
-.03025	.50040	-.01421	.20561	.01218	-.08524	.00225	.07475
-.01078	.12429	-.00063	-.15159	.02289	-.62408	.03210	-.35451

-.02868	.32632	-.00336	-.01731	.00886	.01908	-.00761	.22227
-.01409	.43172	-.01970	.40779	-.00468	.14076	.00768	-.18919
.01638	-.23744	-.00381	-.04919	.00186	-.26314	.02287	-.56161
.02042	-.41989	.00097	-.16648	-.00026	.03337	-.01144	.17920
-.00180	.02224	.00528	-.16156	.01441	-.21600	-.00490	-.05836
-.01005	-.32585	.01373	-.24536	.00181	-.14458	-.00034	-.02795
-.00074	-.17842	.01708	-.34566	.00711	-.34018	.01684	-.41806
.00597	-.28772	.01091	-.44206	.02428	-.32904	-.01532	.43128
-.03590	.42526	.01484	-.27320	.02188	-.36616	.00845	-.21873
-.00646	.04436	-.00698	-.06419	.01653	-.29568	.00602	.04617
-.02171	.44531	-.01432	.17673	.01202	.04181	-.01416	.04413
.01913	-.19339	-.00929	-.07025	.01747	-.19503	-.00144	-.00706
-.01230	.26607	-.00160	-.27731	.02815	-.01287	-.02716	.08672
-.04847	-.35753	-.39107	-.87874	-.39444	-.93169	-.41695	-.57813
-.18235	.17143	.22167	.46409	.32343	.01335	.07294	-.27440
-.11940	.22910	.08246	.52236	-.17581	-.20527	.21118	-.28065
.20263	-.01287	-.08655	-.10374	-.01273	-.35972	.30239	.01908
.36039	.17925	-.06729	.11875	-.36986	-.08147	-.20098	.16688
-.16495	.25539	-.24862	.25834	.18208	.68466	.09140	.04765
.06544	-.13948	.03905	.06028	-.05501	-.41097	-.25076	-.81859
-.30012	-.50734	-.43436	.10782	-.21834	.21887	.11657	.27606
.01889	-.30235	-.04226	-.26579	.10057	.05524	.15859	.38742
.05201	.27737	.11248	.04113	-.01915	-.30639	.17942	.09071
.05774	.44012	-.10814	.16170	.22943	.24432	.26607	-.19254
-.06978	-.48162	.01923	-.04086	.15229	.28884	.27896	.21639
.26871	-.06871	.18991	-.08876	.19458	.36220	.23495	.15479
.17895	.20772	-.30348	-.06376	-.23396	-.08436	.10814	.29158
-.13212	.29057	-.21345	.32616	-.11033	.17654	-.28464	-.37306
-.11551	.23019	.21906	.60482	.14294	.59138	.19560	.62513

PORT 4

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .649192E 00

-.00296	.01584	.03535	-.16214	.19997	-.67253	.22220	-.75109
.22414	-.77788	.20567	-.71518	.21082	-.73345	.18281	-.62486
.13482	-.44869	-.04701	.16494	-.04923	.09143	.09756	-.26673
-.08753	.31521	-.17607	.57382	-.10309	.30749	.09491	-.28261
.04724	-.15094	.06109	-.20458	-.10376	.37793	-.11713	.31358
.06085	-.14821	-.02170	.06394	-.00694	.04226	-.10398	.32617
.05585	-.22151	.11790	-.33546	.00453	-.05031	.01209	-.00747
-.15973	.53028	-.08451	.24935	-.00515	.07708	-.14776	.50710
-.12174	.35963	.06358	-.18536	.02057	-.03664	-.04743	.14011
-.04732	.16867	-.08285	.28658	-.12494	.43783	-.13342	.43997
-.09598	.34834	-.08680	.27141	.05065	-.16392	.05477	-.15706
.00464	-.03223	.02478	-.09698	.05568	-.17077	-.04657	.18162
-.13800	.43580	-.06738	.23251	-.00902	.02605	-.02984	.15697
-.05023	.07911	.05747	-.08611	-.14717	.41592	.01577	-.02522
.09401	-.36504	.03690	.02807	-.23703	.63512	-.01185	.14171
.04584	-.27632	.10559	-.13487	-.09608	.06026	.17425	-.30340
-.12436	.02235	.05152	.20001	-.08464	.59527	-.22952	.27479
.07177	-.17437	-.39097	.34534	-.38947	.22700	-.23272	.43292
-.13177	.52271	-.23842	.36615	.13287	.02073	-.10921	.20768
.30237	-.66589	.15029	-.52960	.54214	-.77314	.36285	-.44664
.45989	-.29684	.28138	-.23969	-.31085	.62563	-.06386	.51819
-.41300	.60836	-.63967	.81610	-.75121	1.00000	-.13794	.56259
-.30489	.45739	-.21801	.19761	.31970	-.03301	.45861	-.25994
.41585	-.69857	-.10465	-.21210	.09323	.18727	.03508	.20689
-.54235	.69238	-.06731	.17182	.45208	-.25859	.51955	-.04580
.31680	.35146	-.03627	.24355	-.00939	.24808	.03650	.17018
.09237	-.02884	.16182	-.03114	.58975	-.44245	.24466	.20844
-.34546	.31950	-.40050	.06846	-.08154	-.41801	-.15516	.13824
-.15186	.26597	.40082	-.51355	.32061	-.20690	-.04533	.01314
.21549	-.04905	-.23131	.47540	-.30352	.38072	-.09611	.09012
-.12405	.13658	.00873	.28514	-.21795	.52607	-.33099	.68736
-.17470	.27933	-.07954	.20127	.07635	.00591	.07800	-.19553
-.44879	.51120	-.17620	.23751	.08608	-.24315	.00871	.05399
-.16748	.55949	-.08391	.23551	.04237	-.08934	-.02153	.07747
-.04589	.14198	.03773	-.16251	.20731	-.68843	.14116	-.41310

-.12048	.35874	-.00788	-.01337	.01130	.01863	-.07175	.24505
-.13895	.47861	-.14179	.46052	-.04794	.16444	.06099	-.20424
.08866	-.26854	.01036	-.05703	.07541	-.28814	.18680	-.62448
.14700	-.47637	.05023	-.19126	-.00808	.03127	-.06484	.20011
-.00983	.02881	.05117	-.17605	.07986	-.24410	.01130	-.06558
.10344	-.36021	.08835	-.27857	.04388	-.16347	.00862	-.03387
.04817	-.19385	.11934	-.38612	.10556	-.37905	.14017	-.46926
.09033	-.32370	.13826	-.49134	.12605	-.37771	-.13637	.46690
-.16798	.48510	.09162	-.29290	.13170	-.41002	.07420	-.24973
-.01369	.04532	.00849	-.06585	.10692	-.32894	-.00121	.04175
-.15865	.49196	-.06570	.20713	-.00558	.04483	-.01667	.05687
.05646	-.21893	.03804	-.07468	.04213	-.22222	.03577	-.01159
-.12377	.29483	.10903	-.29641	.00210	-.02984	-.03301	.10496
.16496	-.39152	.73382	-.87504	.76353	-.92879	.66411	-.55940
.12958	.24419	-.43607	.44676	-.35046	-.06097	.05169	-.33892
.06617	.30714	-.32509	.53684	.25775	-.16454	-.09318	-.33894
-.20494	-.10322	.10071	-.08246	.15784	-.37840	-.31302	-.04792
-.45441	.07760	.02283	.13915	.41817	.01492	.16815	.24748
.06260	.29319	.19729	.37409	-.46432	.70876	-.12961	.02632
.02140	-.15519	-.08634	.04973	.18819	-.44048	.57267	-.84473
.53209	-.49822	.39639	.20758	.14472	.31791	-.24028	.26352
.09280	-.33881	.16572	-.27763	-.12429	.04018	-.31184	.36835
-.17034	.30857	-.13464	.01856	.13824	-.32377	-.18717	.05753
-.25480	.44423	.06081	.23210	-.33727	.23050	-.22295	-.29973
.27436	-.50965	.00499	-.04034	-.28310	.27257	-.38175	.17797
-.26052	-.14536	-.13415	-.13660	-.35186	.34840	-.31043	.12390
-.25391	.16564	.32903	.00397	.29837	-.00039	-.22022	.27237
.02872	.33571	.11364	.43477	.02066	.21827	.46492	-.34237
.06367	.30185	-.47310	.59574	-.35766	.61949	-.43023	.65493

Table 3-2. BCR Simulation Program Output File BCRS:0
Blinking Source Example (See Figure 3-1)

```
-------------------------------------------------------------------------------
                     DIGITAL ADAPTIVE ARRAY PROCESSING
                     ---------------------------------
                                   USING
                                   -----
                         BATCH COVARIANCE RELAXATION
                         ---------------------------

-------------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWS0   -  OPTIONAL OUTPUT OF  S0                  :          1
  IWS1   -  OPTIONAL OUTPUT OF  S1                  :          0
  IWCB   -  OPTIONAL OUTPUT OF  C  AND  B           :          1

BCR PARAMETERS
--------------
  NSX    -  MAXIMUM NUMBER OF SIGNAL SAMPLES        :        256
  IWO    -  INITIAL WEIGHT-VALUE OPTION             :          0
  NWX    -  MAXIMUM NUMBER OF WEIGHTS               :         20
  ISW    -  WEIGHT SELECTOR ARRAY                   :    1 1 1 1 0 0 0 0 0 0
                                                    :    0 0 0 0 0 0 0 0 0 0
  ITERX  -  MAXIMUM NUMBER OF ITERATIONS            :         20
-------------------------------------------------------------------------------
  ITER   -  ACTUAL NUMBER OF ITERATIONS             :          X
-------------------------------------------------------------------------------
```

```
--------------------------------------------------------------------------------
                    SPECIFICATION OF SYSTEM PARAMETERS
                    ----------------------------------
--------------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWANT   -  MAIN ANTENNA ARRAY WEIGHTING
  IWRAP   -  RECEIVER AMPLITUDE AND PHASE            :        0
  IWRI    -  RECEIVER IMPULSE RESPONSE               :        0
  IWCAP   -  CHANNEL AMPLITUDE AND PHASE             :        0
  IWCI    -  CHANNEL IMPULSE RESPONSE                :        0
  IWSC    -  INDIVIDUAL CHANNEL SIGNALS              :        0
                                                     :        0
FILTER PARAMETERS
-----------------
  NPOL    -  NUMBER OF LOWPASS PROTOTYPE POLES
             POL(1)                                  :        2
             POL(2)                                  :  -.70700   -.70700
  FBIF    -  FRACTIONAL BANDWIDTH AT FINAL IF        :  -.70700    .70700
  FBRF    -  FRACTIONAL BANDWIDTH AT RF              :   .10000
  RADRNG  -  NORMALIZED RADIAN FREQUENCY RANGE       :   .00100
  RADIN   -  NORMALIZED INITIAL RADIAN FREQUENCY     :  4.00000
  NF      -  NUMBER OF FREQUENCY SAMPLES             : -2.00000
  LPRP    -  LOWPASS/BANDPASS OPTION                 :       32
             0  :   LOWPASS                          :        1
             1  :   BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS  -  NUMBER OF CHANNELS PER PORT
  ISC     -  CHANNEL SELECTOR ARRAY                  :       20
                                                     :  1 1 0 0 0 0 0 0 0 0
                                                     :  0 0 0 0 0 0 0 0 0 0
CHANNEL  1:
  AN      -  AMPLITUDE OF NOISE SOURCE
  IX0     -  INITIAL RANDU SETTING                   :  1.00000
  N1      -  FIRST-TIME-ON SAMPLE NUMBER             :        1
  NB      -  BLINK DURATION IN SAMPLES               :        1
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)       :    10000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS      : 45.00000
                                                     :   .00000
```

```
CHANNEL  2:
  AN       -  AMPLITUDE OF NOISE SOURCE                      :     1.00000
  IX0      -  INITIAL RANDU SETTING                          :          11
  N1       -  FIRST-TIME-ON SAMPLE NUMBER                    :          64
  NB       -  BLINK DURATION IN SAMPLES                      :          64
  TH       -  AZIMUTH ANGLES OF INCIDENCE (DEG)              :    10.00000
  CDEL     -  CHANNEL DELAY IN SAMPLE-TIME UNITS             :      .00000

PORT PARAMETERS
---------------
  D0       -  ANTENNA-ELEMENT SEPARATION FACTOR              :     1.00000
  NS       -  NUMBER OF SIGNAL SAMPLES                       :         256
  NPORTS   -  NUMBER OF PORTS                                :          21
  ISP      -  PORT SELECTOR ARRAY                            :  1 1 1 1 0 0 0 0 0 0
                                                             :  0 0 0 0 0 0 0 0 0 0

PORT     0:
  NEL      -  NUMBER OF ANTENNA ELEMENTS                     :         255
  LOC1     -  LOCATION OF THE FIRST ELEMENT                  :           1
  BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                     :     1.00000
  BWOFF    -  BANDWIDTH OFFSET FACTOR                        :      .00000
  PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY             :      .00000

PORT     1:
  NEL      -  NUMBER OF ANTENNA ELEMENTS                     :           1
  LOC1     -  LOCATION OF THE FIRST ELEMENT                  :           1
  BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                     :     1.00000
  BWOFF    -  BANDWIDTH OFFSET FACTOR                        :      .00000
  PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY             :      .00000

PORT     2:
  NEL      -  NUMBER OF ANTENNA ELEMENTS                     :           1
  LOC1     -  LOCATION OF THE FIRST ELEMENT                  :         255
  BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                     :     1.00000
  BWOFF    -  BANDWIDTH OFFSET FACTOR                        :      .00000
  PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY             :      .00000

PORT     3:
  NEL      -  NUMBER OF ANTENNA ELEMENTS                     :           1
  LOC1     -  LOCATION OF THE FIRST ELEMENT                  :         128
  BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                     :     1.00000
  BWOFF    -  BANDWIDTH OFFSET FACTOR                        :      .00000
  PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY             :      .00000
```

```
PORT      4:
  NEL     -  NUMBER OF ANTENNA ELEMENTS              :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT           :         97
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR              :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                 :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY      :     .00000
-------------------------------------------------------------------------
```

PORT 0

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .344914E 00

.00573	.00859	-.04709	-.09054	-.01194	-.35142	.01546	-.38613
.01198	-.40043	.01519	-.36700	.00996	-.37752	.02541	-.31855
.04065	-.22620	.04685	.09405	-.06996	.03809	.03776	-.13504
.04276	.17080	-.01136	.29592	-.06901	.14884	-.00516	-.14954
.01320	-.07478	.01380	-.10447	.05337	.20487	-.08377	.15101
.01996	-.07612	-.00393	.03382	.02139	.02428	-.01601	.16832
-.05785	-.12442	.05173	-.16691	-.00815	-.02534	.04238	.00130
.00947	.27782	-.05680	.12022	.03213	.04278	.01682	.26613
-.06523	.17723	-.01004	-.09962	.03341	-.01376	-.00494	.07284
.00753	.08830	.00358	.14884	.00954	.22813	-.02532	.22395
.00052	.17930	-.03955	.13508	-.01979	-.08910	.02608	-.07763
-.00211	-.01634	-.01043	-.05144	.01648	-.08650	.04063	.10060
-.01661	.22515	-.03288	.11055	.00527	.02165	.00812	.07103
-.02686	.05376	.02656	-.05850	.02078	.23930	-.09187	-.04907
.05253	-.15603	.02687	-.01275	.03839	.37230	-.12970	.01342
.07369	-.08983	-.04702	-.12781	.07513	.09918	-.13046	-.24395
.24740	.13446	-.19906	.03104	-.06533	.17537	.06554	.28412
.07762	-.10419	.42961	.45015	.48333	.44110	.15388	.36659
-.00762	.21394	.11306	.32985	-.15617	-.12084	.00465	.14188
-.20557	-.46274	.06536	-.24025	-.53974	-.70610	-.22471	-.45208
-.67290	-.51290	-.14201	-.36826	.14857	.49093	-.14552	.14242
.38237	.53824	.63881	.80607	.62690	1.00000	-.08110	.26225
.29137	.36947	.16512	.30688	-.46263	-.30750	-.63580	-.53182
-.27415	-.60615	.33277	.09949	-.28668	-.03339	-.06053	-.03474
.49376	.70079	-.10112	.13201	-.52192	-.48443	-.71622	-.50218
-.61593	-.26686	-.09291	.07050	-.07621	.09941	-.18222	-.02348
-.08995	-.07929	-.30987	-.16201	-.65158	-.65299	-.37928	-.23394
.33474	.37284	.51056	.42029	.32195	-.00121	.28417	.21228
-.02959	.18669	-.37983	-.48275	-.29315	-.38017	.00971	.05183
-.26371	-.20948	.19034	.30418	.19698	.39682	.12739	.10579
.13262	.18191	-.11733	.05332	.12822	.32677	.15025	.47797
.09767	.23067	.05577	.14271	-.18969	-.08525	.06917	-.13408
.46172	.57038	.04482	.22257	.01098	-.14106	.04026	.03297
-.00780	.27753	-.05780	.12229	.00912	-.05465	.01959	.04948
-.01560	.06675	-.04312	-.08669	-.01674	-.36462	.09917	-.19575

-.00401	.18495	-.03807	-.00944	.03242	.01053	.01083	.12884
.01124	.24960	-.03385	.23288	-.02981	.07963	-.02360	-.11030
.03007	-.13534	-.00096	-.02858	-.03292	-.15392	.00073	-.32400
.04371	-.23959	.01453	-.09561	.02498	.02000	-.01083	.10257
-.01767	.01175	-.01590	-.09401	.02566	-.12352	-.00990	-.03406
-.01343	-.18941	.02940	-.13987	.00399	-.08386	.00835	-.01580
-.03166	-.10493	.01248	-.19856	-.00788	-.19656	.01765	-.24069
.00507	-.16559	-.01261	-.25602	.06632	-.18670	.05328	.25159
-.08166	.24028	-.03650	-.15976	.02407	-.20896	.02550	-.12491
.01298	.02310	-.03782	-.04007	.01171	-.17186	.05393	.02871
-.00275	.26110	-.04911	.09570	.02136	.03048	-.03746	.01572
.01396	-.09923	-.00699	-.05579	.00717	-.09491	.03879	-.01844
-.02385	.16789	-.04315	-.17960	.07955	.01614	-.05392	.03416
-.14536	-.23619	-.73208	-.89993	-.69023	-.97140	-.68303	-.75570
-.22940	-.12453	.38024	.50644	.50302	.32913	.10274	-.06014
-.11849	-.02372	.13491	.43974	-.35689	-.30247	.35906	-.02126
.35205	.23795	-.16448	-.13844	-.03571	-.25998	.54954	.27175
.61324	.50681	-.11482	.04576	-.63714	-.41098	-.29919	-.13432
-.29261	.03964	-.37494	-.13848	.33201	.59174	.08889	.14726
.12184	-.04950	.06007	.09520	-.15404	-.28557	-.47699	-.73488
-.47237	-.61200	-.69652	-.33832	-.34242	-.12826	.17787	.29562
-.02565	-.15289	-.05875	-.21879	.20660	.11016	.30049	.41482
.08415	.21406	.16405	.14420	-.06372	-.21187	.36797	.19835
.10707	.37959	-.17839	-.03502	.39013	.34279	.38586	.18749
-.13078	-.36211	.08341	-.03722	.28159	.33073	.46880	.39598
.42048	.23824	.34860	.11143	.36457	.41940	.39107	.31512
.30864	.33590	-.54464	-.30195	-.36961	-.34344	.20714	.29473
-.22027	.08410	-.33393	-.03709	-.22947	.02295	-.51560	-.50495
-.11638	-.03126	.39826	.59778	.26749	.50261	.35425	.56253

COVARIANCE MATRIX C

NORMALIZATION CONSTANT : .192330E 02

.93493	.00000	.50481	-.55829	.82932	-.35086	.52788	.03722
.50481	.55829	1.00000	.00000	.86348	.35615	.39603	.82937
.82932	.35086	.86348	-.35615	.98264	.00000	.58368	.47575
.52788	-.03722	.39603	-.82937	.58368	-.47575	.98933	.00000

FORCING VECTOR B

NORMALIZATION CONSTANT : .786043E 01

.87598	-.20474	.93374	.15244	1.00000	-.03217	.26835	-.66954

PERFORMANCE SUMMARY OF RCR SIMULATION

INITIAL CONDITIONS

C	:	.93493	.00000	.50481	-.55829	.82932	-.35086	.52788	.03722
	:	.50481	.55829	1.00000	.00000	.86348	.35615	.39603	.82937
	:	.82932	.35086	.86348	-.35615	.98264	.00000	.58368	.47575
	:	.52788	-.03722	.39603	-.82937	.58368	-.47575	.98933	.00000

B	:	.87598	-.20474	.93374	.15244	1.00000	-.03217	.26835	-.66954
BNORM	:	1.79602							
R	:	.87598	-.20474	.93374	.15244	1.00000	-.03217	.26835	-.66954
RNORM	:	1.79602							
P	:	.87598	-.20474	.93374	.15244	1.00000	-.03217	.26835	-.66954
W	:	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
WNORM	:	.00000							
POC (DB)	:	.00000							

MAIN PROCESS

ITER	:	1							
CP	:	2.36006	-1.35676	3.02676	.82390	3.11667	-.35812	1.78487	-2.01166
PCP	:	10.25097							
ALPHA	:	.31467							
W	:	-.27564	.06442	-.29382	-.04797	-.31467	.01012	-.08444	.21068
WNORM	:	.56515							
R	:	.13333	.22219	-.01869	-.10682	.01927	.08052	-.29330	-.03653
RNORM	:	.41608							
RORDR	:	-12.70271							
BETA	:	.05367							
P	:	.18035	.21121	.03142	-.09864	.07294	.07879	-.27890	-.07246

POC (DB)	6.21337							

ITER	2							
CP	.07301	.12124	-.01088	-.05733	.01083	.04451	-.16216	-.02075
PCP	.09511							
ALPHA	1.82012							
W	-.60390	-.32000	-.35102	.13157	-.44744	-.13328	.42319	.34257
WNORM	1.05925							
R	.00044	.00152	.00112	-.00247	-.00043	-.00050	.00186	.00124
RNORM	.00391							
ROBDR	-53.24521							
BETA	.00009							
P	.00045	.00154	.00112	-.00248	-.00042	-.00049	.00183	.00123
POC (DB)	25.30331							

ITER	3							
CP	.00000	.00002	.00000	-.00002	-.00001	-.00000	.00001	.00002
PCP	.00000							
ALPHA	113.66029							
W	-.65553	-.49466	-.47823	.41369	-.39919	-.07727	.21497	.20243
WNORM	1.15185							
R	.00042	-.00055	.00083	.00001	.00122	-.00038	.00052	-.00071
RNORM	.00189							
ROBDR	-59.56723							
BETA	.23324							
P	.00053	-.00019	.00109	-.00057	.00112	-.00049	.00094	-.00042
POC (DB)	27.83153							

ITER	4							
CP	.00199	-.00206	.00333	.00022	.00309	-.00114	.00158	-.00249
PCP	.00001							
ALPHA	.30923							
W	-.65569	-.49460	-.47857	.41386	-.39954	-.07712	.21468	.20256
WNORM	1.15220							
R	-.00020	.00009	-.00020	-.00006	.00026	-.00002	.00003	.00006
RNORM	.00040							
ROBDR	-73.03284							
BETA	.04502							
P	-.00017	.00008	-.00015	-.00008	.00031	-.00005	.00007	.00004
POC (DB)	27.83369							

COMPOSITE WEIGHT-VECTOR ARRAY

ADAPTIVE WEIGHT VECTOR AT INDICATED HCR ITERATION
NORMALIZATION CONSTANT : .267978E 00

.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
-.42039	.09826	-.44811	-.07316	-.47991	.01544	-.12878	.32132
-.92101	-.48803	-.53534	.20066	-.68239	-.20327	.64541	.52246
-.99975	-.75441	-.72935	.63092	-.60881	-.11785	.32786	.30872
-1.00000	-.75432	-.72987	.63119	-.60934	-.11762	.32741	.30892

COMBINED PORT

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .126408E-01

-.01670	-.01049	.14383	-.10235	.14944	-.11248	.06150	-.11227
.07382	-.10519	.03994	-.09959	.06327	-.08707	.00489	-.05748
-.04627	.00542	-.18414	.05696	.15259	-.06112	-.02997	.03696
-.17560	.07747	-.06304	.06596	.13334	-.05205	.07868	-.02439
-.03603	-.02509	.00239	.02126	-.23412	.08834	.15389	-.04995
-.00440	.02746	-.00968	-.01289	-.04717	.05324	-.02649	-.00255
.20711	-.07578	-.08196	.01541	.01355	-.02213	-.09969	.07687
-.13751	.06353	.12372	-.00784	-.06997	.06417	-.12808	.07315
.11987	-.02488	.08726	-.01470	-.08657	.02446	-.01831	.02120
-.04195	.04467	-.05173	.05323	-.09084	.07811	.01416	.03828
-.03394	.05041	.06279	-.02406	.10788	-.03024	-.03759	.00075
-.00085	-.01482	.03556	-.02295	-.00774	.016'3	-.14218	.07335
-.03194	.04391	.07460	-.01148	-.00278	.04103	-.03552	-.00372
.05876	.01367	-.03805	.01610	-.13111	.07019	.22880	-.11424
-.02414	.04605	-.12771	.02026	-.14863	.12014	.22718	-.11243
.01964	.03027	-.05866	-.03662	.05112	-.01256	.00131	-.00930
.15456	-.16619	-.75158	.66713	.59752	-.56878	-.08748	.22881
.21586	-.24867	.19722	-.23537	-.07623	.06640	-.73105	.61691
.32139	-.29233	-.20923	.29246	-.01967	-.00760	.09385	-.04655
.18778	-.09700	-.09134	.04205	-.34222	.33680	.24419	-.22873
-.45724	.46156	1.00000	-.91192	-.61246	.50680	.29627	-.24268
.29625	-.25814	.20903	-.27272	-.62749	.57917	-.37235	.35083
.63456	-.52692	-.69756	.68432	-.39926	.38732	.14926	-.01996
.85295	-.73510	-.16309	.07449	-.66763	.55854	.75212	-.66064
.01341	-.03526	-.69258	.73944	-.24526	.31619	-.23375	.27878
.46264	-.40255	.48592	-.32340	-.34504	.26844	.20783	-.11532
-.08616	.09817	-.23051	.29481	-.31636	.36774	.66015	-.67401
.71557	-.61492	-.10428	.05072	-.14769	.14339	-.33023	.15049
-.13876	.21766	-.35172	.41962	.41336	-.44316	-.00564	.08085
-.21176	.14628	.56232	-.53412	-.30132	.30024	.16136	-.14340
-.35803	.33924	-.09190	.05492	.17061	-.14795	-.07255	.05255
.02042	.05658	-.19098	.16433	-.04454	.10904	.47259	-.45599
-.09366	-.00145	-.30710	.32865	.13550	-.06540	-.25990	.17584
.01721	-.04248	.05339	.04622	.08226	-.05678	-.10841	.06307
.05532	-.04627	.10709	-.05654	.18944	-.12552	-.21587	.08893

-.07324	-.00154	.10222	.00735	-.07733	.02760	-.06023	.06030
-.09777	.07709	.02994	.01491	.06628	-.02415	.11254	-.05851
-.02752	-.00970	-.00931	-.03301	.12070	-.08493	.09387	-.07960
-.05981	-.02029	-.02706	-.00777	-.07293	.03518	-.00850	.00696
.05559	-.01537	.08360	-.05028	-.02128	-.00699	.0134[illegible]	-.04735
.09804	-.05312	-.04834	-.02308	.00419	-.02219	-.03068	-.01036
.10980	-.06506	.02766	-.04004	.06713	-.07550	.02190	-.04442
.00719	-.05677	.09756	-.07357	-.11379	.04412	-.24319	.10245
.12366	-.03743	.17102	-.06580	.00006	-.04051	-.03672	-.00064
-.05894	-.00186	.09358	-.05978	.02086	-.02218	-.15183	.06568
-.10077	.07039	.11396	-.02213	-.05236	.04401	.08974	-.07408
.00656	.03012	.02286	-.07086	.02804	.02239	-.12947	.02208
.02558	-.01040	.14395	-.05864	-.16196	.07126	.08782	-.07619
-.22439	.32045	-.38254	.38641	.31745	-.19790	-.18275	.13129
.83810	-.83896	.37085	-.34375	.09616	-.06534	-.91709	.82068
.26865	-.37550	-.12872	.23669	-.01732	.00807	.94009	-.83172
-.77434	.58803	-.09391	.09209	.35919	-.31995	.61833	-.66909
-.59254	.50054	-.85603	.77148	-.20838	.28414	.56144	-.46373
-.48409	.53126	.44789	-.46634	.47511	-.37896	-.30040	.32135
.10385	-.12835	-.14436	.11038	-.09353	.17408	-.25002	.26283
-.12145	.11054	-.27916	.24058	.88585	-.73991	.18070	-.12501
-.18137	.21236	-.00264	-.05433	.37051	-.36541	-.36117	.26874
.10448	-.10008	-.06810	.11226	.00511	-.01225	.28937	-.33322
-.69197	.61217	.22171	-.20186	.72575	-.65381	-.44997	.45193
-.38829	.28314	.38672	-.39540	.07265	-.12809	.27441	-.27290
-.22538	.19496	-.04970	-.03981	-.08706	.00065	.19438	-.16201
-.64833	.58095	-.70139	.67606	.81407	-.70394	.05017	-.01401
-.68414	.61100	.23980	-.18562	.02752	.12761	-.29388	.31504
.71437	-.74236	.11101	-.06283	-.23040	.13537	.08229	-.05596

SUPPRESSION (DB) : 27.78702

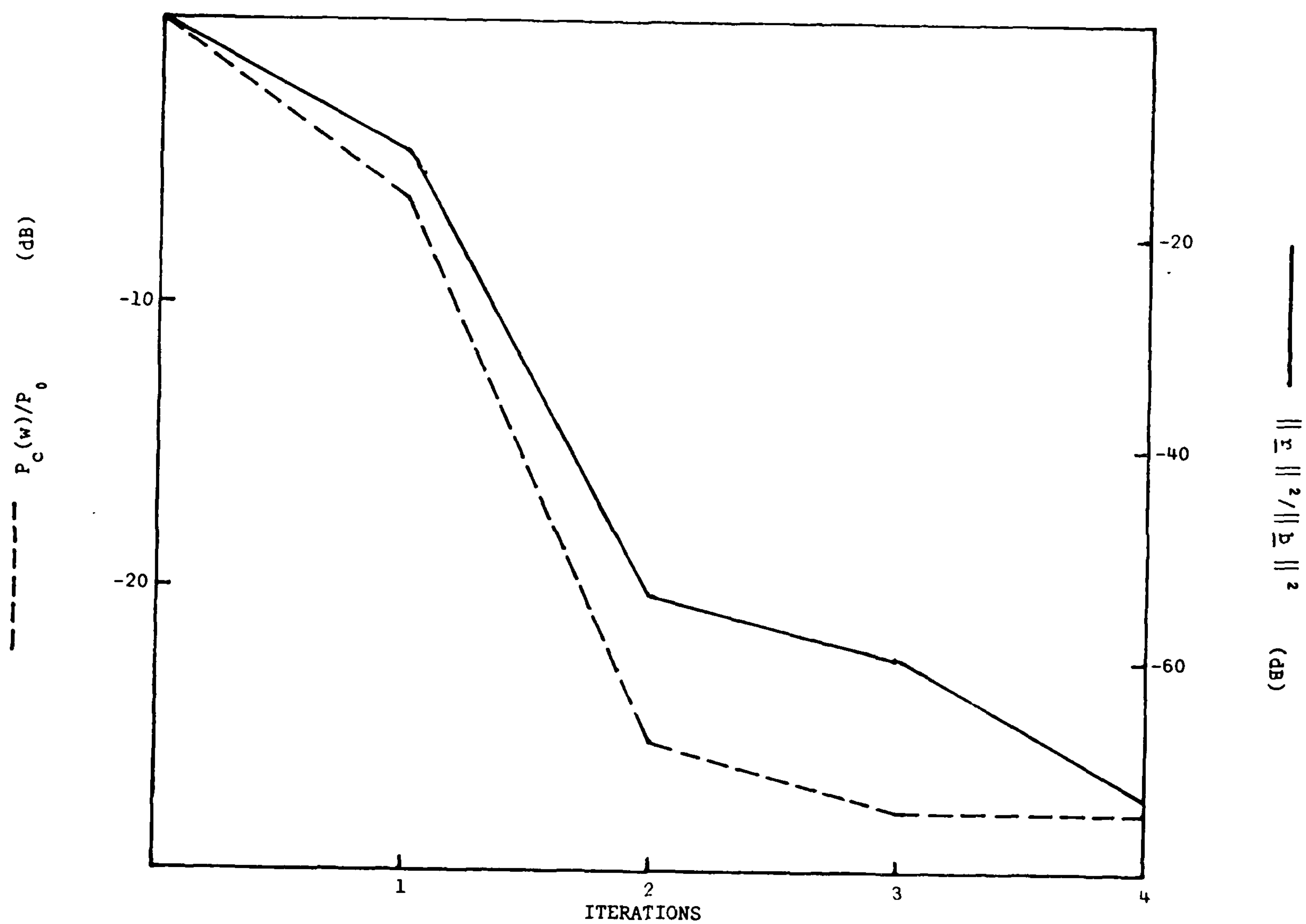


Figure 3-2. BCR Convergence Characteristics
Blinking Source Example
(See Figure 3-1)

The input data file, BCRP:D, for the BCR plotting load module, BCRP:L, is constructed by modifying BCRS:O. The actual number of BCR iterations is entered in the location marked with an X in the BCRS:DO header. Subsequently, the output between the end of the "PORT 0" listing and the beginning of the "COMPOSITE WEIGHT-VECTOR ARRAY" listing is deleted. The resulting input data file, BCRP:D, is used to execute load module BCRP:L yielding an output file BCRP:O containing numerical performance information accompanied by a corresponding graphical representation. Output file BCRP:O is listed in Table 3-3 and is immediately followed by the graphical output in Figure 3-3, which is self-explanatory.

The present example serves to demonstrate the nulling capability of the BCR process in a dynamic interference environment. Although a dynamic algorithm would tend to exhibit a transient behavior in the presence of a blinking source, the BCR process is essentially immune to this undesired phenomenon, as evidenced in the controlled signal amplitude of Figure 3-3(b).

Figure 3-3(c) describes the evolution of the BCR-adapted weight-vector components in the complex plane in four iterations, beginning with an initial estimate of $\underline{0}$. Because the fourth iteration accounts for a relatively infinitesimal change in weight-vector value, only three distinct breaks may be observed in the weight-component loci.

Figure 3-3(d) gives the baseband channel amplitude response for each of the two sources before and after BCR adaptation. Note that, even for the rather low RF bandwidth of 0.1% in this case, the source transfer functions through the main antenna exhibit a non-symmetric amplitude response, a phenomenon which will be more pronounced with increased bandwidth. The adopted channel amplitude responses shown exhibit a stopband behavior within the capability of the four degrees of freedom provided via the four-component adaptive weight vector.

Figure 3-3(e) depicts the field pattern amplitudes, before and after BCR adaptation, over a 0.25° window about each of source angles of arrival, evaluated at the center RF frequency. Since the sources are concentrated at specific angles of arrival, the adapted patterns exhibit sharply defined nulls. Note that, in contrast, the adapted channel amplitude responses in Figure 3-3(d) simply describe the frequency dependence of the adapted pattern nulls over twice the 0.1% RF bandwidth referred to normalized baseband.

Finally, Figure 3-3(f) shows the effect of adaptation to the main bean. Clearly, the adapted main beam differs almost imperceptibly from the original main beam. This desired behavior is essentially due to the fact that the auxiliary antenna gains determined by the adaptive weights were necessarily low so as to match the main antenna sidelobe gains at the two angles of arrival.

3.1.2 Source-Strength Variation

The next two examples address the BCR nulling performance with equal and unequal source strengths. The latter case is of particular interest since it can give rise to an ill-conditioned covariance matrix.

Table 3-3. BCR Plotting Program Output File, BCRP:0
Blinking Source Example (See Figure 3-1)

```
-------------------------------------------------------------------------------
                     DIGITAL ADAPTIVE ARRAY PROCESSING
                     ---------------------------------
                                  USING
                                  -----
                        BATCH COVARIANCE RELAXATION
                        ---------------------------

-------------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWS0   -  OPTIONAL OUTPUT OF  S0                  :          1
  IWS1   -  OPTIONAL OUTPUT OF  S1                  :          0
  IWCB   -  OPTIONAL OUTPUT OF  C  AND  B           :          1

BCR PARAMETERS
--------------
  NSX    -  MAXIMUM NUMBER OF SIGNAL SAMPLES        :        256
  IWO    -  INITIAL WEIGHT-VALUE OPTION             :          0
  NWX    -  MAXIMUM NUMBER OF WEIGHTS               :         20
  ISW    -  WEIGHT SELECTOR ARRAY                   :    1 1 1 1 0 0 0 0 0 0
                                                    :    0 0 0 0 0 0 0 0 0 0
  ITERX  -  MAXIMUM NUMBER OF ITERATIONS            :         20
-------------------------------------------------------------------------------
  ITER   -  ACTUAL NUMBER OF ITERATIONS             :          4
-------------------------------------------------------------------------------
```

```
--------------------------------------------------------------------------------
                    SPECIFICATION OF SYSTEM PARAMETERS
                    ----------------------------------

--------------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWANT  -  MAIN ANTENNA ARRAY WEIGHTING                :         0
  IWRAP  -  RECEIVER AMPLITUDE AND PHASE                :         0
  IWRI   -  RECEIVER IMPULSE RESPONSE                   :         0
  IWCAP  -  CHANNEL AMPLITUDE AND PHASE                 :         0
  IWCI   -  CHANNEL IMPULSE RESPONSE                    :         0
  IWSC   -  INDIVIDUAL CHANNEL SIGNALS                  :         0

FILTER PARAMETERS
-----------------
  NPOL   -  NUMBER OF LOWPASS PROTOTYPE POLES           :         2
            POL(1)                                      :   -.70700   -.70700
            POL(2)                                      :   -.70700    .70700
  FBIF   -  FRACTIONAL BANDWIDTH AT FINAL IF            :    .10000
  FBRF   -  FRACTIONAL BANDWIDTH AT RF                  :    .00100
  RADRNG -  NORMALIZED RADIAN FREQUENCY RANGE           :   4.00000
  RADIN  -  NORMALIZED INITIAL RADIAN FREQUENCY         :  -2.00000
  NF     -  NUMBER OF FREQUENCY SAMPLES                 :        32
  LPBP   -  LOWPASS/BANDPASS OPTION                     :         1
            0  :   LOWPASS
            1  :   BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS -  NUMBER OF CHANNELS PER PORT                 :        20
  ISC    -  CHANNEL SELECTOR ARRAY                      :   1 1 0 0 0 0 0 0 0 0
                                                        :   0 0 0 0 0 0 0 0 0 0

CHANNEL  1:
  AN     -  AMPLITUDE OF NOISE SOURCE                   :   1.00000
  IX0    -  INITIAL RANDU SETTING                       :         1
  N1     -  FIRST-TIME-ON SAMPLE NUMBER                 :         1
  NB     -  BLINK DURATION IN SAMPLES                   :     10000
  TH     -  AZIMUTH ANGLES OF INCIDENCE (DEG)           :  45.00000
  CDEL   -  CHANNEL DELAY IN SAMPLE-TIME UNITS          :    .00000
```

```
CHANNEL  2:
  AN      -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IXO     -  INITIAL RANDU SETTING                     :          11
  N1      -  FIRST-TIME-ON SAMPLE NUMBER               :          64
  NB      -  BLINK DURATION IN SAMPLES                 :          64
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :    10.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :      .00000

PORT PARAMETERS
---------------
  D0      -  ANTENNA-ELEMENT SEPARATION FACTOR         :     1.00000
  NS      -  NUMBER OF SIGNAL SAMPLES                  :         256
  NPORTS  -  NUMBER OF PORTS                           :          21
  ISP     -  PORT SELECTOR ARRAY                       :     1 1 1 1 0 0 0 0 0 0
                                                       :     0 0 0 0 0 0 0 0 0 0

PORT     0:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :         255
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :      .00000

PORT     1:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :      .00000

PORT     2:
  NEL     -  NUMBER OF ANTFNNA ELEMENTS                :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :         255
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :      .00000

PORT     3:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :         128
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :      .00000
```

```
PORT      4:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                 :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT              :         97
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                 :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                    :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY         :     .00000
-------------------------------------------------------------------------
```

PORT 0

RECEIVED BASEBAND PORT SIGNAL

NORMALIZATION CONSTANT : .344914E 00

.00573	.00859	-.04709	-.09054	-.01194	-.35142	.01546	-.38613
.01198	-.40043	.01519	-.36700	.00996	-.37752	.02541	-.31855
.04065	-.22620	.04685	.09405	-.06996	.03809	.03776	-.13504
.04276	.17080	-.01136	.29592	-.06901	.14884	-.00516	-.14954
.01320	-.07478	.01380	-.10447	.05337	.20487	-.08377	.15101
.01996	-.07612	-.00393	.03382	.02139	.02428	-.01601	.16832
-.05785	-.12442	.05173	-.16691	-.00815	-.02534	.04238	.00130
.00947	.27782	-.05680	.12022	.03213	.04278	.01682	.26613
-.06523	.17723	-.01004	-.09962	.03341	-.01376	-.00494	.07284
.00753	.08830	.00358	.14884	.00954	.22813	-.02532	.22395
.00052	.17930	-.03955	.13508	-.01979	-.08910	.02608	-.07763
-.00211	-.01634	-.01043	-.05144	.01648	-.08650	.04063	.10060
-.01661	.22515	-.03288	.11055	.00527	.02165	.00812	.07103
-.02686	.05376	.02656	-.05850	.02078	.23930	-.09187	-.04907
.05253	-.15603	.02687	-.01275	.03839	.37230	-.12970	.01342
.07369	-.08983	-.04702	-.12781	.07513	.09918	-.13046	-.24395
.24740	.13446	-.19906	.03104	-.06533	.17537	.06554	.28412
.07762	-.10419	.42961	.45015	.48333	.44110	.15388	.36659
-.00762	.21394	.11306	.32985	-.15617	-.12084	.00465	.14188
-.20557	-.46274	.06536	-.24025	-.53974	-.70610	-.22471	-.45208
-.67290	-.51290	-.14201	-.36826	.14857	.49093	-.14552	.14242
.38237	.53824	.63881	.80607	.62690	1.00000	-.08110	.26225
.29137	.36947	.16512	.30688	-.46263	-.30750	-.63580	-.53182
-.27415	-.60615	.33277	.09949	-.28668	-.03339	-.06053	-.03474
.49376	.70079	-.10112	.13201	-.52192	-.48443	-.71622	-.50218
-.61593	-.26686	-.09291	.07050	-.07621	.09941	-.18222	-.02348
-.08995	-.07929	-.30987	-.16201	-.65158	-.65299	-.37928	-.23394
.33474	.37284	.51056	.42029	.32195	-.00121	.28417	.21228
-.02959	.18669	-.37983	-.48275	-.29315	-.38017	.00971	.05183
-.26371	-.20948	.19034	.30418	.19698	.39682	.12739	.10579
.13262	.18191	-.11733	.05332	.12822	.32677	.15025	.47797
.09767	.23067	.05577	.14271	-.18969	-.08525	.06917	-.13408
.46172	.57038	.04482	.22257	.01098	-.14106	.04026	.03297
-.00780	.27753	-.05780	.12229	.00912	-.05465	.01959	.04948
-.01560	.06675	-.04312	-.08669	-.01674	-.36462	.09917	-.19575

-.00401	.18495	-.03807	-.00944	.03242	.01053	.01083	.12884
.01124	.24960	-.03385	.23288	-.02981	.07963	-.02360	-.11030
.03007	-.13534	-.00096	-.02858	-.03292	-.15392	.00073	-.32400
.04371	-.23959	.01453	-.09561	.02498	.02000	-.01083	.10257
-.01767	.01175	-.01590	-.09401	.02566	-.12352	-.00990	-.03406
-.01343	-.18941	.02940	-.13987	.00399	-.08386	.00835	-.01580
-.03166	-.10493	.01248	-.19856	-.00788	-.19656	.01765	-.24069
.00507	-.16559	-.01261	-.25602	.06632	-.18670	.05328	.25159
-.08166	.24028	-.03650	-.15976	.02407	-.20896	.02550	-.12491
.01298	.02310	-.03782	-.04007	.01171	-.17186	.05393	.02871
-.00275	.26110	-.04911	.09570	.02136	.03048	-.03746	.01572
.01396	-.09923	-.00699	-.05579	.00717	-.09491	.03879	-.01844
-.02385	.16789	-.04315	-.17960	.07955	.01614	-.05392	.03416
-.14536	-.23619	-.73208	-.89993	-.69023	-.97140	-.68303	-.75570
-.22940	-.12453	.38024	.50644	.50302	.32913	.10274	-.06014
-.11849	-.02372	.13491	.43974	-.35689	-.30247	.35906	-.02126
.35205	.23795	-.16448	-.13844	-.03571	-.25998	.54954	.27175
.61324	.50681	-.11482	.04576	-.63714	-.41098	-.29919	-.13432
-.29261	.03964	-.37494	-.13848	.33201	.59174	.08889	.14726
.12184	-.04950	.06007	.09520	-.15404	-.28557	-.47699	-.73488
-.47237	-.61200	-.69652	-.33832	-.34242	-.12826	.17787	.29562
-.02565	-.15289	-.05875	-.21879	.20660	.11016	.30049	.41482
.08415	.21406	.16405	.14420	-.06372	-.21187	.36797	.19835
.10707	.37959	-.17839	-.03502	.39013	.34279	.38586	.18749
-.13078	-.36211	.08341	-.03722	.28159	.33073	.46880	.39598
.42048	.23824	.34860	.11143	.36457	.41940	.39107	.31512
.30864	.33590	-.54464	-.30195	-.36961	-.34344	.20714	.29473
-.22027	.08410	-.33393	-.03709	-.22947	.02295	-.51560	-.50495
-.11638	-.03126	.39826	.59778	.26749	.50261	.35425	.56253

COMPOSITE WEIGHT-VECTOR ARRAY

ADAPTIVE WEIGHT VECTOR AT INDICATED RCR ITERATION
NORMALIZATION CONSTANT : .267978E 00

.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
-.42039	.09826	-.44811	-.07316	-.47991	.01544	-.12878	.32132
-.92101	-.48803	-.53534	.20066	-.68239	-.20327	.64541	.52246
-.99975	-.75441	-.72935	.63092	-.60881	-.11785	.32786	.30872
-1.00000	-.75432	-.72987	.63119	-.60934	-.11762	.32741	.30892

COMBINED PORT

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .126408E-01

-.01670	-.01049	.14383	-.10235	.14944	-.11248	.06150	-.11227
.07382	-.10519	.03994	-.09959	.06327	-.08707	.00489	-.05748
-.04627	.00542	-.18414	.05696	.15259	-.06112	-.02997	.03696
-.17560	.07747	-.06304	.06596	.13334	-.05205	.07868	-.02439
-.03603	-.02509	.00239	.02126	-.23412	.08834	.15389	-.04995
-.00440	.02746	-.00968	-.01289	-.04717	.05324	-.02649	-.00255
.20711	-.07578	-.08196	.01541	.01355	-.02213	-.09969	.07687
-.13751	.06353	.12372	-.00784	-.06997	.06417	-.12808	.07315
.11987	-.02488	.08726	-.01470	-.08657	.02446	-.01831	.02120
-.04195	.04467	-.05173	.05323	-.09084	.07811	.01416	.03828
-.03394	.05041	.06279	-.02406	.10788	-.03024	-.03759	.00075
-.00085	-.01482	.03556	-.02295	-.00774	.01613	-.14218	.07335
-.03194	.04391	.07460	-.01148	-.00278	.04103	-.03552	-.00372
.05876	.01367	-.03805	.01610	-.13111	.07019	.22880	-.11424
-.02414	.04605	-.12771	.02026	-.14863	.12014	.22718	-.11243
.01964	.03027	-.05866	-.03662	.05112	-.01256	.00131	-.00930
.15456	-.16619	-.75158	.66713	.59752	-.56878	-.08748	.22881
.21586	-.24867	.19722	-.23537	-.07623	.06640	-.73105	.61691
.32139	-.29233	-.20923	.29246	-.01967	-.00760	.09385	-.04655
.18778	-.09700	-.09134	.04205	-.34222	.33680	.24419	-.22873
-.45724	.46156	1.00000	-.91192	-.61246	.50680	.29627	-.24268
.29625	-.25814	.20903	-.27272	-.62749	.57917	-.37235	.35083
.63456	-.52692	-.69756	.68432	-.39926	.38732	.14926	-.01996
.85295	-.73510	-.16309	.07449	-.66763	.55854	.75212	-.66064
.01341	-.03526	-.69258	.73944	-.24526	.31619	-.23375	.27878
.46264	-.40255	.48592	-.32340	-.34504	.26844	.20783	-.11532
-.08616	.09817	-.23051	.29481	-.31636	.36774	.66015	-.67401
.71557	-.61492	-.10428	.05072	-.14769	.14339	-.33023	.15049
-.13876	.21766	-.35172	.41962	.41336	-.44316	-.00564	.08085
-.21176	.14628	.56232	-.53412	-.30132	.30024	.16136	-.14340
-.35803	.33924	-.09190	.05492	.17061	-.14795	-.07255	.05255
.02042	.05658	-.19098	.16433	-.04454	.10904	.47259	-.45599
-.09366	-.00145	-.30710	.32865	.13550	-.06540	-.25990	.17584
.01721	-.04248	.05339	.04622	.08226	-.05678	-.10841	.06307
.05532	-.04627	.10709	-.05654	.18944	-.12552	-.21587	.08893

-.07324	-.00154	.10222	.00735	-.07733	.02760	-.06023	.06030
-.09777	.07709	.02994	.01491	.06628	-.02415	.11254	-.05851
-.02752	-.00970	-.00931	-.03301	.12070	-.08493	.09387	-.07960
-.05981	-.02029	-.02706	-.00777	-.07293	.03518	-.00850	.00696
.05559	-.01537	.08360	-.05028	-.02128	-.00699	.01390	-.04735
.09804	-.05312	-.04834	-.02308	.00419	-.02219	-.03068	-.01036
.10980	-.06506	.02766	-.04004	.06713	-.07550	.02190	-.04442
.00719	-.05677	.09756	-.07357	-.11379	.04412	-.24319	.10245
.12366	-.03743	.17102	-.06580	.00006	-.04051	-.03672	-.00064
-.05894	-.00186	.09358	-.05978	.02086	-.02218	-.15183	.06568
-.10077	.07039	.11396	-.02213	-.05236	.04401	.08974	-.07408
.00656	.03012	.02286	-.07086	.02804	.02239	-.12947	.02208
.02558	-.01040	.14395	-.05864	-.16196	.07126	.08782	-.07619
-.22439	.32045	-.38254	.38641	.31745	-.19790	-.18275	.13129
.83810	-.83896	.37085	-.34375	.09616	-.06534	-.91709	.82068
.26865	-.37550	-.12872	.23669	-.01732	.00807	.94009	-.83172
-.77434	.58803	-.09391	.09209	.35919	-.31995	.61833	-.66909
-.59254	.50054	-.85603	.77148	-.20838	.28414	.56144	-.46373
-.48409	.53126	.44789	-.46634	.47511	-.37896	-.30040	.32135
.10385	-.12835	-.14436	.11038	-.09353	.17408	-.25002	.26283
-.12145	.11054	-.27916	.24058	.88585	-.73991	.18070	-.12501
-.18137	.21236	-.00264	-.05433	.37051	-.36541	-.36117	.26874
.10448	-.10008	-.06810	.11226	.00511	-.01225	.28937	-.33322
-.69197	.61217	.22171	-.20186	.72575	-.65381	-.44997	.45193
-.38829	.28314	.38672	-.39540	.07265	-.12809	.27441	-.27290
-.22538	.19496	-.04970	-.03981	-.08706	.00065	.19438	-.16201
-.64833	.58095	-.70139	.67606	.81407	-.70394	.05017	-.01401
-.68414	.61100	.23980	-.18562	.02752	.12761	-.29388	.31504
.71437	-.74236	.11101	-.06283	-.23040	.13537	.08229	-.05596

SUPPRESSION (DB) : 27.78702

TAYLOR WEIGHTING FOR MAIN ANTENNA ARRAY

.50004	.50034	.50095	.50186	.50307	.50458	.50640	.50851
.51093	.51364	.51664	.51994	.52353	.52741	.53157	.53602
.54076	.54577	.55105	.55661	.56244	.56853	.57489	.58150
.58837	.59549	.60285	.61046	.61830	.62637	.63467	.64319
.65193	.66088	.67004	.67939	.68894	.69868	.70860	.71870
.72897	.73941	.75000	.76074	.77163	.78266	.79382	.80511
.81651	.82802	.83964	.85136	.86317	.87506	.887[illegible]	.89906
.91115	.92330	.93550	.94773	.96000	.97229	.98460	.99692
1.00924	1.02155	1.03385	1.04613	1.05838	1.07060	1.08278	1.09490
1.10697	1.11897	1.13090	1.14275	1.15451	1.16618	1.17774	1.18920
1.20055	1.21177	1.22287	1.23383	1.24465	1.25531	1.26583	1.27618
1.28637	1.29638	1.30621	1.31585	1.32531	1.33456	1.34362	1.35246
1.36109	1.36950	1.37769	1.38565	1.39337	1.40086	1.40810	1.41509
1.42183	1.42832	1.43454	1.44051	1.44620	1.45162	1.45677	1.46164
1.46624	1.47054	1.47457	1.47830	1.48175	1.48490	1.48776	1.49032
1.49258	1.49454	1.49621	1.49757	1.49863	1.49939	1.49985	1.50000
1.49985	1.49939	1.49863	1.49757	1.49621	1.49454	1.49258	1.49032
1.48776	1.48490	1.48175	1.47830	1.47457	1.47054	1.46624	1.46164
1.45677	1.45162	1.44620	1.44051	1.43454	1.42832	1.42183	1.41509
1.40810	1.40086	1.39337	1.38565	1.37769	1.36950	1.36109	1.35246
1.34362	1.33456	1.32531	1.31585	1.30621	1.29638	1.28637	1.27618
1.26583	1.25531	1.24465	1.23383	1.22287	1.21177	1.20055	1.18920
1.17774	1.16618	1.15451	1.14275	1.13090	1.11897	1.10697	1.09490
1.08278	1.07060	1.05838	1.04613	1.03385	1.02155	1.00924	.99692
.98460	.97229	.96000	.94773	.93550	.92330	.91115	.89906
.88702	.87506	.86317	.85136	.83964	.82802	.81651	.80511
.79382	.78266	.77163	.76074	.75000	.73941	.72897	.71870
.70860	.69868	.68894	.67939	.67004	.66088	.65193	.64319
.63467	.62637	.61830	.61046	.60285	.59549	.58837	.58150
.57489	.56853	.56244	.55661	.55105	.54577	.54076	.53602
.53157	.52741	.52353	.51994	.51664	.51364	.51093	.50851
.50640	.50458	.50307	.50186	.50095	.50034	.50004	

BASEBAND CHANNEL AMPLITUDE RESPONSE 1

MAIN-PORT BASEBAND CHANNEL AMPLITUDE RESPONSE

-32.00455	-30.16212	-28.35980	-26.52783	-24.70956	-22.88368	-21.04797	-19.30652
-17.64091	-16.16315	-14.87927	-13.88285	-13.13818	-12.61402	-12.23330	-11.88884
-11.59961	-11.33837	-11.07288	-10.89269	-10.80316	-10.89853	-11.19811	-11.72890
-12.44590	-13.30921	-14.25012	-15.21103	-16.18156	-17.13153	-18.03403	-18.89853

COMPOSITE BASEBAND CHANNEL AMPLITUDE RESPONSE

-50.84145	-49.78555	-49.00256	-47.98703	-47.16280	-46.40019	-45.26120	-44.83762
-44.23944	-44.37779	-43.94479	-44.22459	-44.42810	-45.48190	-47.53238	-47.79137
-50.10378	-51.23555	-51.51321	-49.73190	-49.08292	-47.04507	-45.78110	-45.19345
-44.86234	-44.57906	-44.33565	-44.74121	-44.76089	-44.84375	-45.24339	-45.57330

BASEBAND CHANNEL AMPLITUDE RESPONSE 2

MAIN-PORT BASEBAND CHANNEL AMPLITUDE RESPONSE

-16.65526	-15.43969	-14.17362	-12.85739	-11.49469	-10.09601	-8.68230	-7.28719
-5.96633	-4.79063	-3.82824	-3.12086	-2.65897	-2.38732	-2.23139	-2.13645
-2.05608	-1.97848	-1.91513	-1.90377	-1.99607	-2.25224	-2.71445	-3.39295
-4.25410	-5.24691	-6.31034	-7.40009	-8.48114	-9.53406	-10.54722	-11.51543

COMPOSITE BASEBAND CHANNEL AMPLITUDE RESPONSE

-33.25093	-32.68465	-32.10396	-31.51437	-30.92793	-30.36092	-29.83827	-29.43158
-29.19894	-29.22334	-29.63283	-30.51628	-31.95538	-34.08572	-37.33257	-42.19189
-54.88530	-47.43739	-39.73276	-35.82310	-33.25366	-31.51688	-30.37544	-29.74704
-29.46498	-29.48735	-29.67073	-29.98752	-30.36397	-30.77847	-31.20366	-31.63120

FIELD PATTERN BEFORE AND AFTER BCK ADAPTATION ABOUT INTERFERENCE 1

MAIN FIELD PATTERN

44.87498	-22.91161	44.87889	-24.32169	44.88280	-26.02716	44.88670	-28.07201
44.89061	-30.58412	44.89452	-35.12675	44.89842	-43.23663	44.90233	-47.90111
44.90623	-37.76054	44.91014	-32.07393	44.91405	-28.60013	44.91795	-26.51216
44.92186	-24.73946	44.92577	-23.33789	44.92967	-22.00999	44.93358	-20.88618
44.93748	-19.96809	44.94139	-19.09592	44.94530	-18.28661	44.94920	-17.57083
44.95311	-16.91776	44.95702	-16.31866	44.96092	-15.72820	44.96483	-15.22130
44.96873	-14.71785	44.97264	-14.34896	44.97655	-13.82789	44.98045	-13.37713
44.98436	-13.01956	44.98827	-12.62630	44.99217	-12.26140	44.99608	-11.94249
44.99998	-11.59961	45.00389	-11.44618	45.00780	-11.01364	45.01170	-10.73277
45.01561	-10.45701	45.01952	-10.23641	45.02342	-9.96329	45.02733	-9.72684
45.03123	-9.48957	45.03514	-9.26974	45.03905	-9.06605	45.04295	-8.86177
45.04686	-8.66108	45.05077	-8.49413	45.05467	-8.29583	45.05858	-8.09089
45.06248	-7.96554	45.06639	-7.80245	45.07030	-7.65774	45.07420	-7.50201
45.07811	-7.36855	45.08202	-7.23990	45.08592	-7.07059	45.08983	-6.96996
45.09373	-6.84608	45.09764	-6.73148	45.10155	-6.61768	45.10545	-6.51336
45.10936	-6.40608	45.11327	-6.31509	45.11717	-6.21940	45.12108	-6.12637

COMPOSITE FIELD PATTERN

44.87498	-38.14034	44.87889	-37.82819	44.88280	-37.48940	44.88670	-37.46150
44.89061	-37.80243	44.89452	-36.88557	44.89842	-36.86397	44.90233	-36.70296
44.90623	-37.66563	44.91014	-37.00244	44.91405	-36.08327	44.91795	-36.53522
44.92186	-36.61916	44.92577	-37.02135	44.92967	-36.86353	44.93358	-36.83510
44.93748	-37.28210	44.94139	-37.59386	44.94530	-37.66040	44.94920	-38.01903
44.95311	-38.39398	44.95702	-38.89070	44.96092	-39.15666	44.96483	-40.00496
44.96873	-40.52791	44.97264	-43.32147	44.97655	-42.41611	44.98045	-42.45590
44.98436	-44.66858	44.98827	-45.33760	44.99217	-46.30151	44.99608	-48.75017
44.99998	-50.10388	45.00389	-45.33383	45.00780	-51.83588	45.01170	-51.48274
45.01561	-47.48781	45.01952	-44.20503	45.02342	-43.99231	45.02733	-41.66835
45.03123	-40.93889	45.03514	-39.39371	45.03905	-38.19415	45.04295	-37.44106
45.04686	-36.47298	45.05077	-35.37852	45.05467	-34.83871	45.05858	-34.27171
45.06248	-33.17728	45.06639	-32.55371	45.07030	-31.78815	45.07420	-31.25069
45.07811	-30.60608	45.08202	-29.98918	45.08592	-29.78421	45.08983	-29.11967
45.09373	-28.64655	45.09764	-28.12364	45.10155	-27.74188	45.10545	-27.27008
45.10936	-26.93004	45.11327	-26.45731	45.11717	-26.08177	45.12108	-25.75650

FIELD PATTERN BEFORE AND AFTER RCR ADAPTATION ABOUT INTERFERENCE 2

MAIN FIELD PATTERN

9.87499	-2.32133	9.87890	-2.86640	9.88281	-3.45642	9.88671	-4.09233
9.89062	-4.78596	9.89452	-5.54609	9.89843	-6.38737	9.90234	-7.32136
9.90625	-8.37942	9.91015	-9.58602	9.91406	-10.99620	9.91796	-12.68318
9.92187	-14.79176	9.92578	-17.56418	9.92968	-21.75583	9.93359	-29.92409
9.93749	-34.78946	9.94140	-23.30136	9.94531	-18.53484	9.94921	-15.47561
9.95312	-13.23025	9.95703	-11.45302	9.96093	-9.98432	9.96484	-8.73459
9.96875	-7.65038	9.97265	-6.68963	9.97656	-5.83354	9.98046	-5.05833
9.98437	-4.35577	9.98828	-3.70984	9.99218	-3.11591	9.99609	-2.56558
10.00000	-2.05608	10.00390	-1.57937	10.00781	-1.13508	10.01171	-.71762
10.01562	-.32580	10.01953	.03987	10.02343	.38935	10.02734	.71681
10.03124	1.02605	10.03515	1.31871	10.03906	1.59518	10.04296	1.85667
10.04687	2.10461	10.05078	2.33906	10.05468	2.56164	10.05859	2.77231
10.06250	2.97135	10.06640	3.15682	10.07031	3.33700	10.07421	3.50537
10.07812	3.66414	10.08203	3.81384	10.08593	3.95449	10.08984	4.08673
10.09374	4.21071	10.09765	4.32641	10.10156	4.43475	10.10546	4.53541
10.10937	4.62833	10.11328	4.71412	10.11718	4.79284	10.12109	4.86454

COMPOSITE FIELD PATTERN

9.87499	1.36065	9.87890	1.12533	9.88281	.87754	9.88671	.61987
9.89062	.34903	9.89452	.06506	9.89843	-.23371	9.90234	-.54608
9.90625	-.87606	9.91015	-1.22179	9.91406	-1.58666	9.91796	-1.97075
9.92187	-2.37772	9.92578	-2.80782	9.92968	-3.26701	9.93359	-3.74768
9.93749	-4.26434	9.94140	-4.81832	9.94531	-5.41165	9.94921	-6.05264
9.95312	-6.74499	9.95703	-7.49954	9.96093	-8.32817	9.96484	-9.24529
9.96875	-10.26889	9.97265	-11.43480	9.97656	-12.77526	9.98046	-14.36363
9.98437	-16.29410	9.98828	-18.78267	9.99218	-22.25862	9.99609	-28.11920
10.00000	-54.88399	10.00390	-28.96068	10.00781	-22.77150	10.01171	-19.20776
10.01562	-16.70735	10.01953	-14.79958	10.02343	-13.22743	10.02734	-11.91891
10.03124	-10.79452	10.03515	-9.80941	10.03906	-8.93699	10.04296	-8.15520
10.04687	-7.44717	10.05078	-6.80340	10.05468	-6.21220	10.05859	-5.66830
10.06250	-5.16668	10.06640	-4.70848	10.07031	-4.27060	10.07421	-3.86693
10.07812	-3.49012	10.08203	-3.13747	10.08593	-2.80760	10.08984	-2.49797
10.09374	-2.20736	10.09765	-1.93501	10.10156	-1.67816	10.10546	-1.43707
10.10937	-1.21109	10.11328	-.99872	10.11718	-.79923	10.12109	-.61231

MAIN BEAM PATTERN BEFORE AND AFTER BCR ADAPTATION

MAIN FIELD PATTERN

-.12500	47.35248	-.12109	47.40089	-.11719	47.44769	-.11328	47.49290
-.10937	47.53650	-.10547	47.57854	-.10156	47.61899	-.09766	47.65788
-.09375	47.69519	-.08984	47.73094	-.08594	47.76514	-.08203	47.79778
-.07812	47.82887	-.07422	47.85843	-.07031	47.88644	-.06641	47.91292
-.06250	47.93787	-.05859	47.96129	-.05469	47.98318	-.05078	48.00356
-.04687	48.02240	-.04297	48.03973	-.03906	48.05554	-.03[illegible]	48.06985
-.03125	48.08266	-.02734	48.09395	-.02344	48.10371	-.01953	48.11198
-.01562	48.11876	-.01172	48.12401	-.00781	48.12778	-.00391	48.13004
.00000	48.13077	.00391	48.13004	.00781	48.12778	.01172	48.12402
.01562	48.11876	.01953	48.11198	.02344	48.10371	.02734	48.09395
.03125	48.08266	.03516	48.06985	.03906	48.05554	.04297	48.03973
.04687	48.02240	.05078	48.00356	.05469	47.98318	.05859	47.96129
.06250	47.93787	.06641	47.91292	.07031	47.88644	.07422	47.85843
.07812	47.82887	.08203	47.79778	.08594	47.76514	.08984	47.73094
.09375	47.69519	.09766	47.65788	.10156	47.61899	.10547	47.57854
.10937	47.53650	.11328	47.49290	.11719	47.44769	.12109	47.40089

COMPOSITE FIELD PATTERN

-.12500	47.34837	-.12109	47.39626	-.11719	47.44250	-.11328	47.48715
-.10937	47.53024	-.10547	47.57175	-.10156	47.61169	-.09766	47.65004
-.09375	47.68686	-.08984	47.72211	-.08594	47.75581	-.08203	47.78796
-.07812	47.81857	-.07422	47.84767	-.07031	47.87521	-.06641	47.90123
-.06250	47.92572	-.05859	47.94868	-.05469	47.97014	-.05078	47.99008
-.04687	48.00851	-.04297	48.02544	-.03906	48.04085	-.03516	48.05473
-.03125	48.06714	-.02734	48.07803	-.02344	48.08743	-.01953	48.09534
-.01562	48.10175	-.01172	48.10664	-.00781	48.11006	-.00391	48.11197
.00000	48.11238	.00391	48.11131	.00781	48.10876	.01172	48.10468
.01562	48.09914	.01953	48.09207	.02344	48.08353	.02734	48.07347
.03125	48.06194	.03516	48.04889	.03906	48.03433	.04297	48.01830
.04687	48.00073	.05078	47.98167	.05469	47.96109	.05859	47.93901
.06250	47.91539	.06641	47.89027	.07031	47.86365	.07422	47.83548
.07812	47.80579	.08203	47.77454	.08594	47.74179	.08984	47.70747
.09375	47.67162	.09766	47.63422	.10156	47.59525	.10547	47.55473
.10937	47.51263	.11328	47.46898	.11719	47.42374	.12109	47.37691

Title Page

BCR ADAPTIVE PROCESSING

BLINKING SOURCE EXAMPLE
(See Figure 3-1)

SOURCES	:	1 CONTINOUS (10°) 1 BLINKING (45°)
RF BANDWIDTH	:	0.1%
AUXILIARY PORTS	:	4
TAP WEIGHTS	:	1/PORT

SIMULATION BY : S. M. DANIEL & I. KERTESZ
ADVANCED TECHNOLOGY AND SYSTEMS ANALYSIS
MOTOROLA, INC. - GED

16:09 FEB 17,'81

Figure 3-3

Relative Signal Amplitude Before and After BCR Adaptation

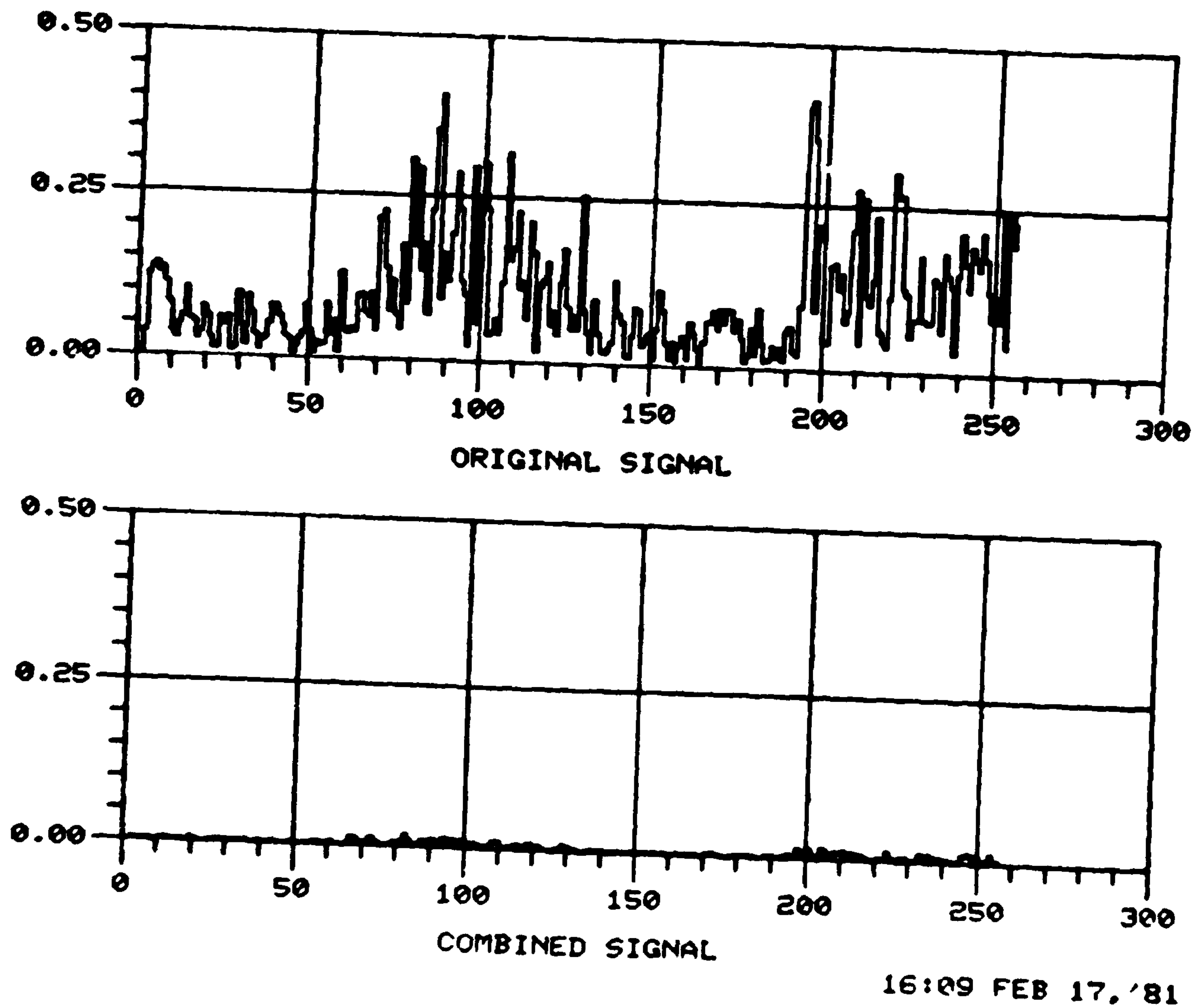


Figure 3-3 (Cont'd)

BCR Adaptive Weight Evolution in the Complex Plane

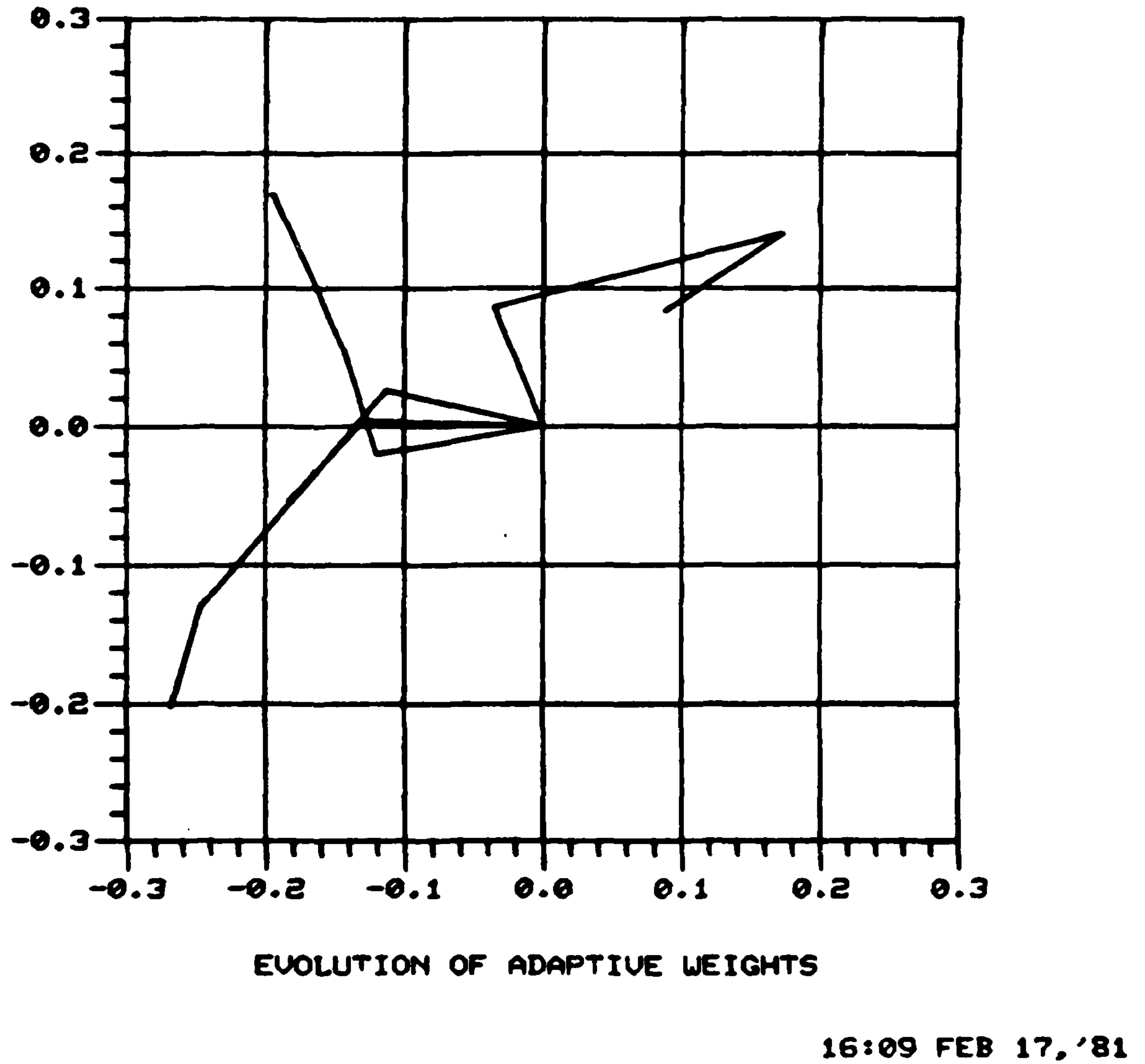


Figure 3-3 (Cont'd)

Source Transfer Function Amplitude Responses Before and After BCR Adaptation

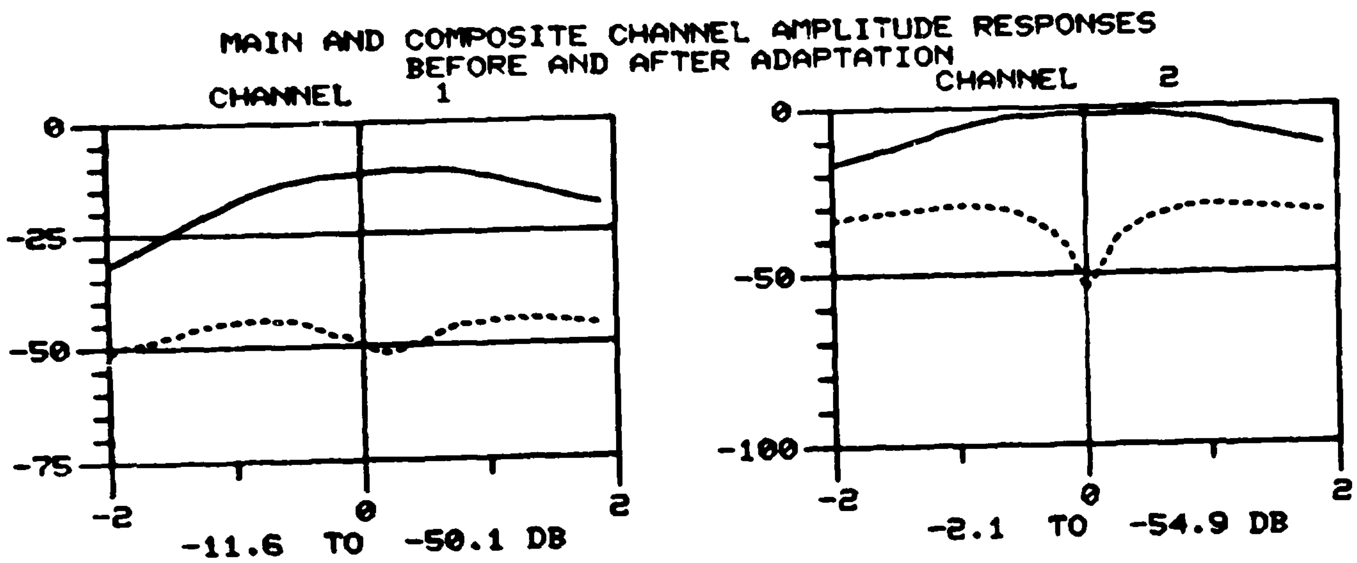


Figure 3-3 (Cont'd)

16:09 FEB 17,'81

Field Pattern Amplitudes Over 0.25 Neighborhoods About Source Angles of Arrival at RF Center

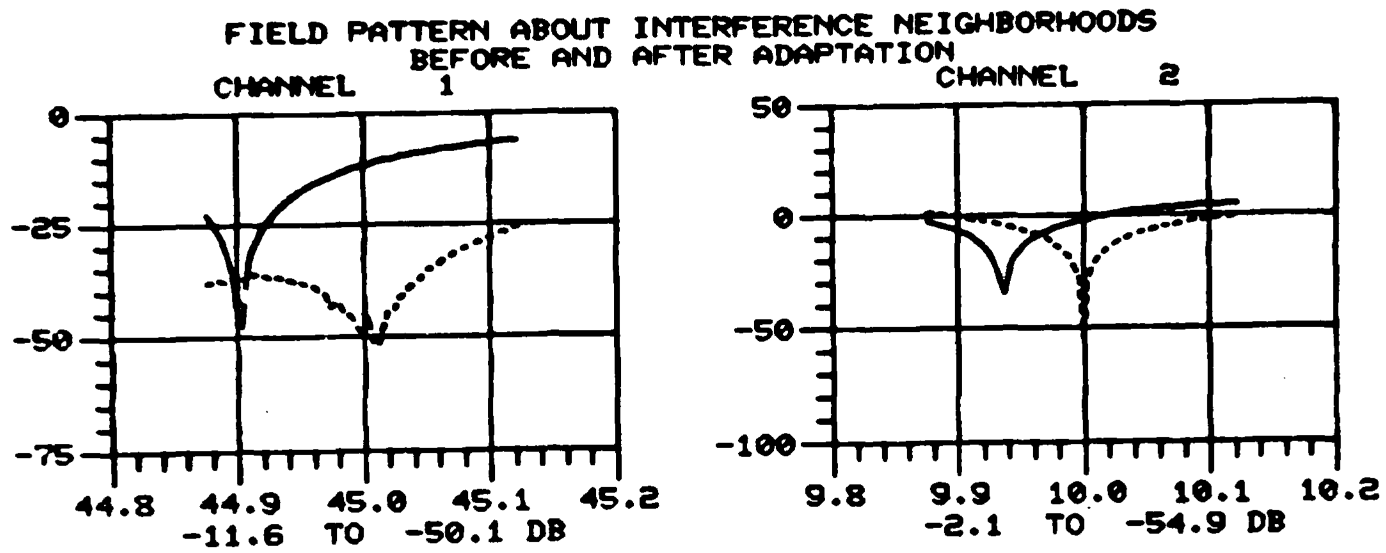


Figure 3-3 (Cont'd)

16:09 FEB 17,'81

Main Beam Field Pattern Amplitude at Center RF Frequency, Before and After BCR Adaptation

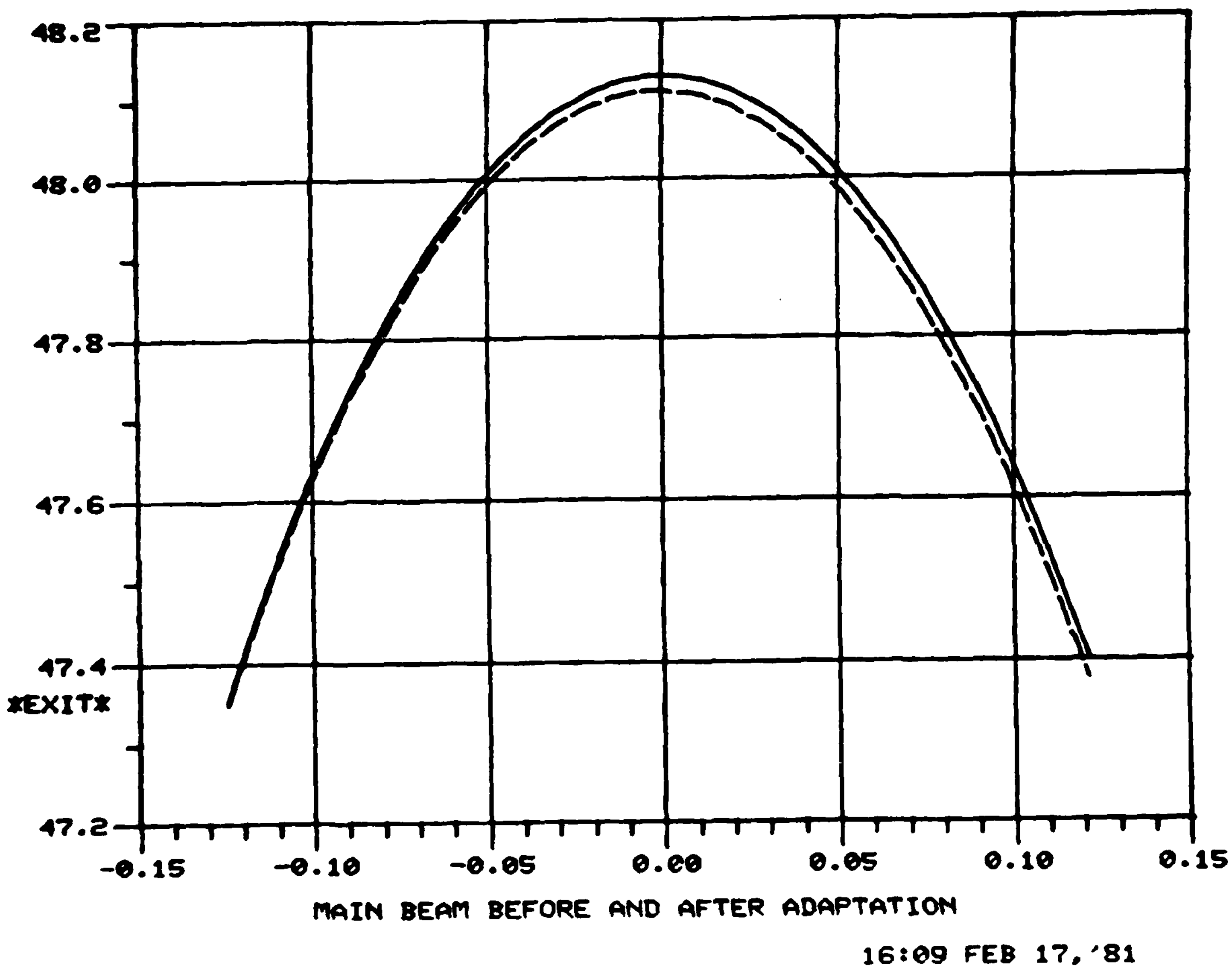


Figure 3-3 (Cont'd)

3.1.2.1 Equal Source Strengths

Figure 3-4 defines the scenario and system description involving six sources of equal strength and six 1-tap ports operating at a 0.1% RF bandwidth. Figure 3-5 shows the observed BCR convergence behavior. Note that from the practical viewpoint of combined power suppression, the BCR process has converged in the sixth iteration. However, it takes an additional iteration to cross below the relative gradient threshold of -60 dB to declare convergence. Figure 3-6 summarizes, graphically, the overall BCR performance for this example.

3.1.2.2 Unequal Source Strengths

Figure 3-7 defines the scenario and system description that is very similar to that of the previous example with the exception that it involves unequal source strengths. Of interest is the BCR convergence behavior shown in Figure 3-8. Note the distinct rise in the relative gradient metric at the fifth iteration accompanied by a rather insignificant drop in relative combined power. The slow convergence behavior beyond the fourth iteration is a manifestation of an ill-conditioned covariance matrix which is attributed to the large dynamic range over the incident source powers. It should also be noted that the nulling performance in the present example is approximately 1.5 dB worst than that of the previous example. This may also be due to the numerically ill-conditioned nature of the present example.

In evaluating the nulling performance in the two examples above, it is interesting to compare the null depths obtained at 30° in each case. In the equal source-strength example, the null depth observed is -32.2 dB. In the unequal source-strength example, the null depth obtained is -11.9 dB. Compensating for the -15 dB power level in the second case, the relative power level incident after adaptation is 5.3 dB higher in the second case. Of course, since this observation is limited to the center RF frequency, it is not an accurate measure of nulling degradation but, rather, an indication. As noted above, the actual nulling degradation over the frequency band of operation has been computed to be 1.5 dB.

3.1.3 Wideband Performance

Given a fixed number of adaptive weights, the adaptive nulling performance of the adaptive array system under consideration will tend to deteriorate with increased bandwidth. The fundamental cause for the degradation in this case is the fact that the channel transfer functions for incident sources will exhibit increasingly irregular characteristics due to the frequency sensitivity in the sidelobe structure of the main antenna pattern. As such, the smooth transfer function associated with an omnidirectional auxiliary antenna cannot be expected to exhibit the necessary matching for a reasonable nulling over an increasing bandwidth of operation. For this reason, a larger number of degrees of freedom than sources is necessary to provide adequate nulling performance. The degrees of freedom may be obtained in number of ports with single or multiple taps.

Of the four wideband examples presented below the first three involve ten 1-tap ports, while the fourth involves two ports with 5 taps each. It is seen that for the same number of adaptive weights, the nulling performance of the multitap approach is inferior to the single-tap multiport option. This

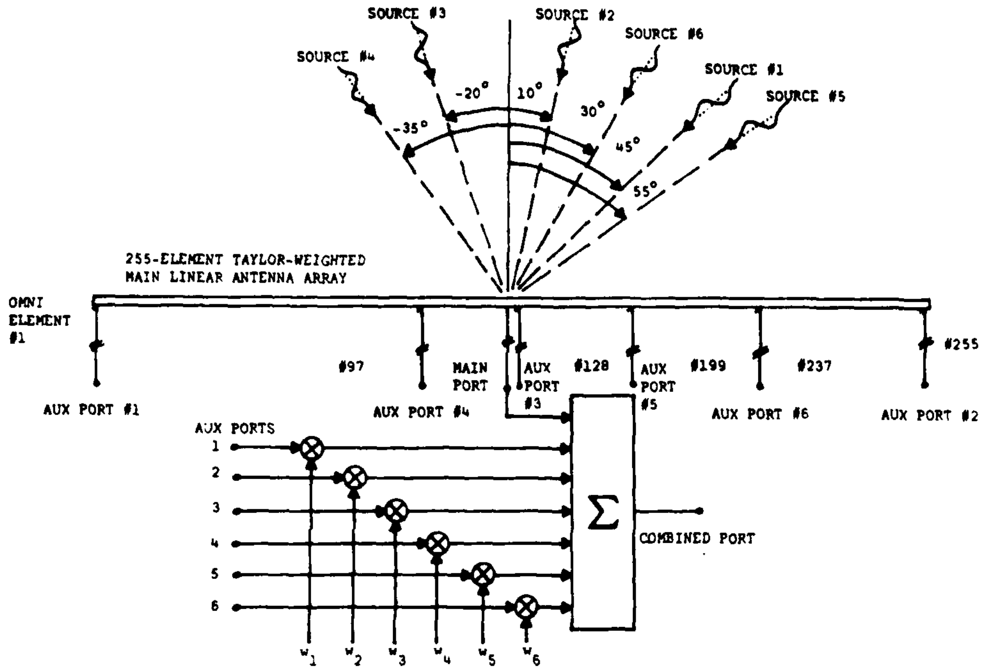


Figure 3-4. Scenario and System Definition
Equal Source-Strength Example
0.1% RF Bandwidth

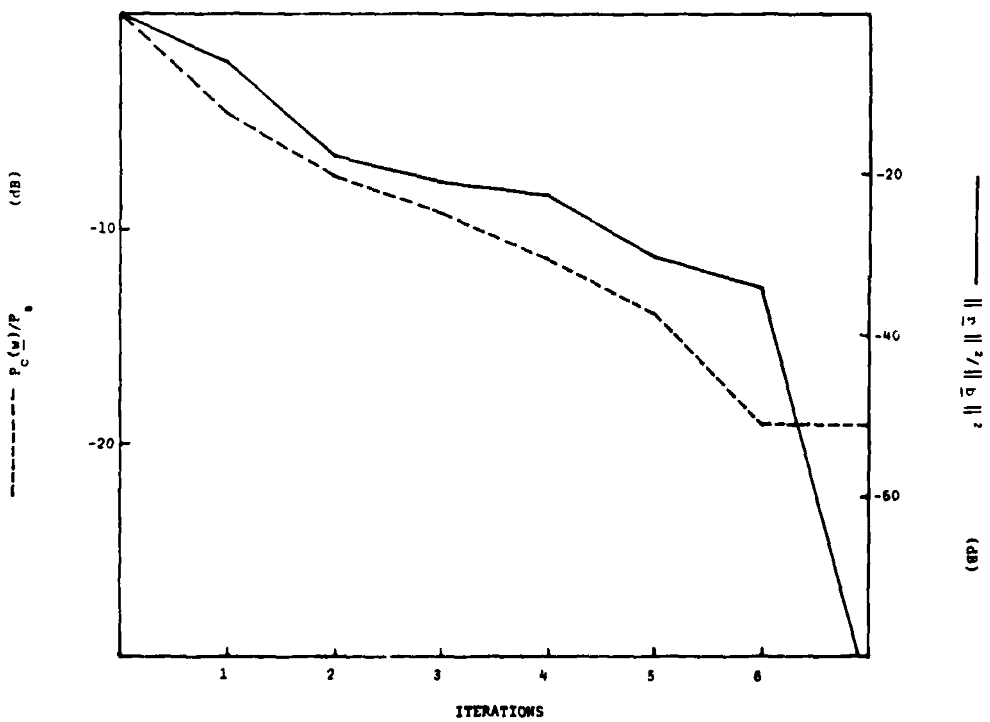


Figure 3-5. BCR Convergence Characteristics
Equal Source Strength Example
(See Figure 3-4)

Title Page

BCR ADAPTIVE PROCESSING

EQUAL SOURCE-STRENGTH EXAMPLE
(See Figure 3-4)

SOURCES	:	6 CONTINUOUS (45°,10°,-20°,-35°,55°,30°)
RF BANDWIDTH	:	0.1%
AUXILIARY PORTS	:	6
TAP WEIGHTS	:	1/PORT

SIMULATION BY : S. M. DANIEL & I. KERTESZ
ADVANCED TECHNOLOGY AND SYSTEMS ANALYSIS
MOTOROLA, INC. - GED

23:58 FEB 15,'81

Figure 3-6

Relative Signal Amplitude Before and After BCR Adaptation

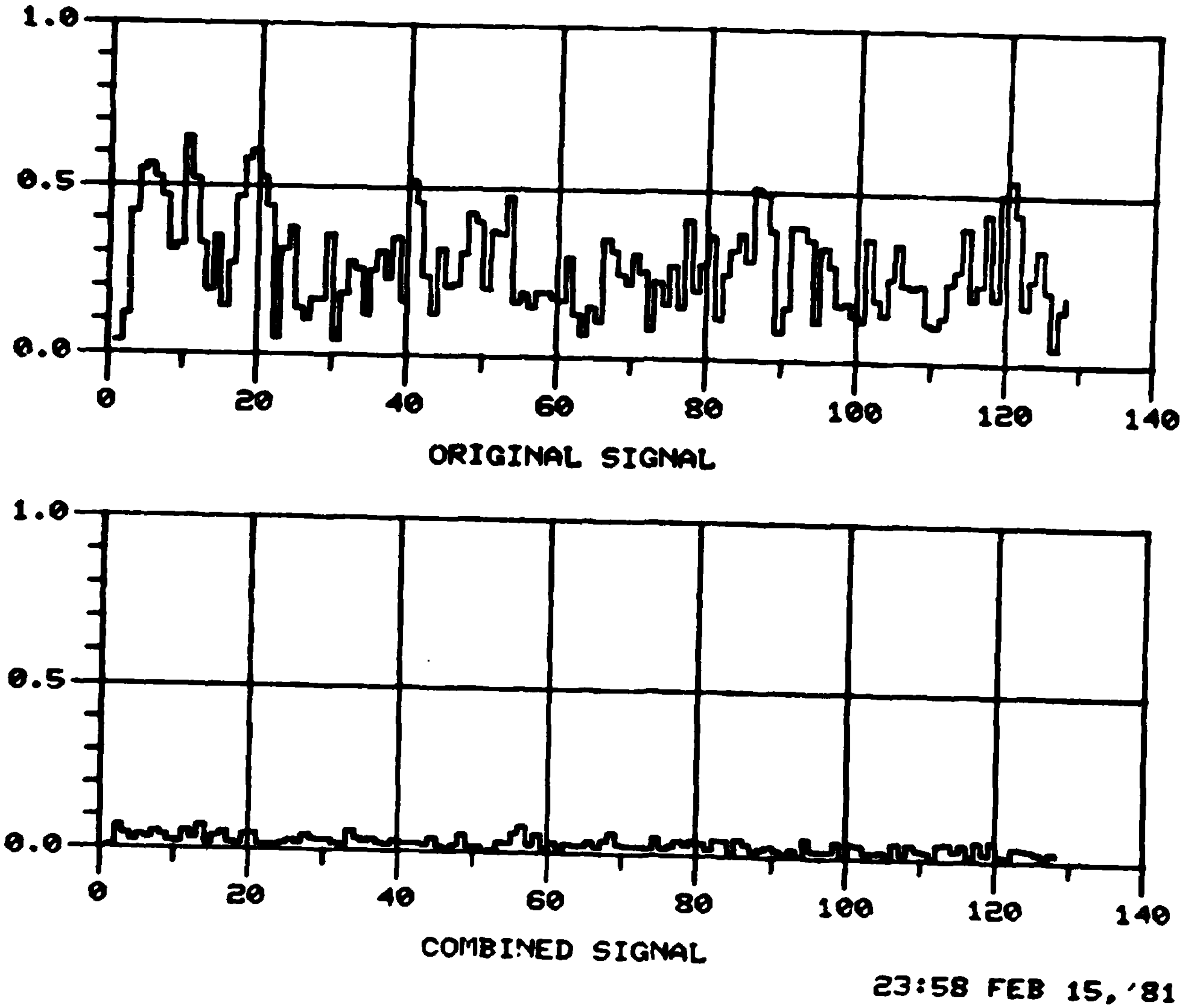


Figure 3-6 (Cont'd)

BCR Adaptive Weight Evolution in the Complex Plane

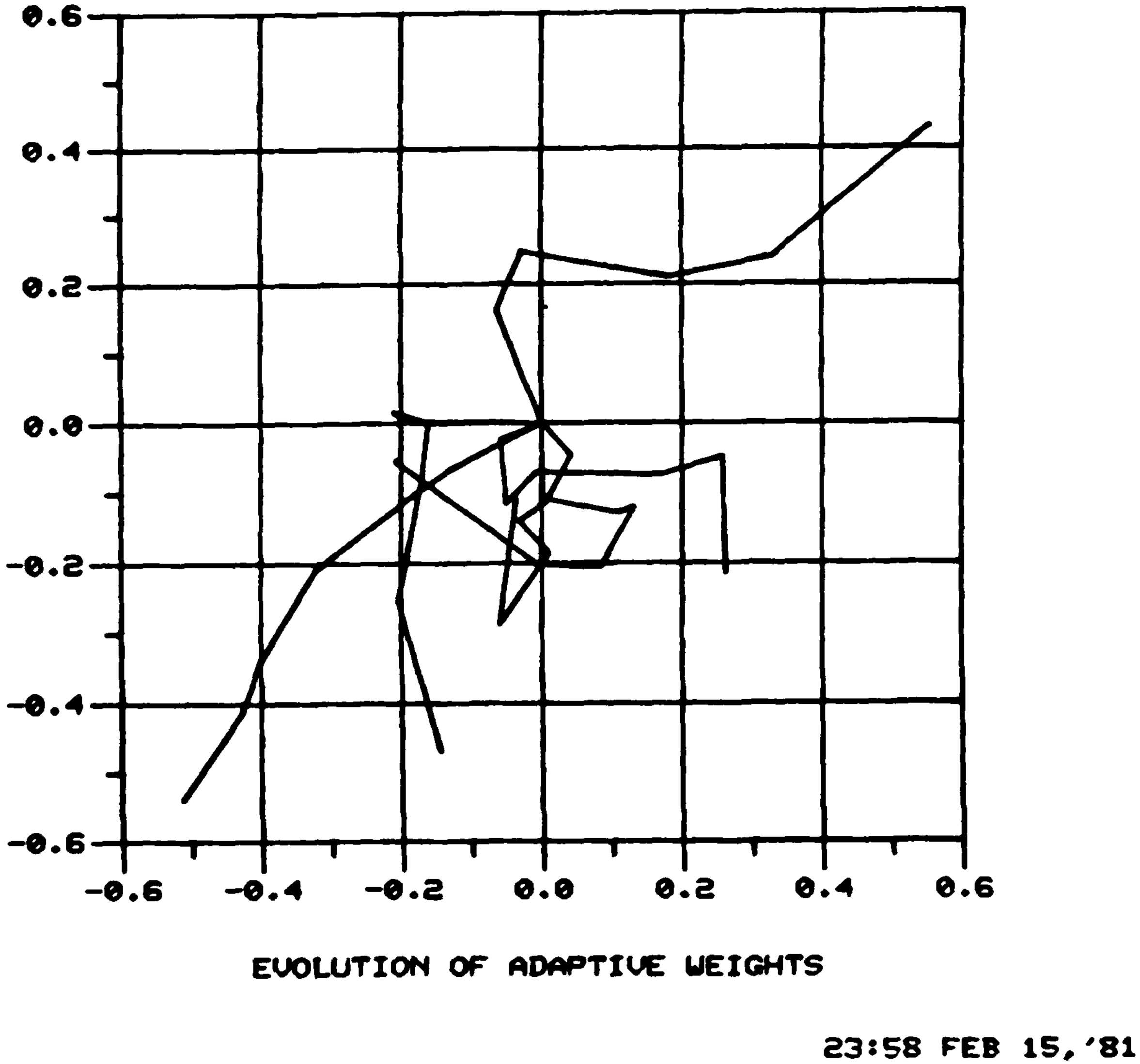
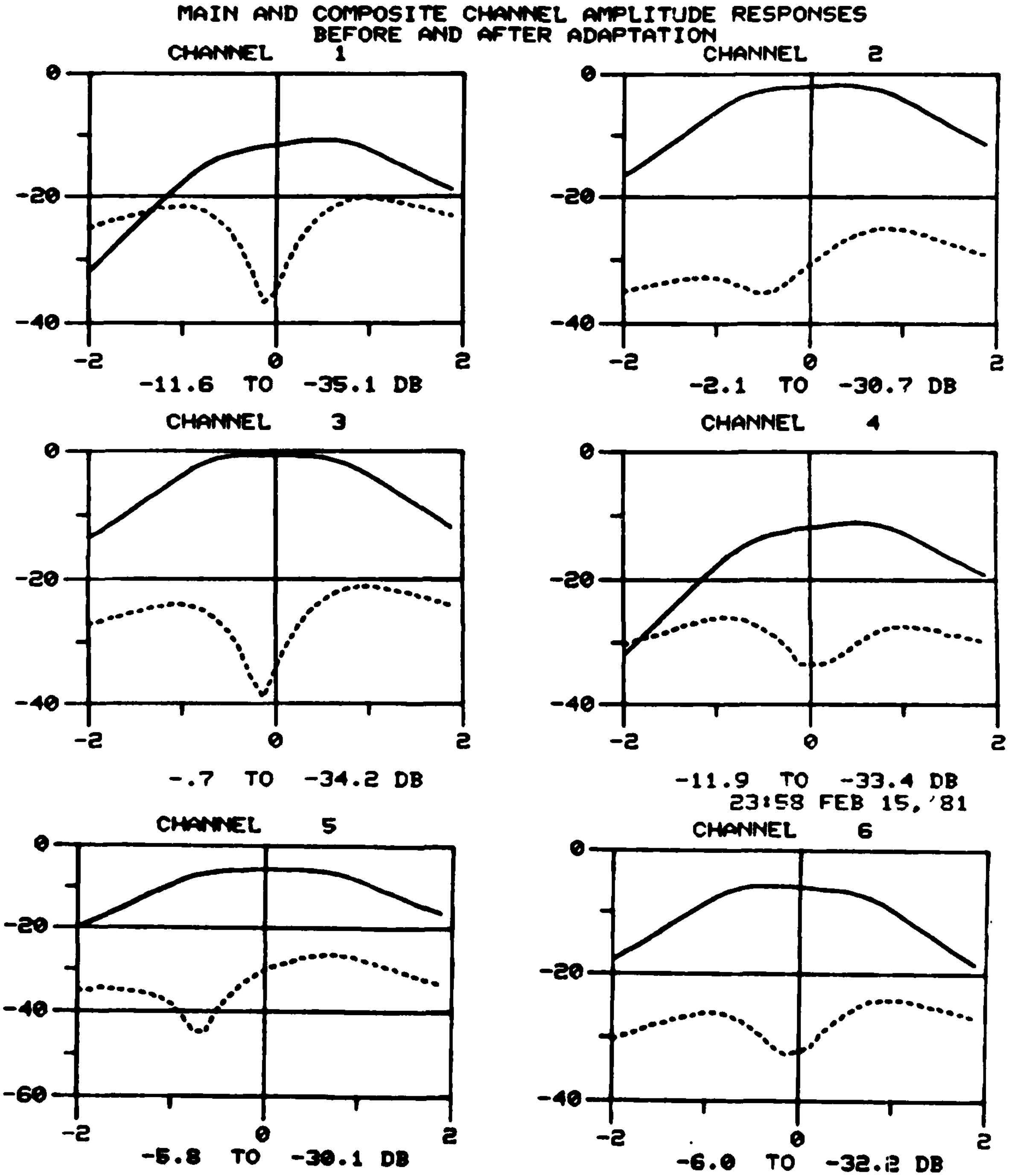


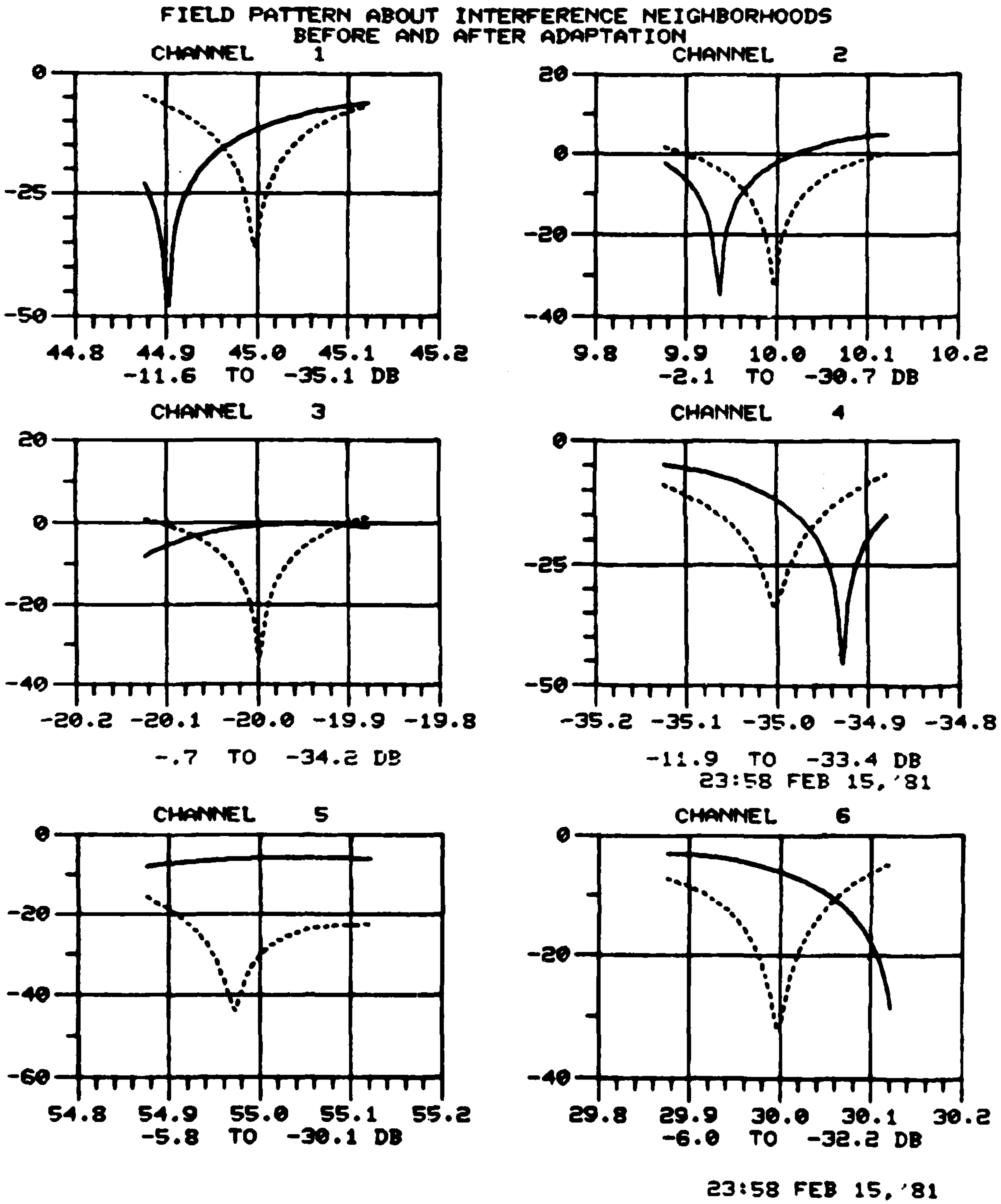
Figure 3-3 (Cont'd)

Source Transfer Function Amplitude Responses Before and After BCR Adaptation



23:58 FEB 15,'81

Field Pattern Amplitudes Over 0.25 Neighborhoods About Source Angles of Arrival at RF Center



Main Beam Field Pattern Amplitude at Center RF Frequency, Before and After BCR Adaptation

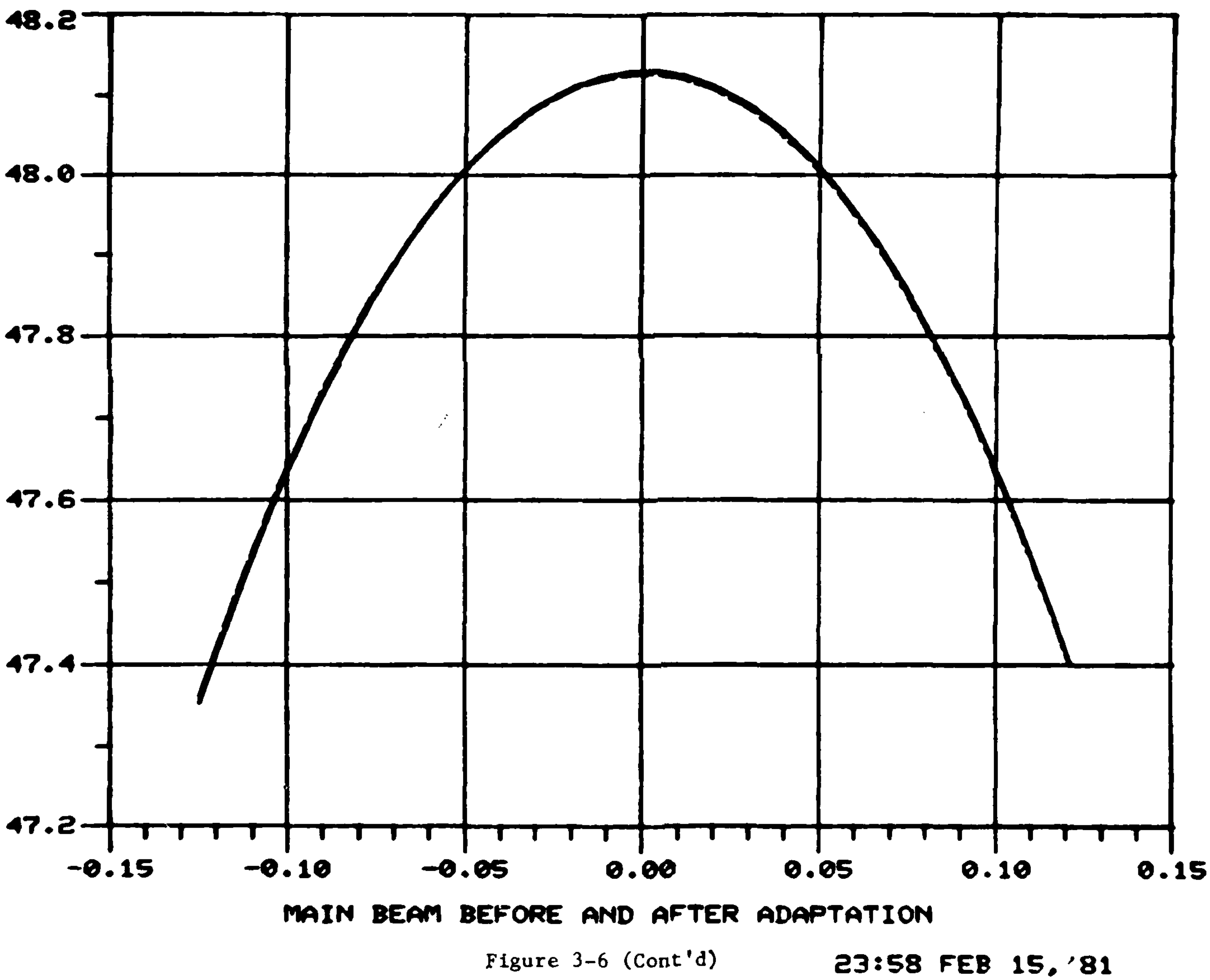
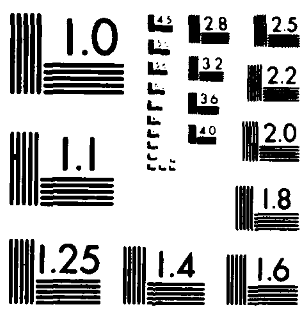


Figure 3-6 (Cont'd)

AD-A109 927   MOTOROLA INC TEMPE AZ GOVERNMENT ELECTRONICS DIV   F/G 20/14
BATCH COVARIANCE RELAXATION (BCR) ADAPTIVE PROCESSING.(U)
AUG 81   S M DANIEL, I KERTESZ   F30602-80-C-0031
UNCLASSIFIED   8512-F   RADC-TR-81-212   NL

6 of 7


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5

Title Page

BCR ADAPTIUE PROCESSING

UNEQUAL SOURCE-STRENGTH EXAMPLE
(See Figure 3-7)

SOURCES	:	6 CONTINUOUS (0,03,-6,-9,-12,-15,dB) (-45°,10°,-20°,35°,55°,30°)
RF BANDWIDTH	:	0.1%
AUXILIARY PORTS	:	6
TAP WEIGHTS	:	1/PORT

SIMULATION BY : S. M. DANIEL & I. KERTESZ
ADVANCED TECHNOLOGY AND SYSTEMS ANALYSIS
MOTOROLA, INC. - GED

20:34 FEB 18,'81

Figure 3-9

Relative Signal Amplitude Before and After BCR Adaptation

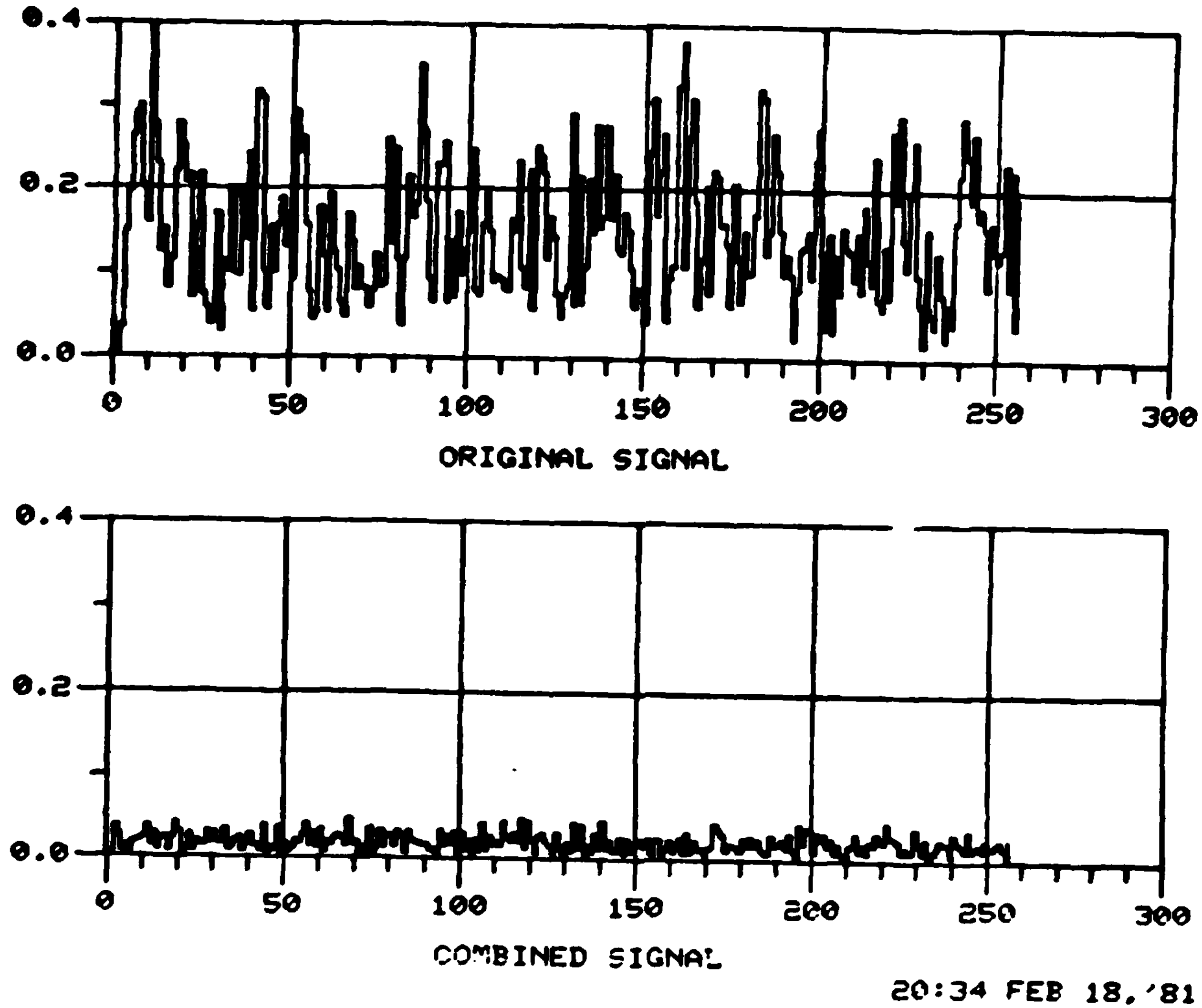


Figure 3-9 (Cont'd)

BCR Adaptive Weight Evolution in the Complex Plane

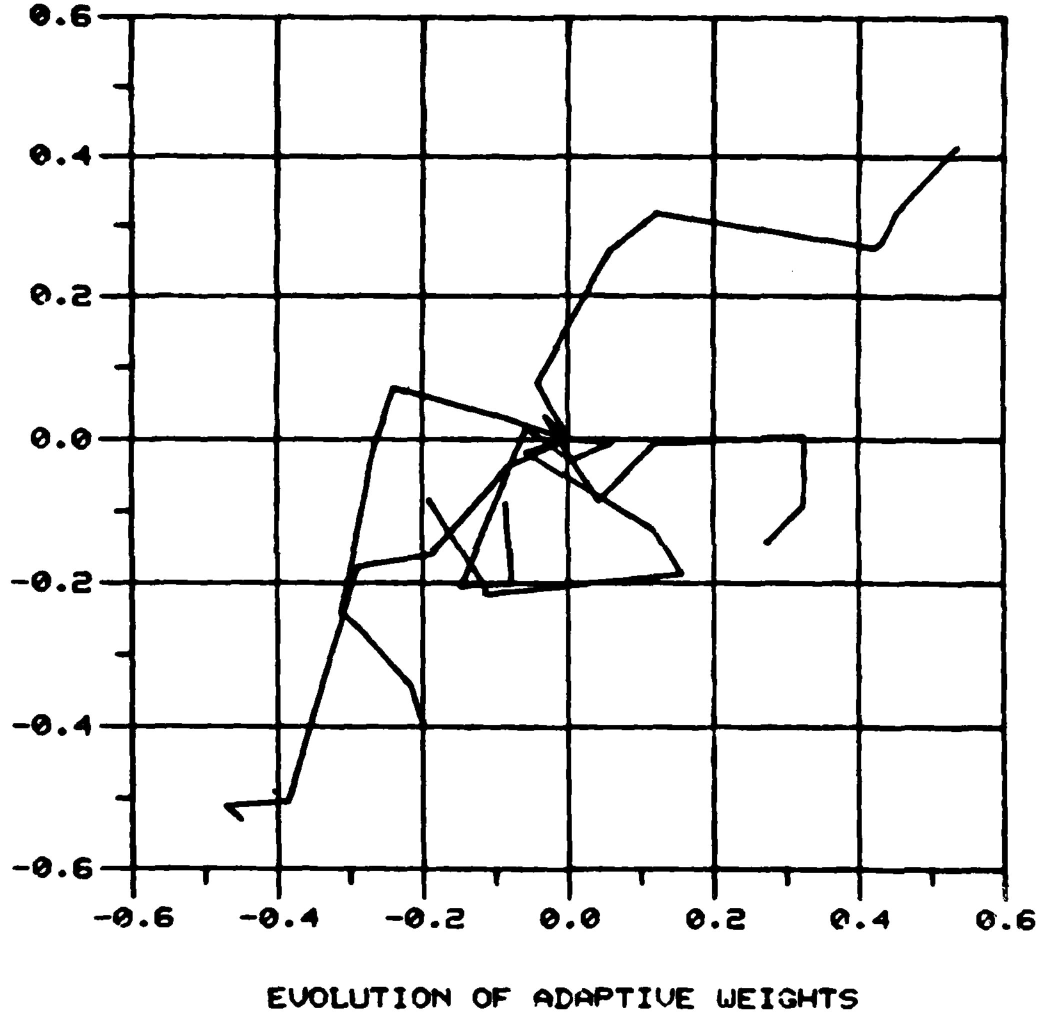


Figure 3-9 (Cont'd)

20:34 FEB 18, '81

Source Transfer Function Amplitude Responses Before and After BCR Adaptation

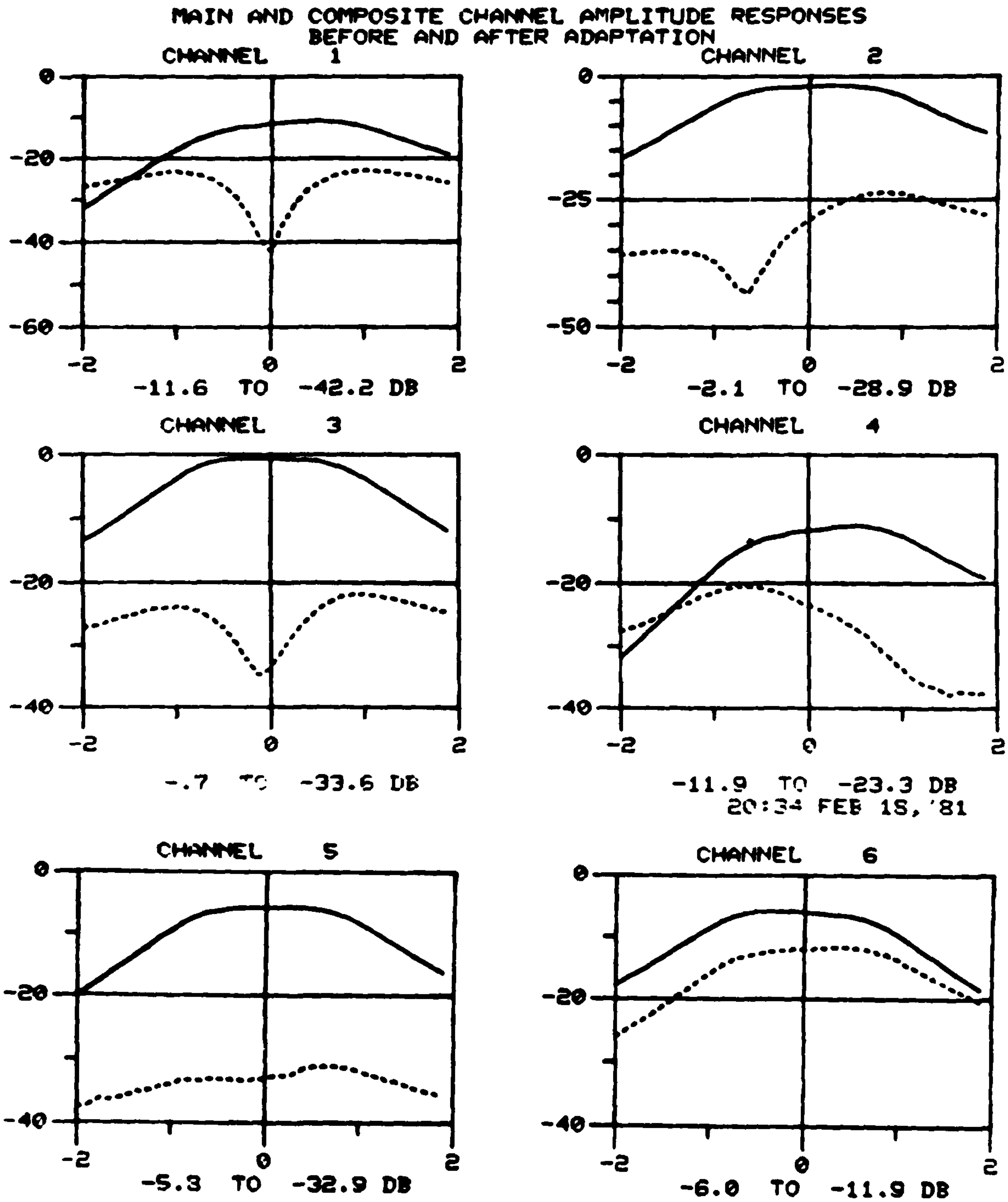
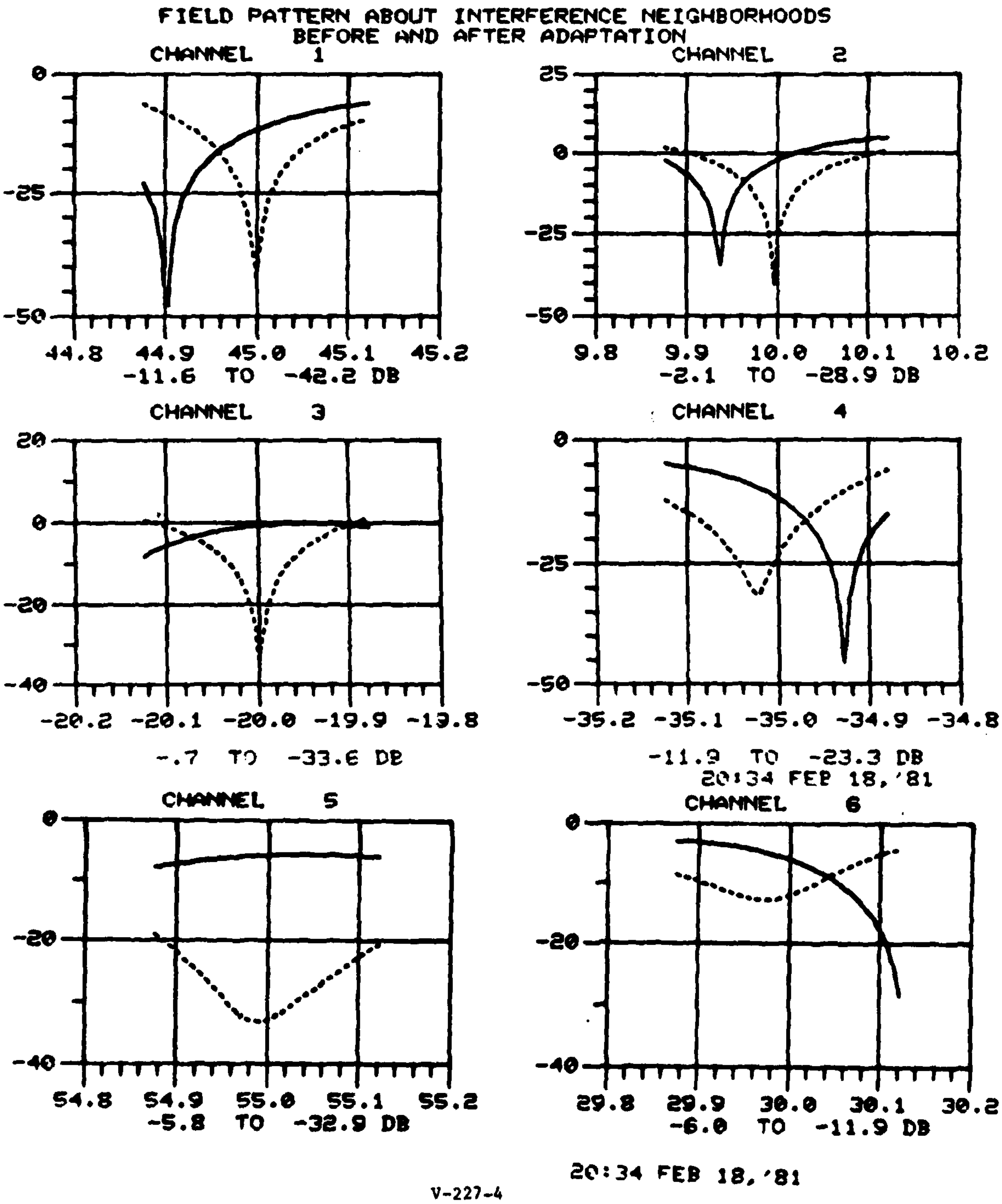


Figure 3-9 (Cont'd) 20:34 FEB 18, '81

Field Pattern Amplitudes Over 0.25 Neighborhoods About Source Angles of Arrival at RF Center



Main Beam Field Pattern Amplitude at Center RF Frequency, Before and After BCR Adaptation

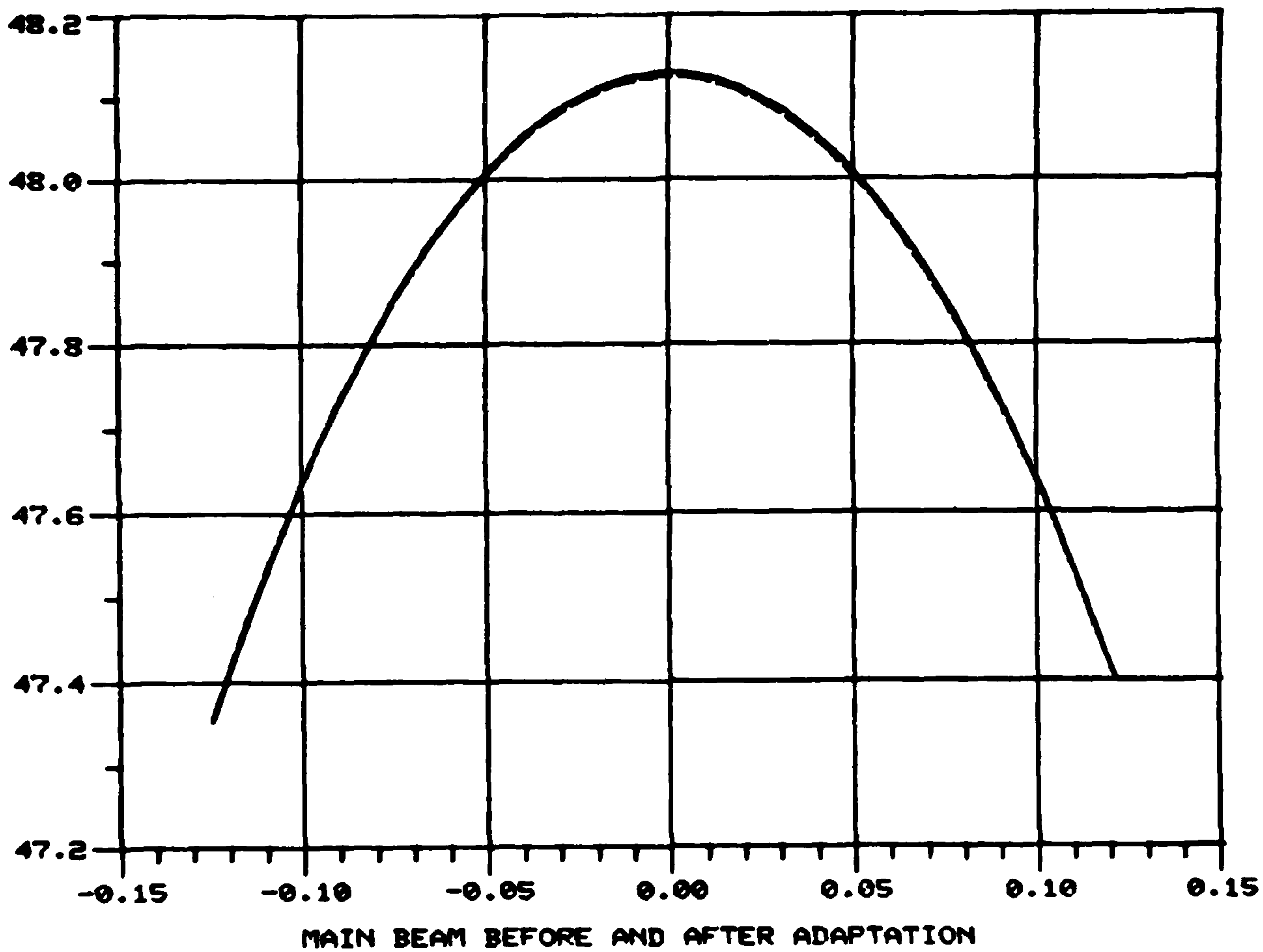


Figure 3-9 (Cont'd)

isolated observation, however, does not necessarily imply that multitap configurations are generally inferior. A more extensive investigation would be needed to draw a general conclusion in this regard.

3.1.3.1 One Source with 1% RF Bandwidth

Figure 3-10 gives the scenario and system description for the first wideband example. As indicated, it involves a single wideband source incident from 45° on to the adaptive array system consisting of the main linear antenna array in combination with 10 single-tap auxiliaries.

The BCR convergence behavior for this example is shown in Figure 3-11. Note that even though only a single source was involved, convergence occurred in four iterations. It appears that the relatively large RF bandwidth of 1% has given rise to a covariance matrix with rank greater or equal to 4.

Figure 3-12 is the graphical output that summarizes the BCR nulling performance for the present example. Of greatest interest here is baseband channel amplitude response before and after adaptation in Figure 3-12(d). The channel amplitude response (solid line) before adaptation is certainly not smooth. Note the sharp dip near normalized radian frequency -0.4. In combination with the 10 weighted auxiliary ports, the composite channel amplitude (dotted line) exhibits a nearly constant level ripple across the baseband.

3.1.3.2 One Source with 10% RF Bandwidth

The scenario and system description for the present example given in Figure 3-13 is identical to the previous one, except for the fact that the RF bandwidth is 10% . The convergence behavior of Figure 3-14 indicates convergence in 6 iterations, although the relative combined power has reached its minimum in the 5th iteration. As expected, the final combined power level is noticeably higher in the present wider band example than in the immediately previous case.

Figure 3-15 is the graphical output that describes the BCR nulling performance. As previously, to be noted here is the rather rough amplitude response of the baseband channel before adaptation in Figure 3-15(d). Clearly, it is considerably rougher in the present wider band case than in the previous one. The adapted composite channel is relatively smoother.

3.1.3.3 Two Sources with 2% RF Bandwidth

Figure 3-16 defines the scenario and system configuration involving two wideband sources and 10 single-tap auxiliary ports. Figure 3-17 gives the BCR convergence behavior. As shown, combined power convergence has been achieved at the 10th iteration, although it has been detected at the 11th via the relative gradient reduction below the -60 dB threshold level.

Figure 3-18 summarizes the BCR nulling performance. As before, the most interesting results here are the baseband channel amplitude response in Figure 3-18(d) associated with the transfer functions of each of the two sources before and after adaptation. As expected, the original channel amplitude response at 45° is considerably rougher than that at 10°. The adapted composite channel amplitude responses are considerably smoother.

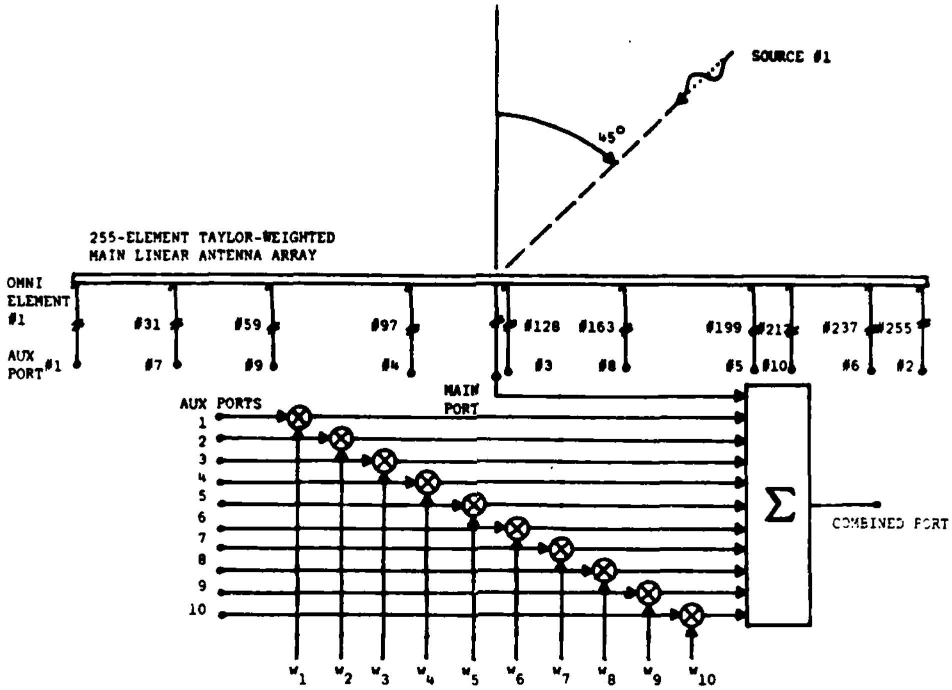


Figure 3-10. Scenario and System Description
Wideband Source Example
1% RF Bandwidth

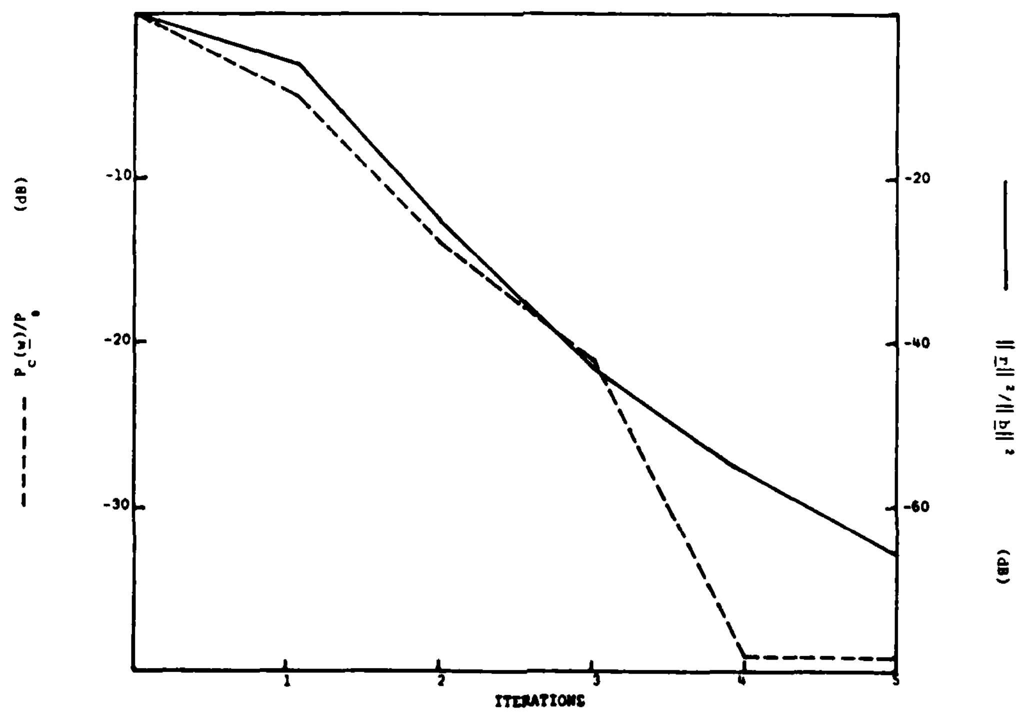


Figure 3-11. BCR Convergence Characteristics
Wideband Source Example
1% RF Bandwidth
(See Figure 3-10)

Title Page

BCR ADAPTIVE PROCESSING

WIDEBAND SOURCE EXAMPLE
(See Figure 3-10)

SOURCES	:	1 CONTINUOUS (45°)
RF BANDWIDTH	:	1%
AUXILIARY PORTS	:	10
TAP WEIGHTS	:	1/PORT

SIMULATION BY : S. M. DANIEL & I. KERTESZ
ADVANCED TECHNOLOGY AND SYSTEMS ANALYSIS
MOTOROLA, INC. - GED

22:37 FEB 18,'81

Figure 3-12

Relative Signal Amplitude Before and After BCR Adaptation

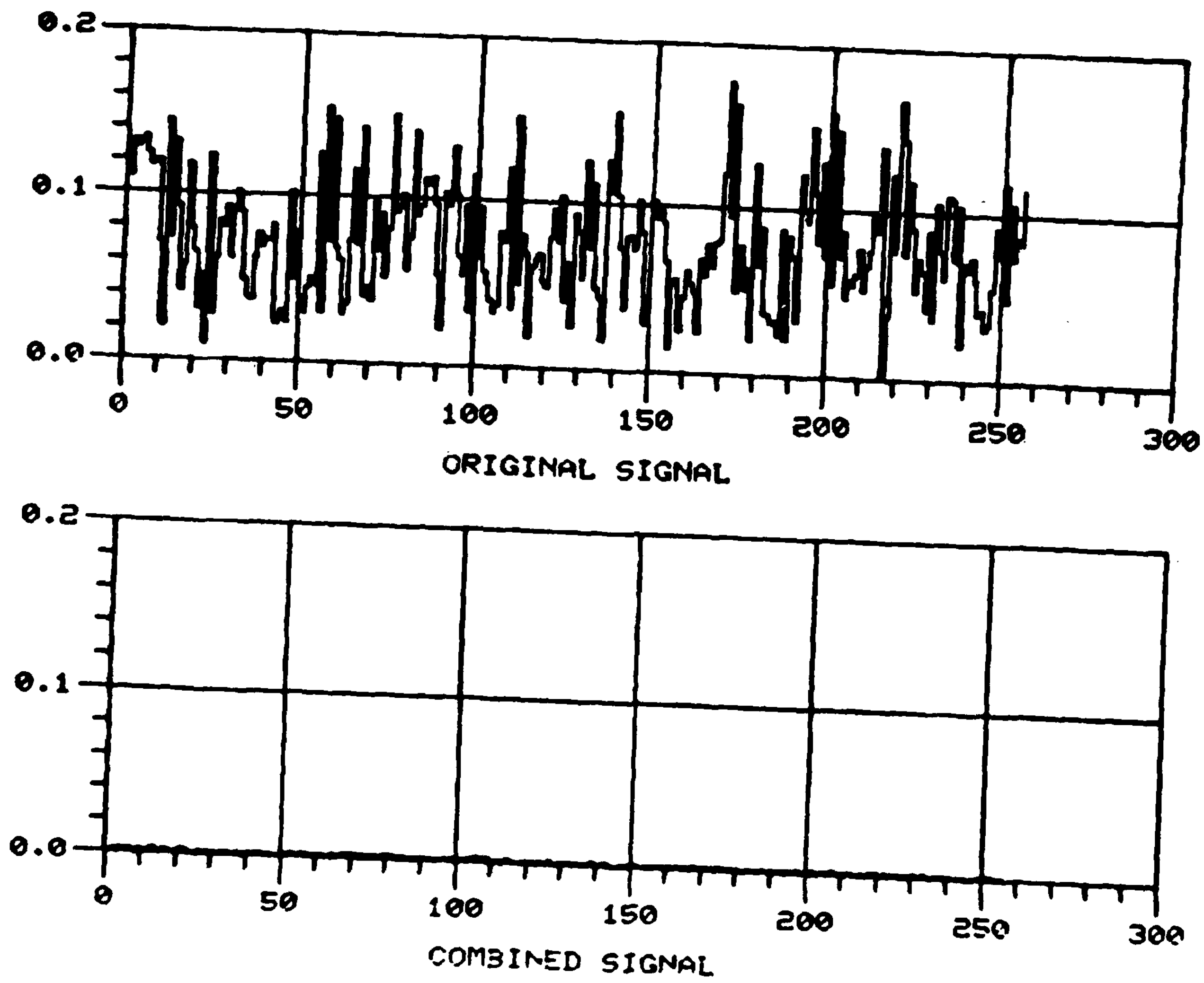


Figure 3-12 (Cont'd)

BCR Adaptive Weight Evolution in the Complex Plane

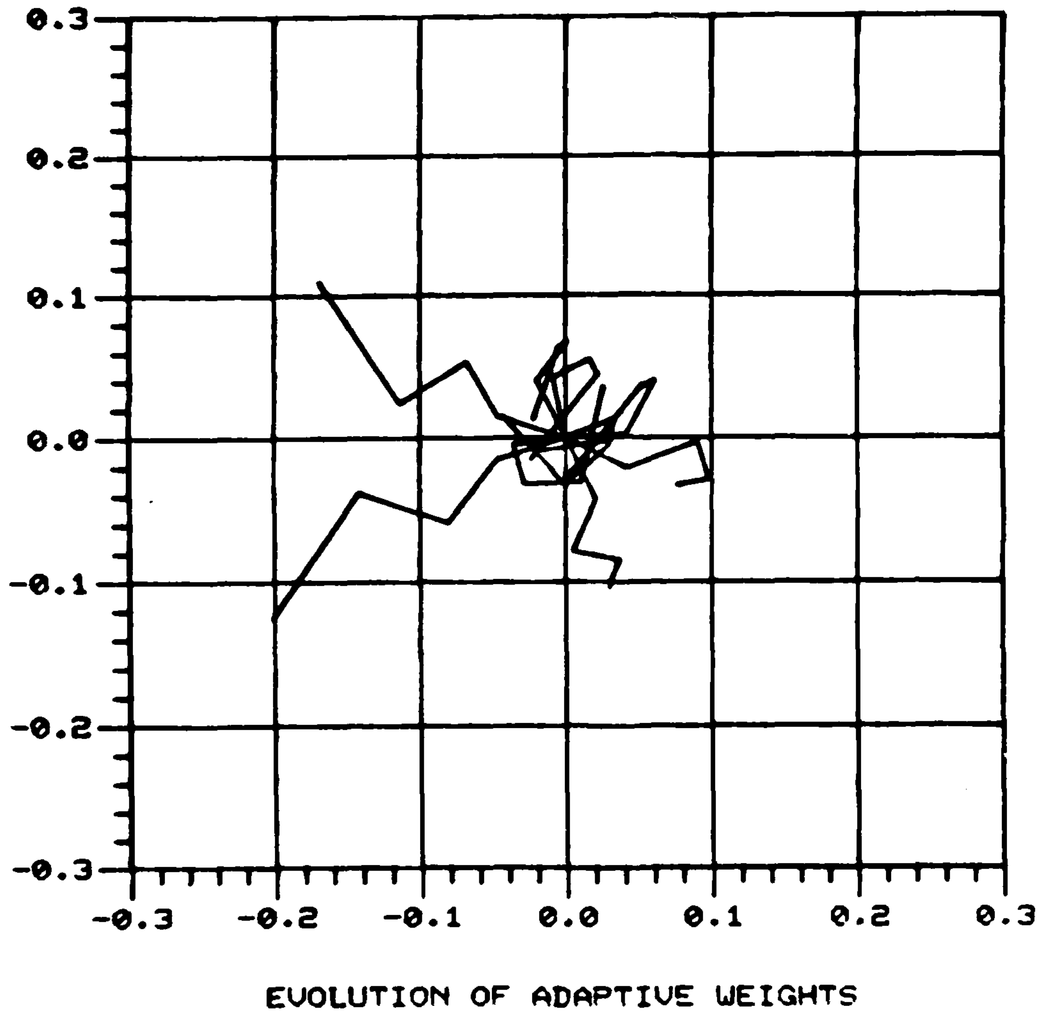


Figure 3-12 (Cont'd)

22:37 FEB 18, '81

Source Transfer Function Amplitude Responses Before and After BCR Adaptation

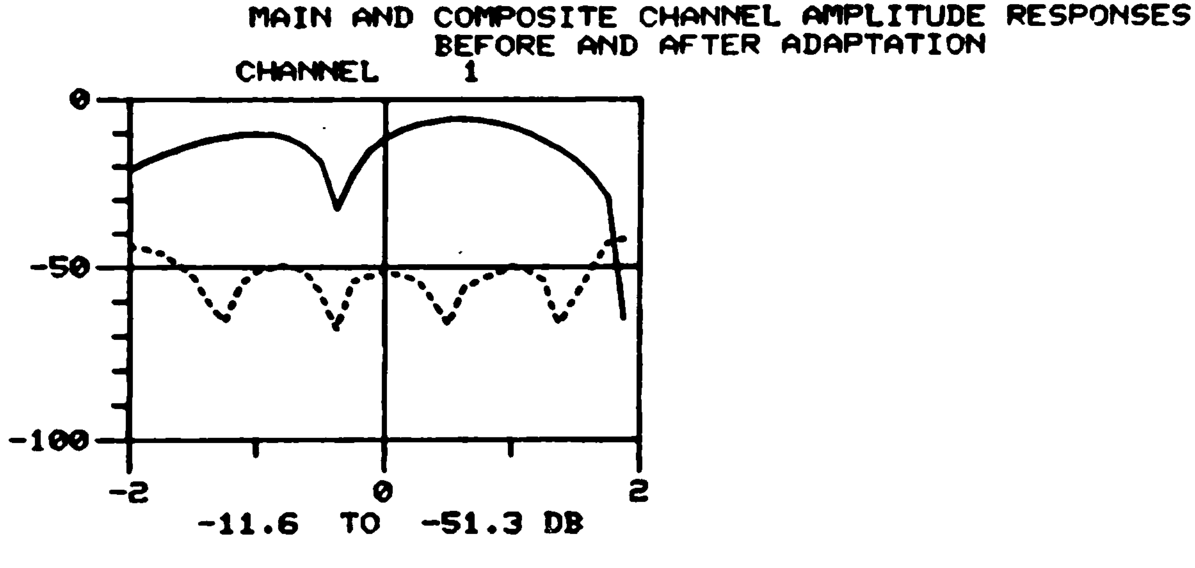


Figure 3-12 (Cont'd)

22:37 FEB 18,'81

Field Pattern Amplitudes Over 0.25 Neighborhoods About Source Angles of Arrival at RF Center

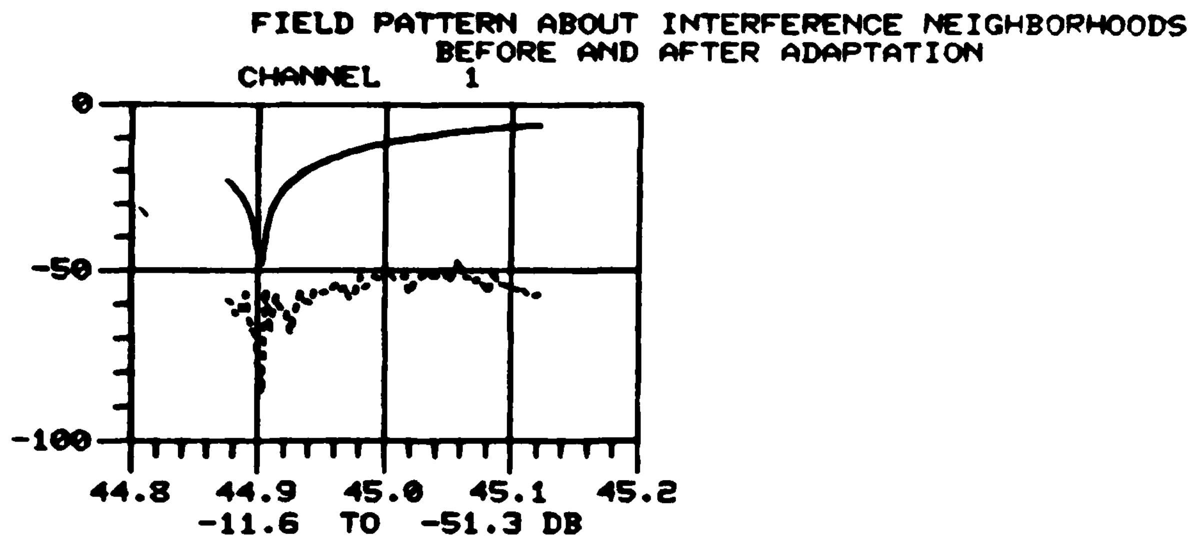


Figure 3-12 (Cont'd)

22:37 FEB 18, '81

Main Beam Field Pattern Amplitude at Center RF Frequency, Before and After BCR Adaptation

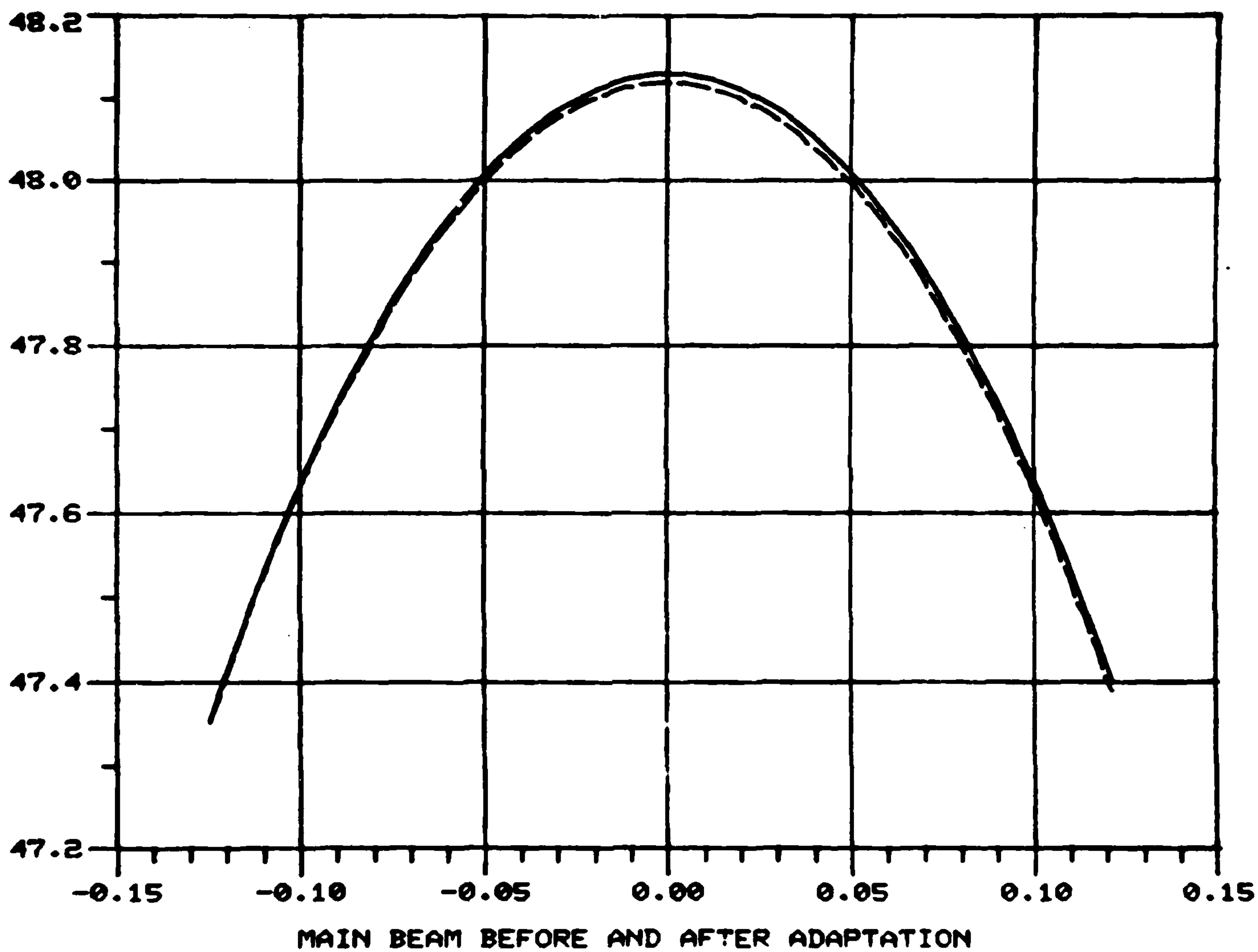


Figure 3-12 (Cont'd)

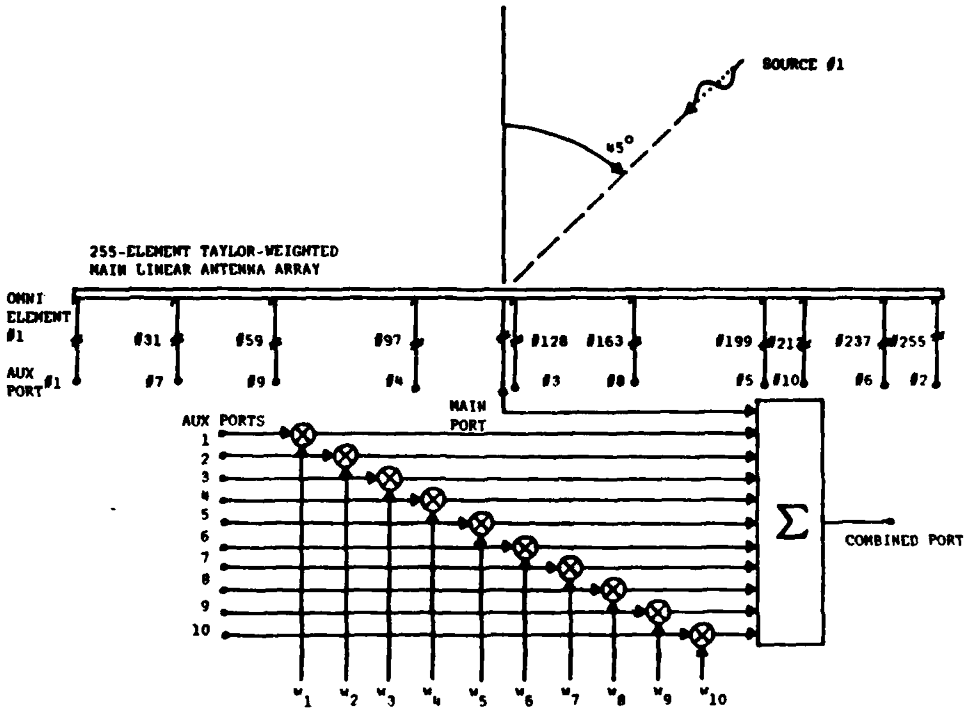


Figure 3-13. BCR Convergence Characteristics
Wideband Source Example
10% RF Bandwidth

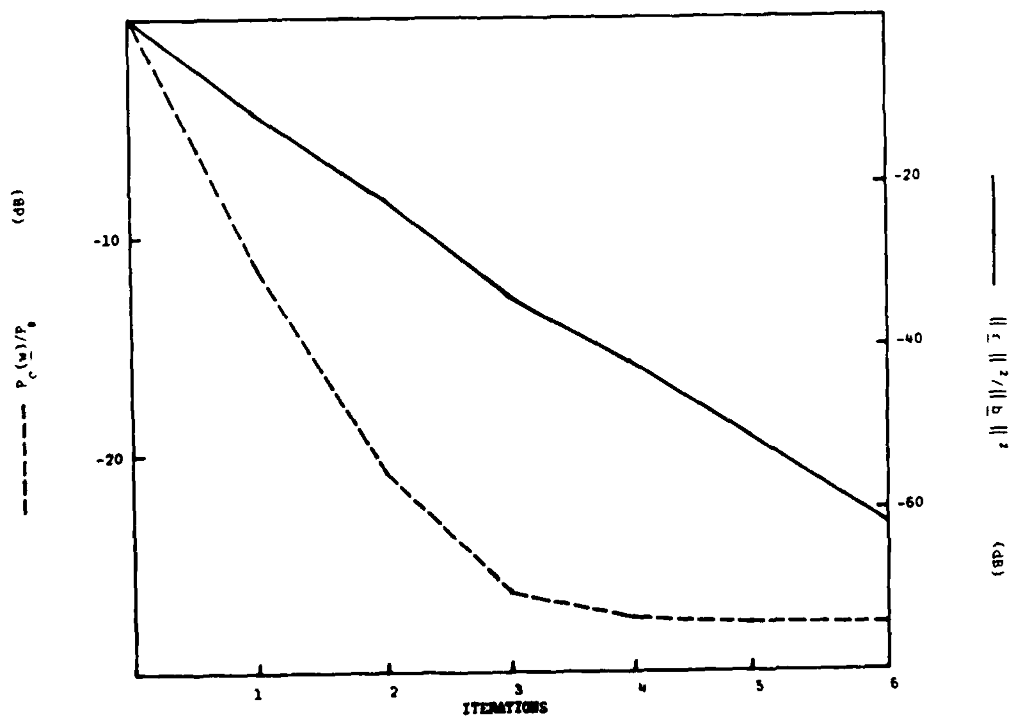


Figure 3-14. BCR Convergence Characteristics
Wideband Source Example
10% RF Bandwidth

Title Page

BCR ADAPTIVE PROCESSING

WIDEBAND SOURCE EXAMPLE

(See Figure 3-13)

SOURCES	:	1 CONTINUOUS (45°)
RF BANDWIDTH	:	10%
AUXILIARY PORTS	:	10
TAP WEIGHTS	:	1/PORT

SIMULATION BY : S. M. DANIEL & I. KERTESZ
ADVANCED TECHNOLOGY AND SYSTEMS ANALYSIS
MOTOROLA, INC. - GED

23:06 FEB 18,'81

Figure 3-15

Relative Signal Amplitude Before and After BCR Adaptation

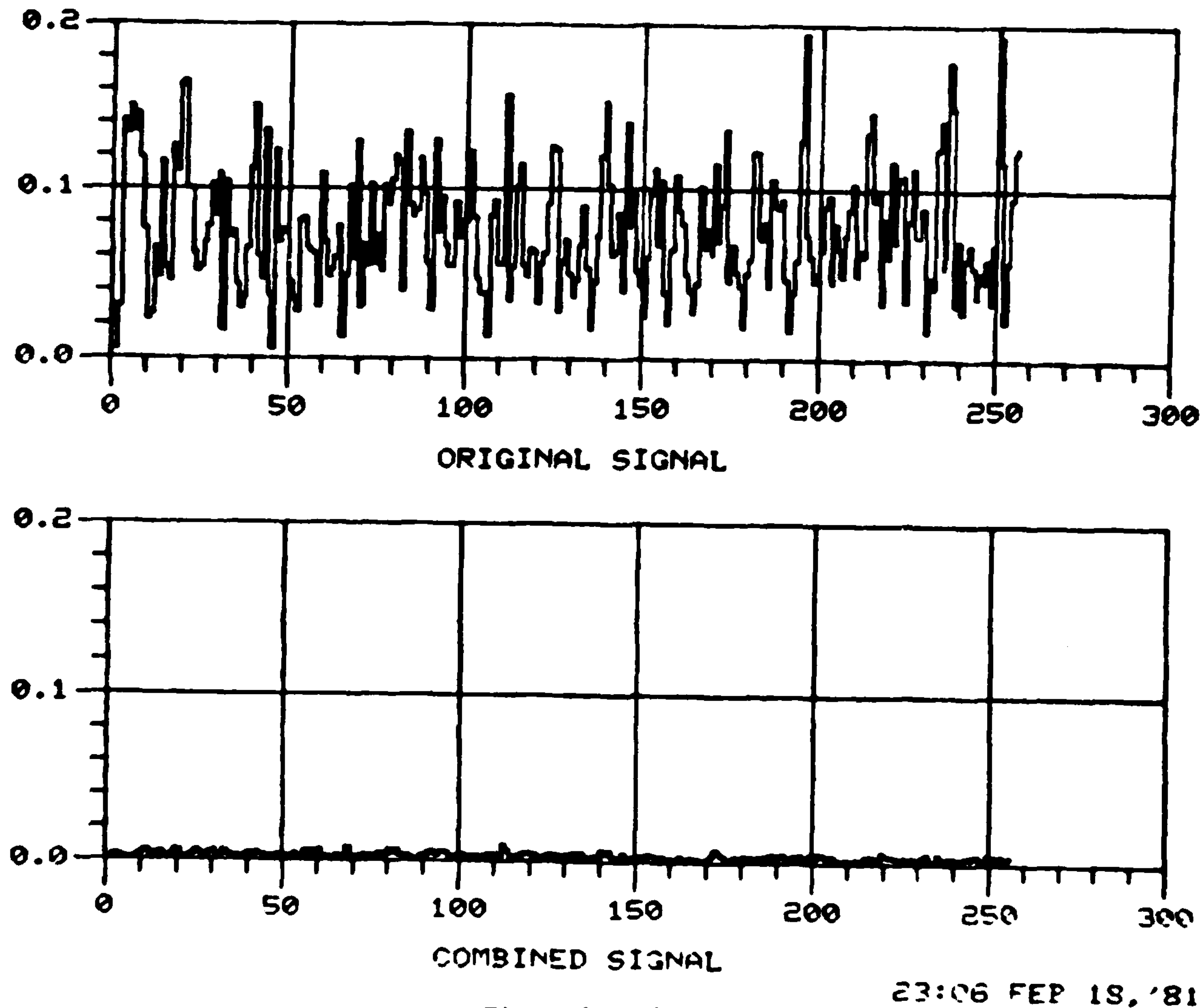
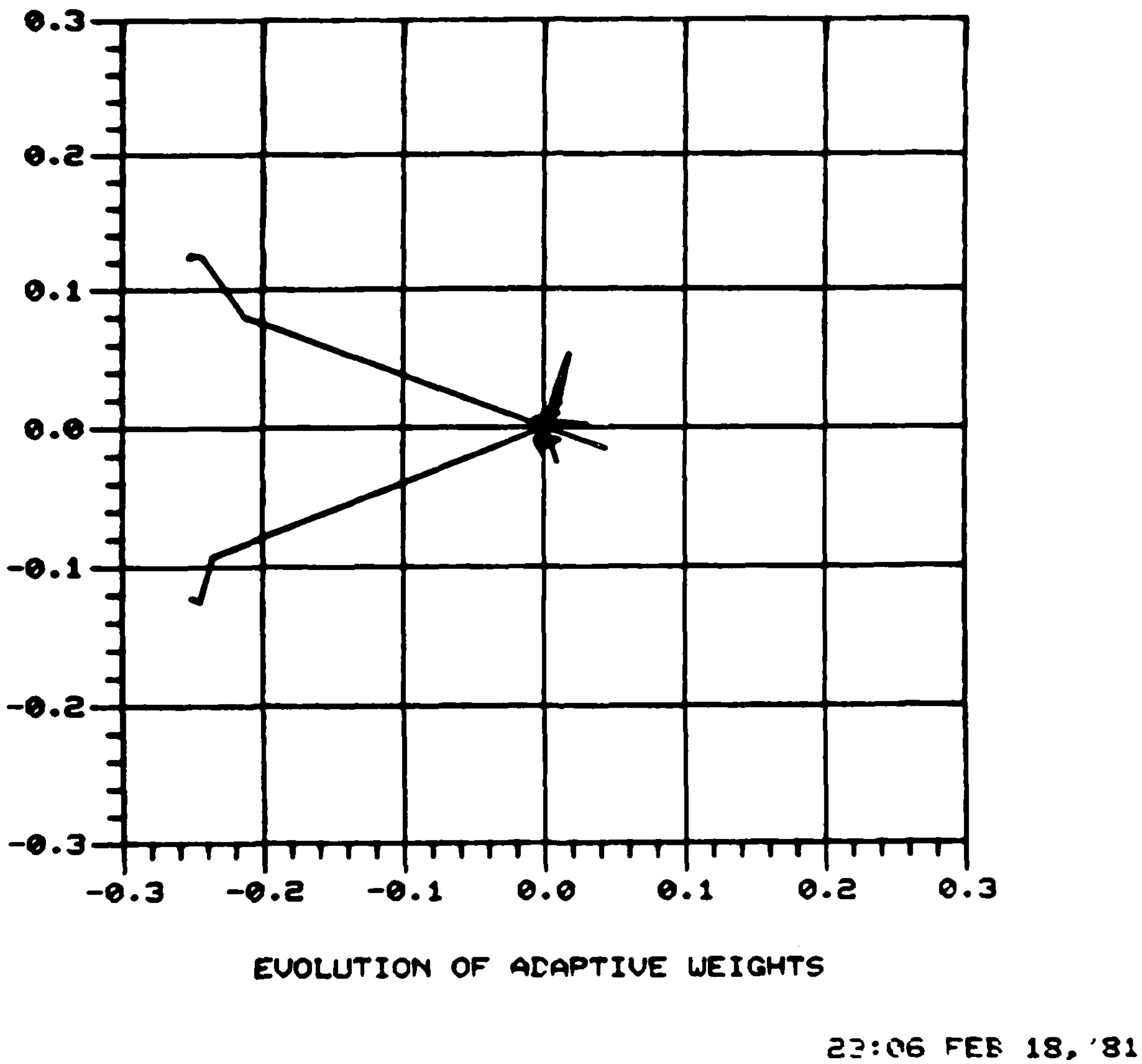
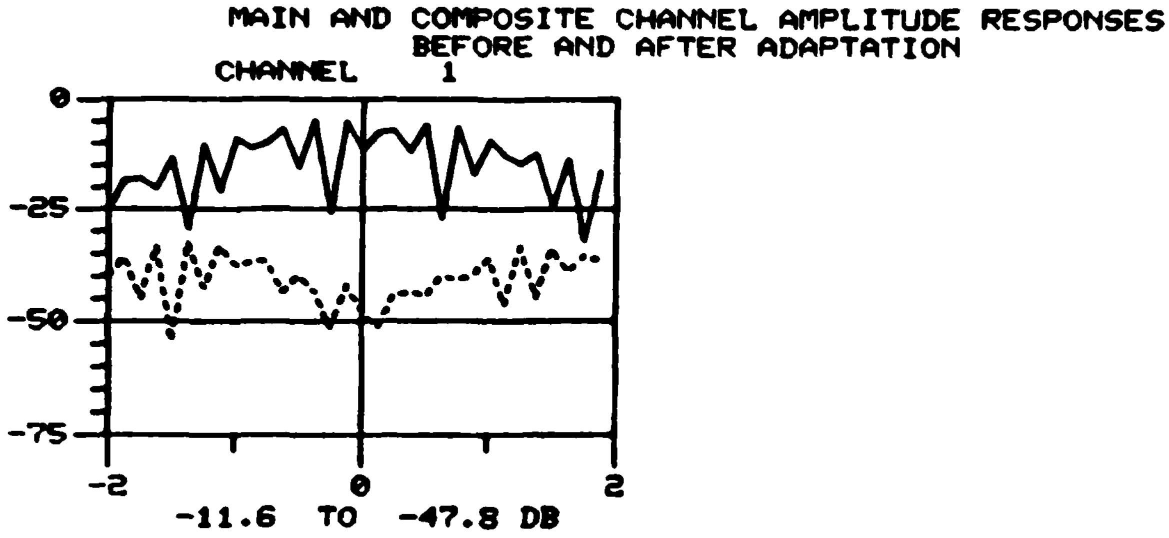


Figure 3-15 (Cont'd)

BCR Adaptive Weight Evolution in the Complex Plane

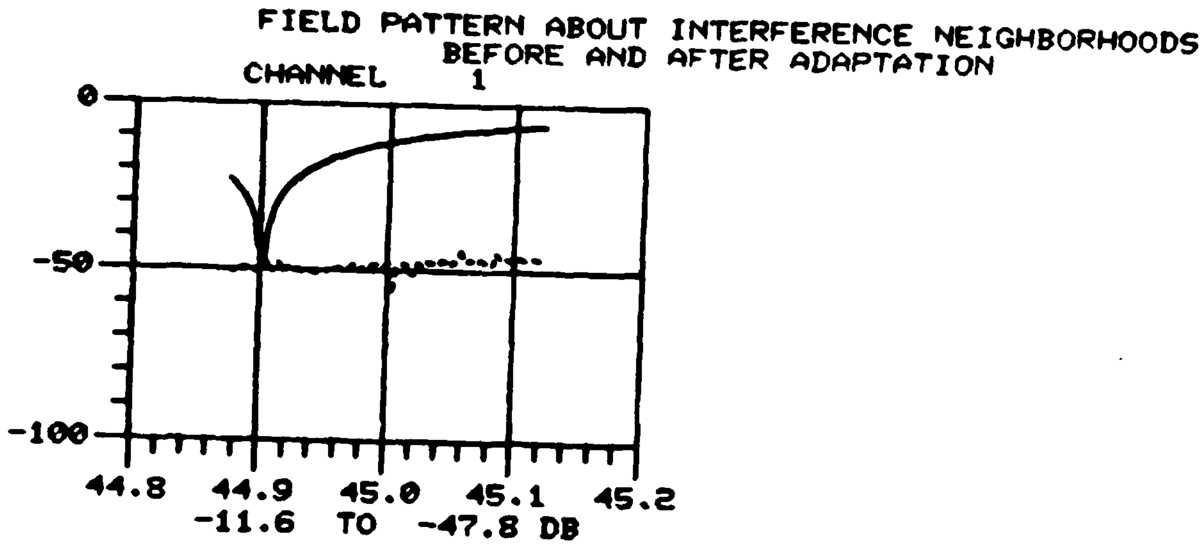


Source Transfer Function Amplitude Responses Before and After BCR Adaptation



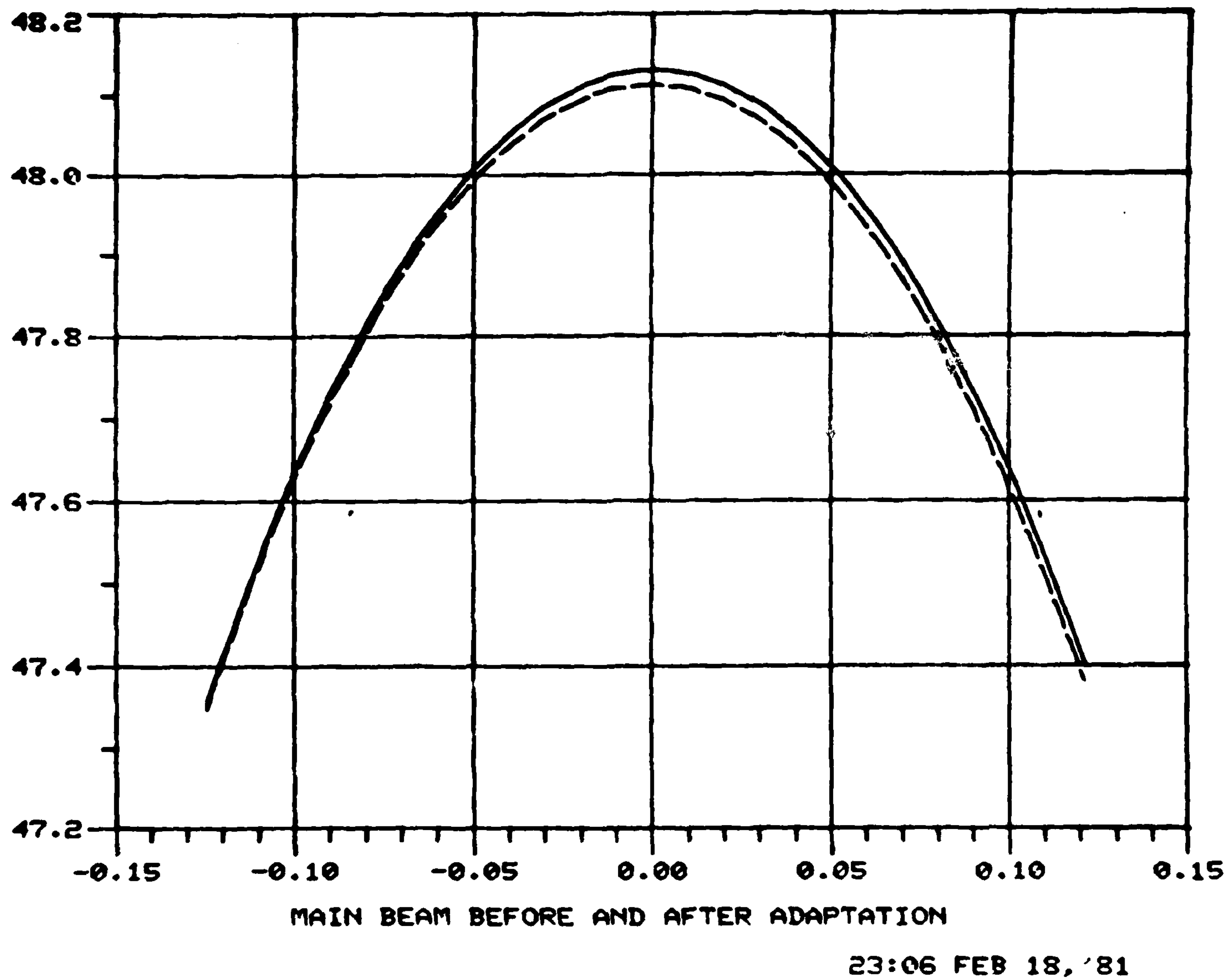
23:06 FEB 18, '81

Field Pattern Amplitudes Over 0.25 Neighborhoods About Source Angles of Arrival at RF Center



23:06 FEB 18, '81

Main Beam Field Pattern Amplitude at Center RF Frequency, Before and After BCR Adaptation



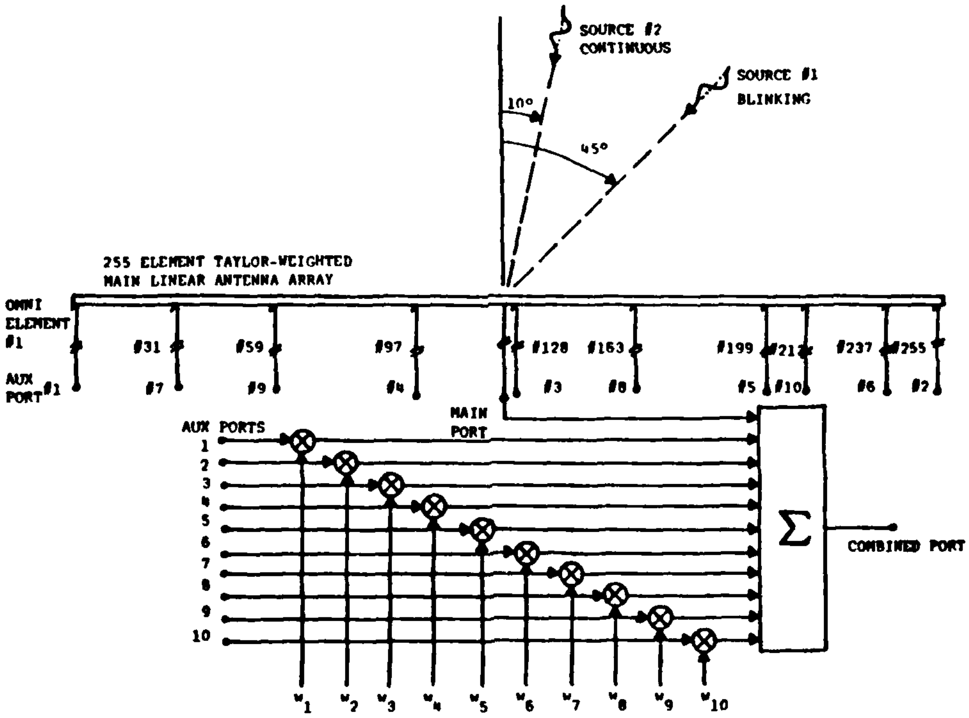


Figure 3-16. Scenario and System Description
Blinking Source Example
2% RF Bandwidth

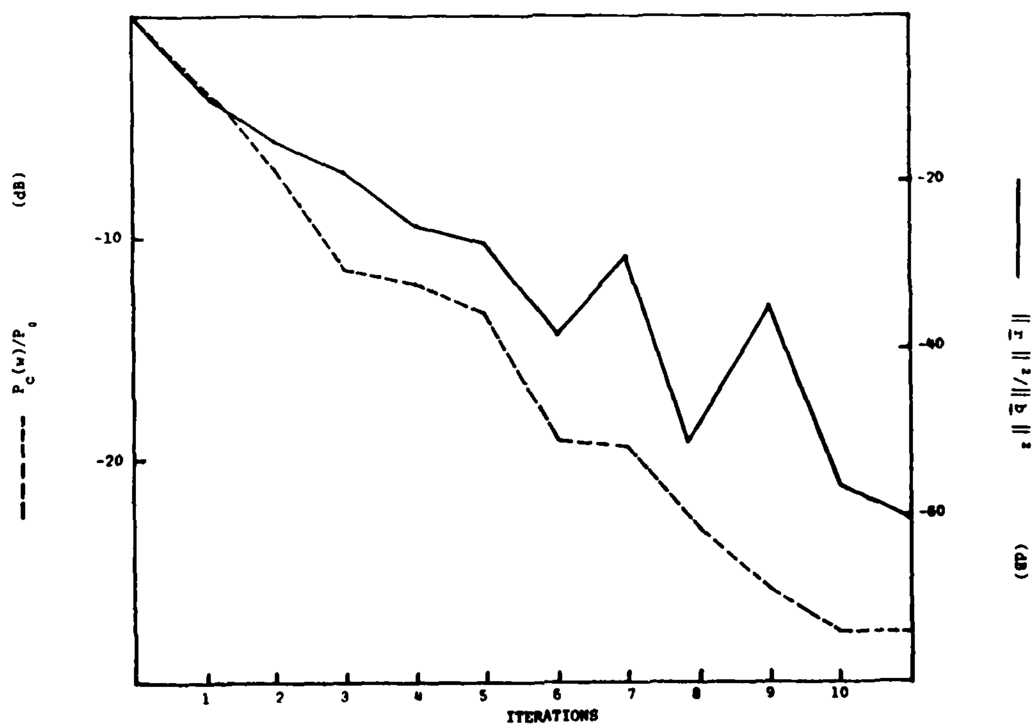


Figure 3-17. BCR Convergence Characteristics
Wideband Source Example
(See Figure 3-16)

Title Page

BCR ADAPTIVE PROCESSING

WIDEBAND SOURCE EXAMPLE
(See Figure 3-16)

SOURCES	:	2 CONTINUOUS (45°, 10°)
RF BANDWIDTH	:	2%
AUXILIARY PORTS	:	10
TAP WEIGHTS	:	1/PORT

SIMULATION BY : S. M. DANIEL & I. KERTESZ
ADVANCED TECHNOLOGY AND SYSTEMS ANALYSIS
MOTOROLA, INC. - GED

02:07 FEB 19,'81

Figure 3-18

Relative Signal Amplitude Before and After BCR Adaptation

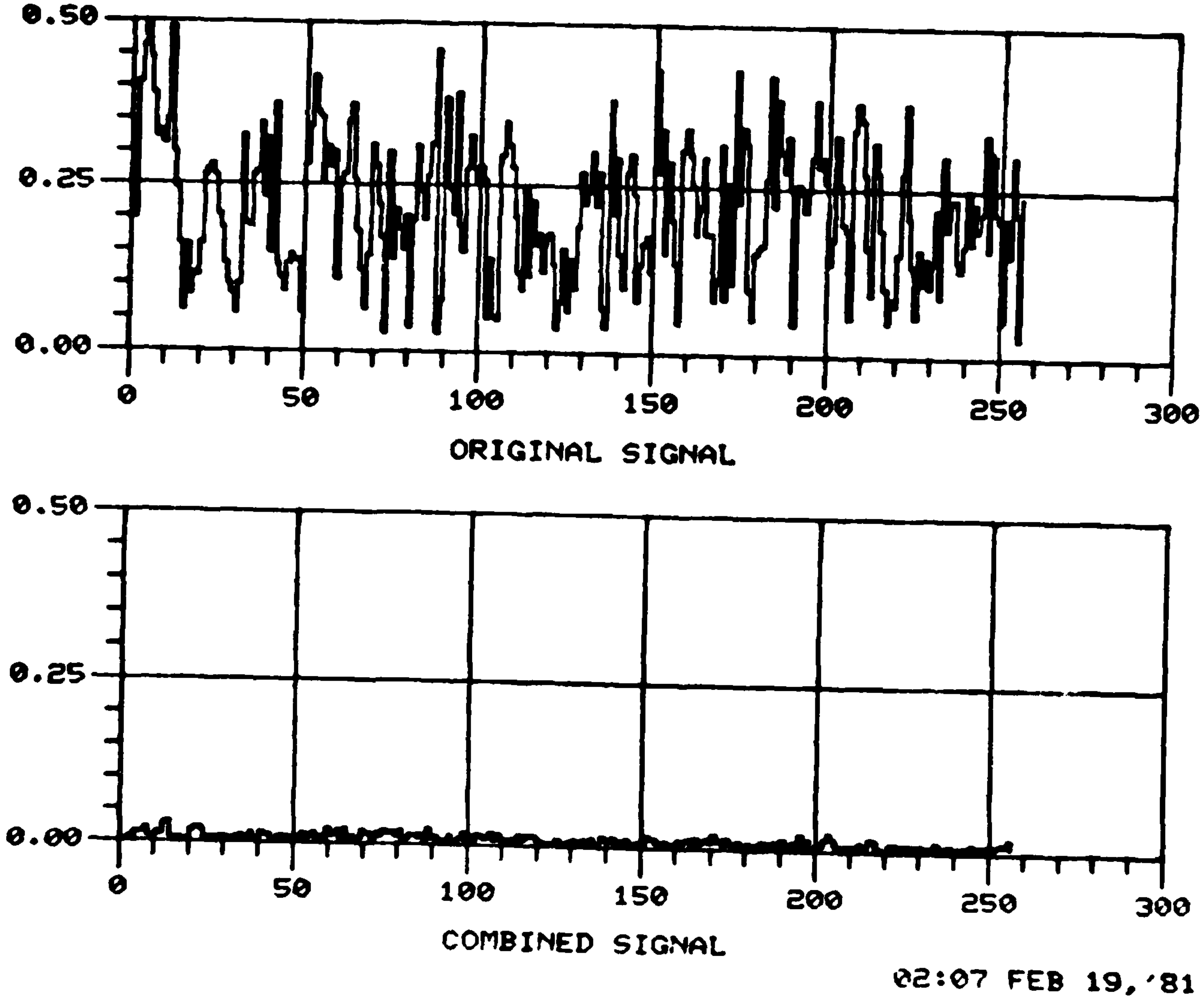


Figure 3-18 (Cont'd)

BCR Adaptive Weight Evolution in the Complex Plane

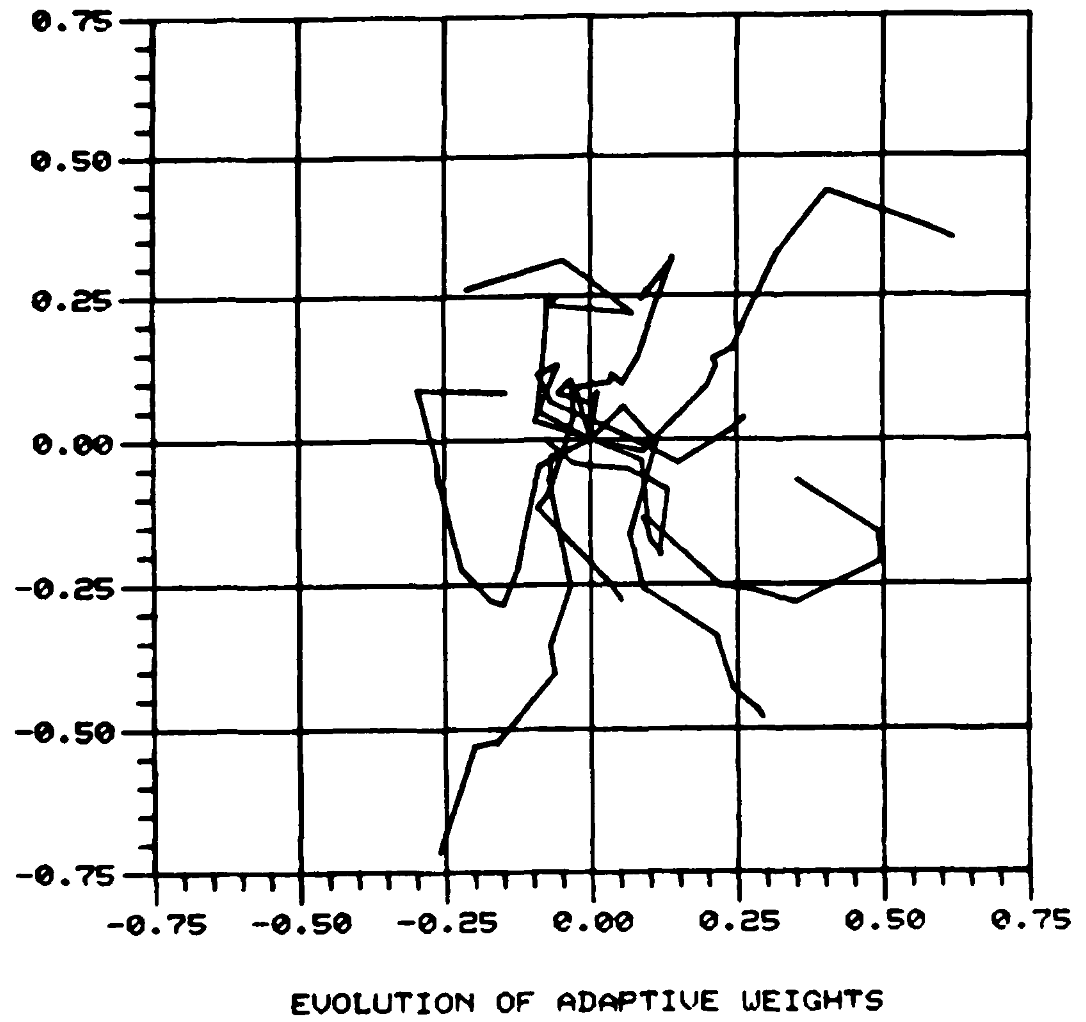


Figure 3-18 (Cont'd)

02:07 FEB 19, '81

Source Transfer Function Amplitude Responses Before and After BCR Adaptation

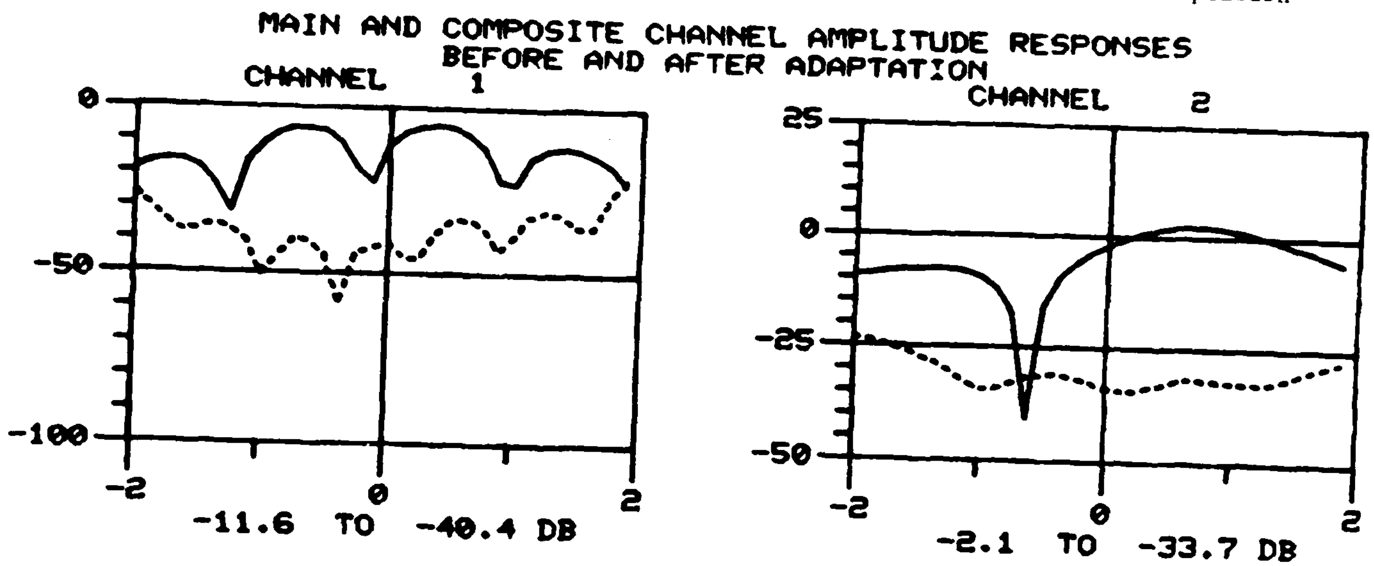


Figure 3-18 (Cont'd) 02:07 FEB 19, '81

Field Pattern Amplitudes Over 0.25 Neighborhoods About Source Angles of Arrival at RF Center

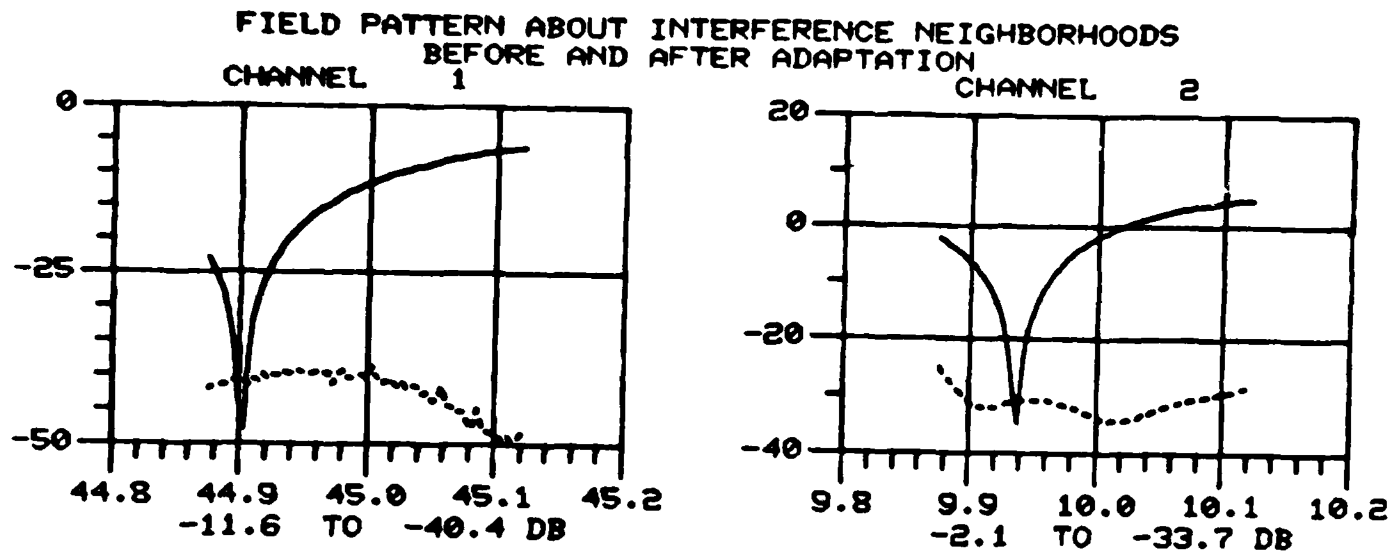


Figure 3-18 (Cont'd)

02:07 FEB 19,'81

Main Beam Field Pattern Amplitude at Center RF Frequency, Before and After BCR Adaptation

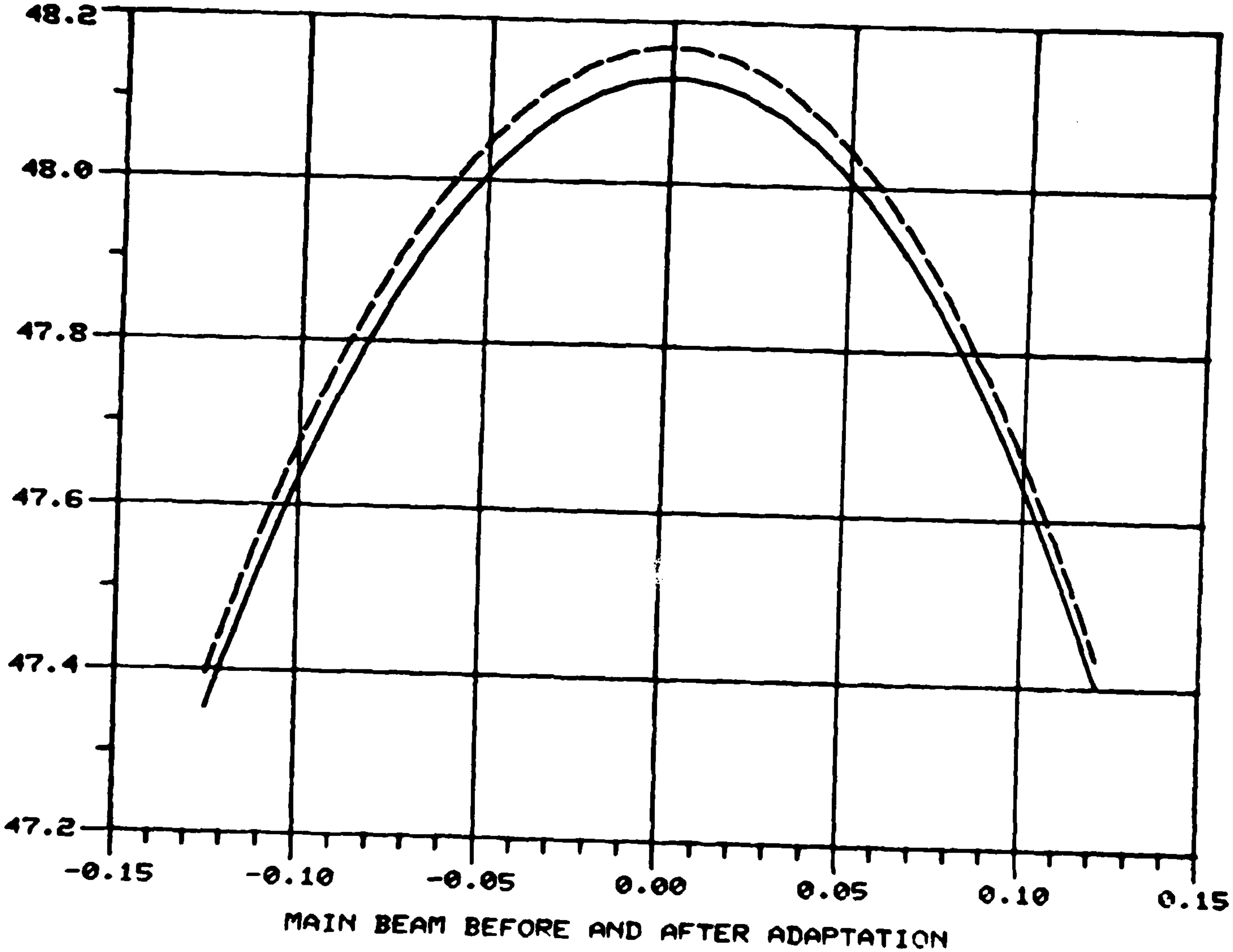


Figure 3-18 (Cont'd)

02:07 FEB 19, '81

3.1.3.4 Multitap Weighting

The previous two-source example was reconsidered with a 2-port/5-tap system configuration as shown in Figure 3-19. In an attempt to cover the antenna aperture dispersion at a 45° incidence, the unit delay D was chosen to be 0.1414 of the maximum aperture delay. Note that the five unit delays in the two auxiliary 5-tap delay lines are accompanied with 2.5D of delay in the main antenna for the purpose of providing proper delay balance.

Figure 3-20 shows the BCR convergence behavior for the present example. Note that, although convergence has been indicated at the 12th iteration, no more than 1 dB additional combined power suppression was achieved beyond the 4th.

Figure 3-21 summarizes the BCR adaptive nulling performance. Clearly manifested in the adapted channel amplitude responses of Figure 3-21(d) is the poor nulling performance that has been achieved.

The results in this example appear to imply that, given a fixed number of adaptive weights, they will be most effective as single taps over the same number of ports rather than multiple taps over an appropriately reduced number of ports. The observed relative combined power suppression of -9.6 dB was improved to only -10.6 when the number of taps was doubled to 10 per port and spaced D/2 apart. Of course, further investigation is necessary in order to substantiate or refute the validity of this observation in general. It would be appropriate, in such an investigation, to evaluate the usefulness of non-uniform tap-delay spacing, varying the extent of each delay line and increasing the number of multitap ports.

3.1.4 Multipath Examples

The two examples included here are two extreme cases of the multipath phenomenon. Both examples involve six multipaths from a given source, consisting of a direct path at 45° and five indirect paths at 0.05° intervals beyond 45°, with equal signal strengths. The bandwidth is chosen to be 0.1% and six 1-tap ports are used.

The first example addresses a case where the six paths are highly correlated. As such, the channel delays chosen are 0.00, 0.05, ..., 0.25 of a sample-time interval ΔT, respectively, where, in the present simulation, ΔT is half of the Nyquist-rate interval. Figure 3-22 shows the scenario and system description for this example. Figure 3-23 gives the observed BCR convergence characteristics. The overall BCR performance is summarized in graphical output presented in Figure 3-24. The most interesting feature to be noted here is the collection of field patterns in Figure 3-24(e). Indeed, the high correlation among the six multipath signals has given rise to a narrow composite pattern null, apparently, at the "centroid" of the six signal paths. Another outcome of the high correlation among the multipath signals is the excellent nulling performance observed.

In an attempt to evaluate the extreme case of highly uncorrelated multipath signals, each signal was generated from an independent noise source. Figure 3-25 shows the scenario and system description for this case. Figure 3-26 contains the observed BCR convergence characteristics. Figure 3-27 is the graphical output obtained in this case.

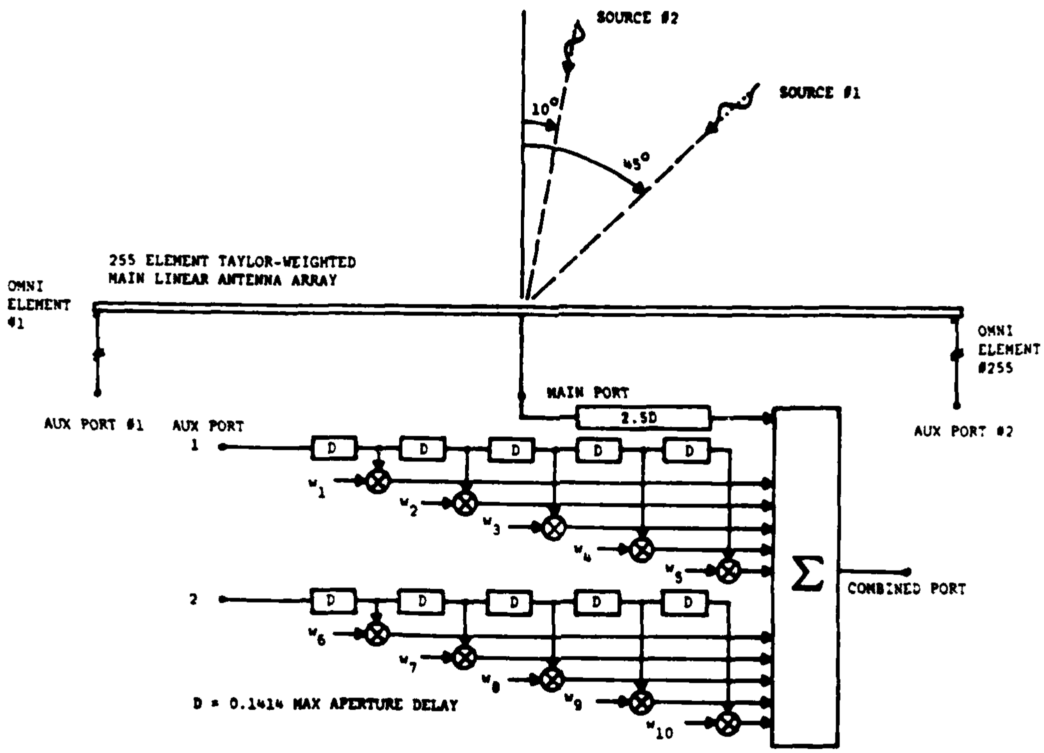


Figure 3-19. Scenario and System Description
Two Wideband Sources/Two 5-Tap Ports
2% RF Bandwidth

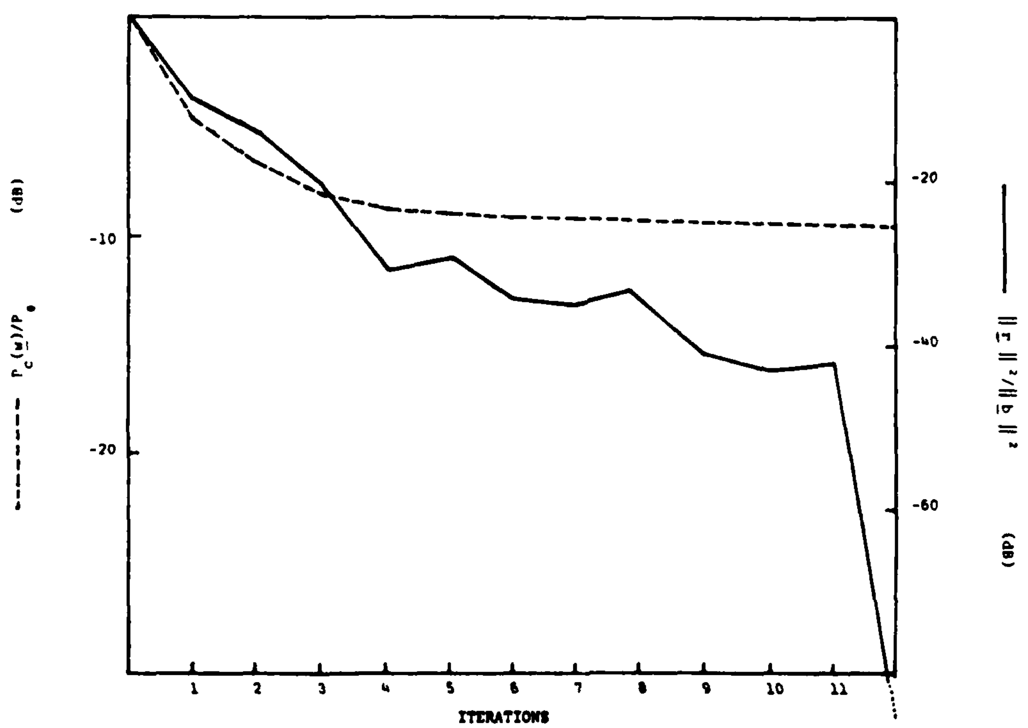


Figure 3-20. BCR Convergence Characteristics
Wideband Source Example
0.2% RF Bandwidth
(See Figure 3-19)

Title Page

BCR ADAPTIVE PROCESSING

WIDEBAND SOURCE EXAMPLE
(See Figure 3-19)

SOURCES	:	2 CONTINUOUS (45°, 10°)
RF BANDWIDTH	:	2%
AUXILIARY PORTS	:	2
TAP WEIGHTS	:	5/PORT

SIMULATION BY : S. M. DANIEL & I. KERTESZ
ADVANCED TECHNOLOGY AND SYSTEMS ANALYSIS
MOTOROLA, INC. - GED

02:59 FEB 19,'81

Figure 3-21

Relative Signal Amplitude Before and After BCR Adaptation

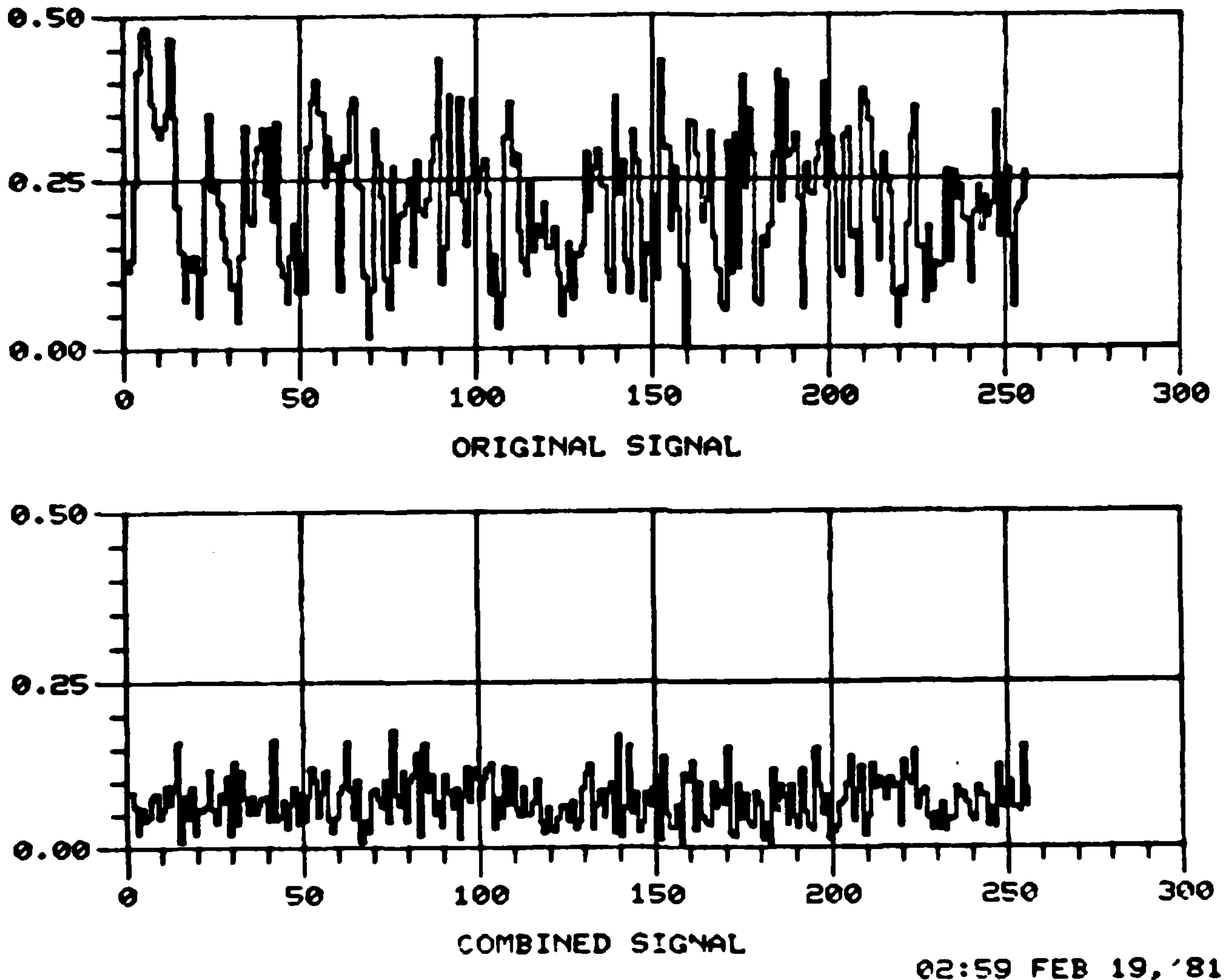


Figure 3-21 (Cont'd)

BCR Adaptive Weight Evolution in the Complex Plane

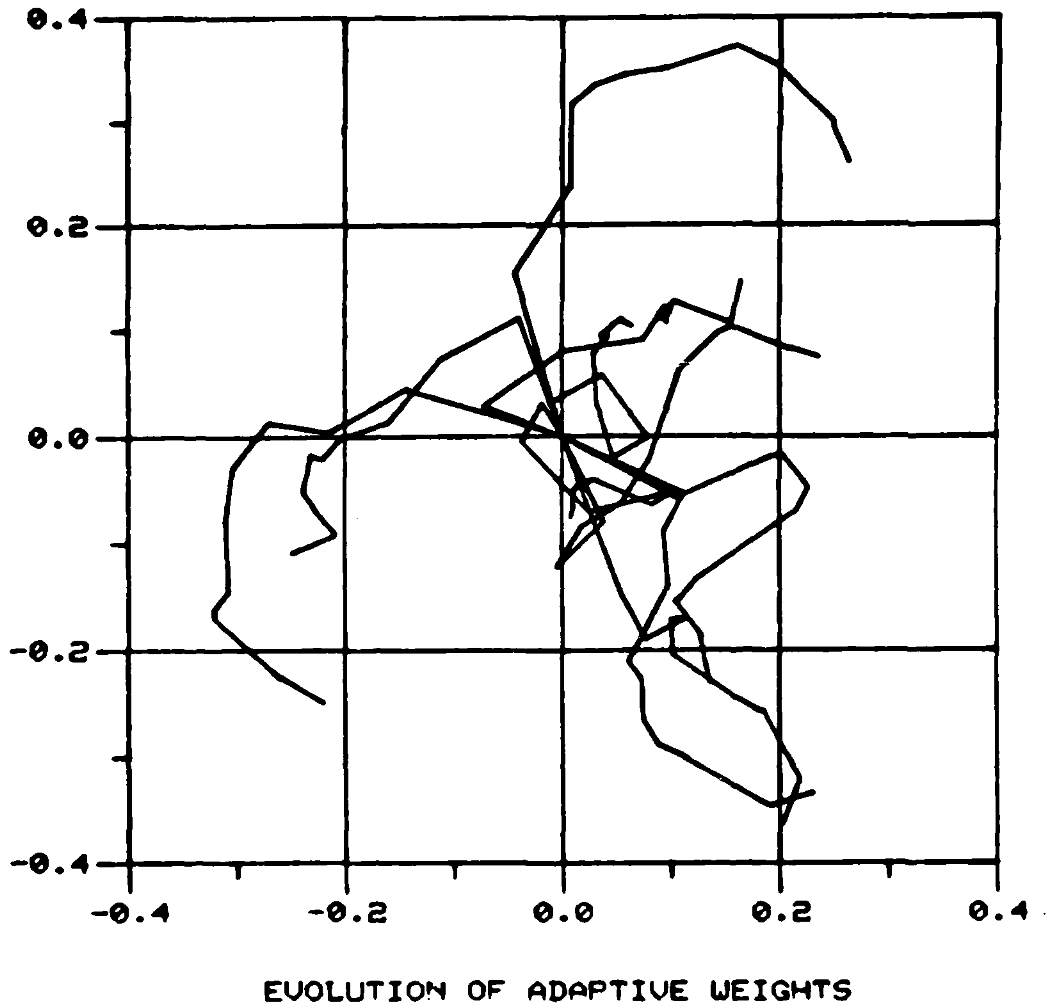


Figure 3-21 (Cont'd)

02:59 FEB 19,'81

Source Transfer Function Amplitude Responses Before and After BCR Adaptation

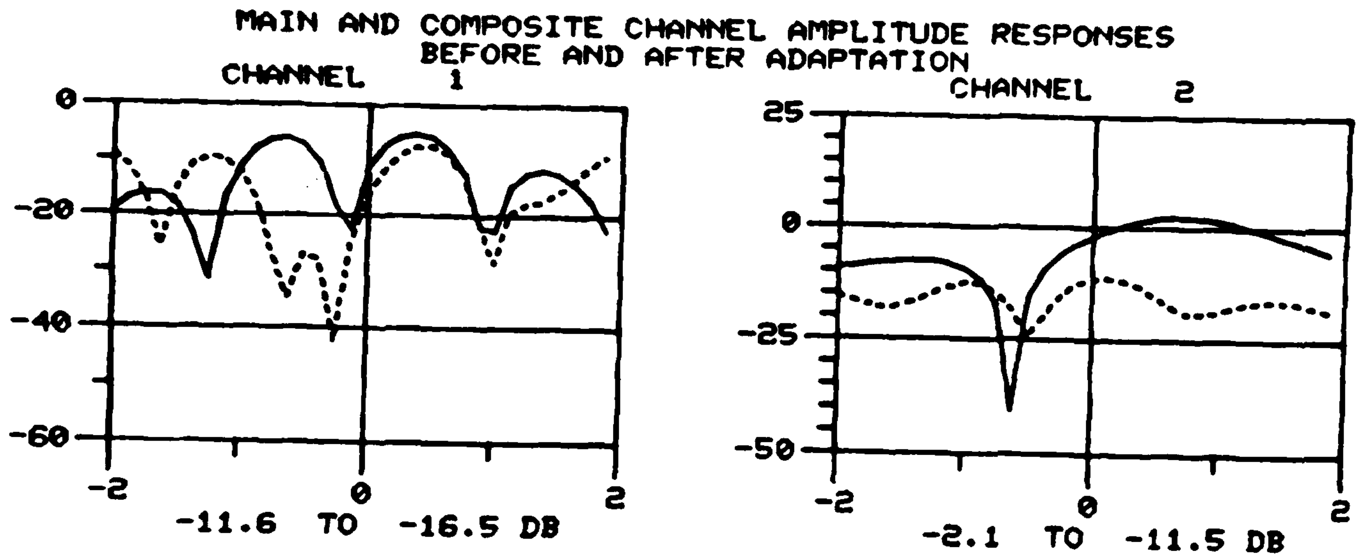


Figure 3-21 (Cont'd)

02:59 FEB 19,'81

Field Pattern Amplitudes Over 0.25 Neighborhoods About Source Angles of Arrival at RF Center

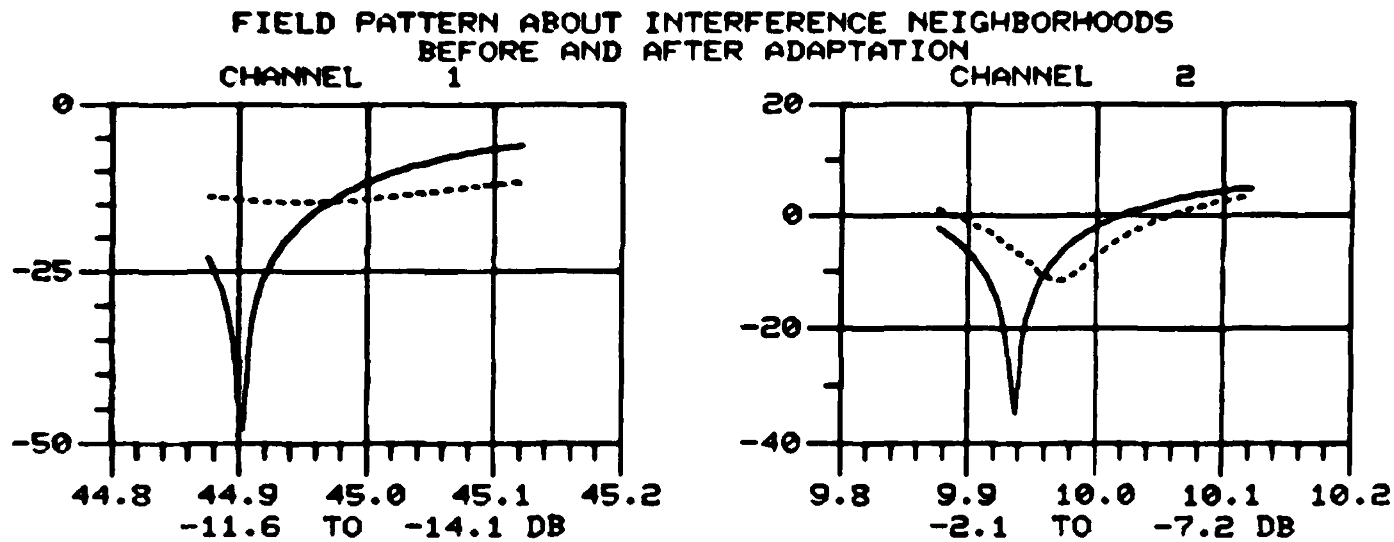


Figure 3-21 (Cont'd)

02:59 FEB 19,'81

Main Beam Field Pattern Amplitude at Center RF Frequency, Before and After BCR Adaptation

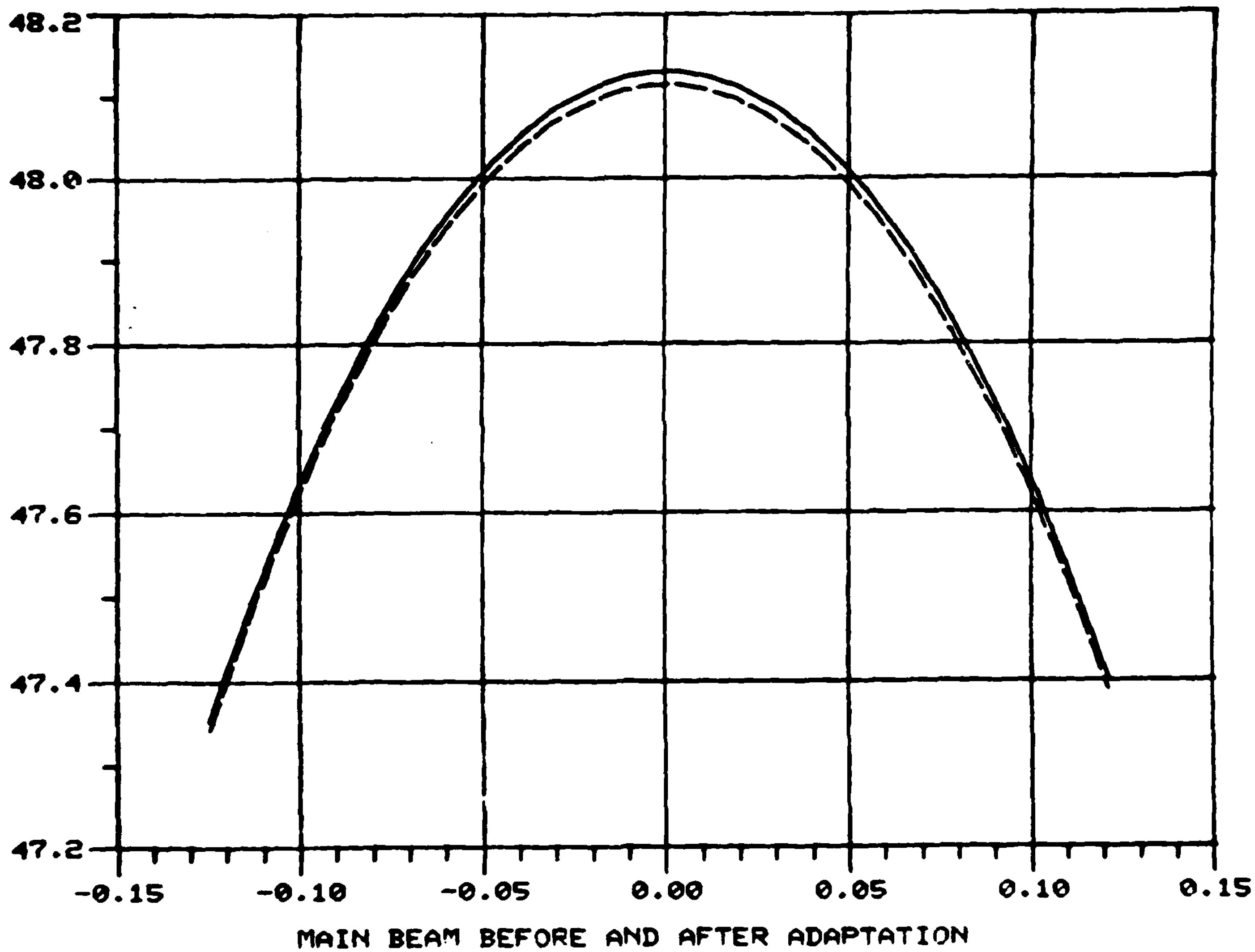


Figure 3-21 (Cont'd) 02:59 FEB 19,'81

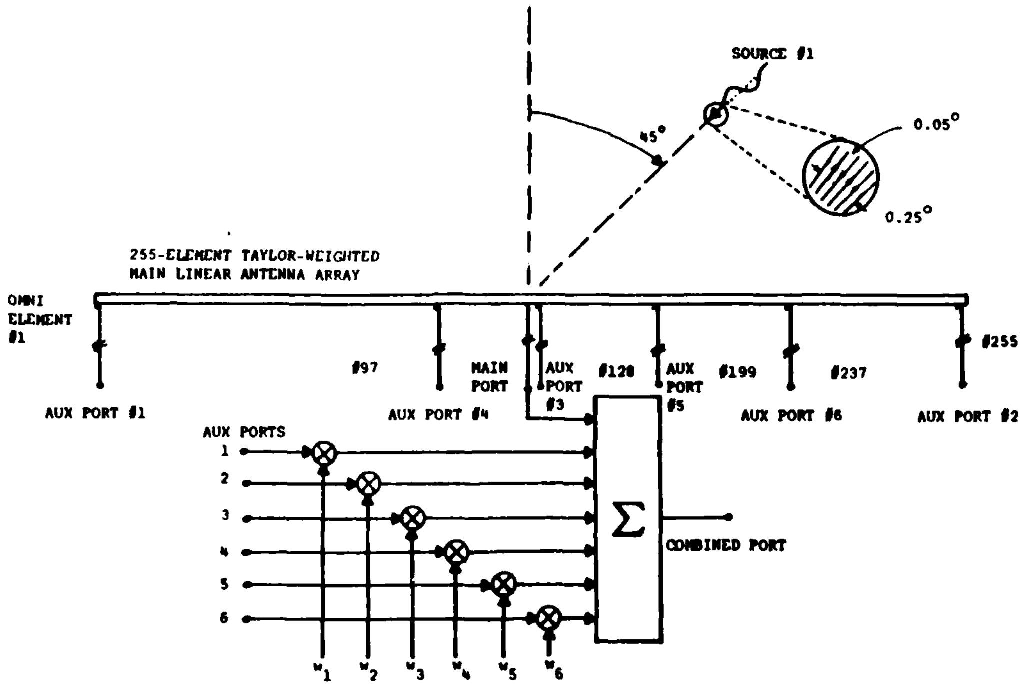


Figure 3-22. Scenario and System Description
Multipath Example
Highly Correlated Case
0.1% RF Bandwidth

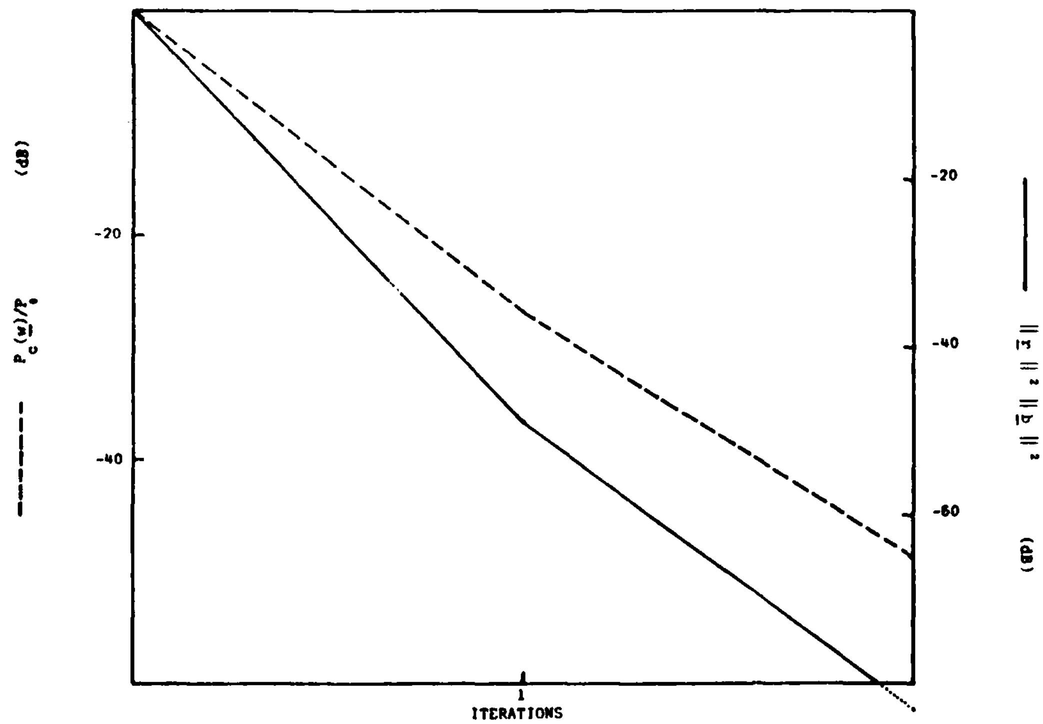


Figure 3-23. BCR Convergence Characteristics
Multipath Example
(See Figure 3-22)

Title Page

BCR ADAPTIVE PROCESSING

MULTIPATH EXAMPLE
(See Figure 3-22)

(Highly Correlated Multipath Signals)

SOURCES	:	6 CONTINUOUS MULTIPATHS (45.0°, 45.05°, ..., 45.25°) (0.00ΔT, 0.05ΔT, ..., 0.25ΔT)
RF BANDWIDTH	:	0.1%
AUXILIARY PORTS	:	6
TAP WEIGHTS	:	1/PORT

SIMULATION BY : S. M. DANIEL & I. KERTESZ
ADVANCED TECHNOLOGY AND SYSTEMS ANALYSIS
MOTOROLA, INC. - GED

16:07 FEB 23, '81

Figure 3-21

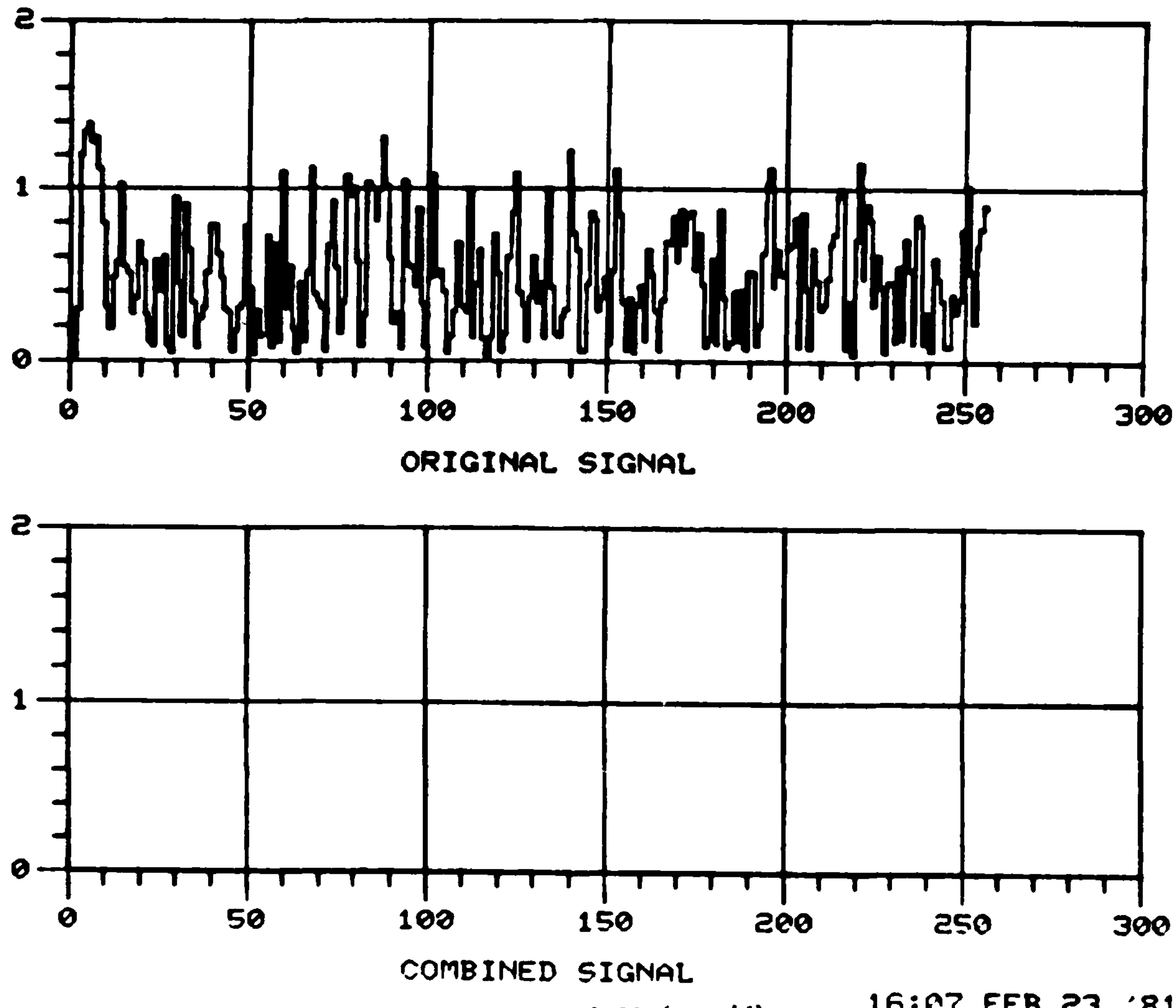


Figure 3-21 (Cont'd)

BCR Adaptive Weight Evolution in the Complex Plane

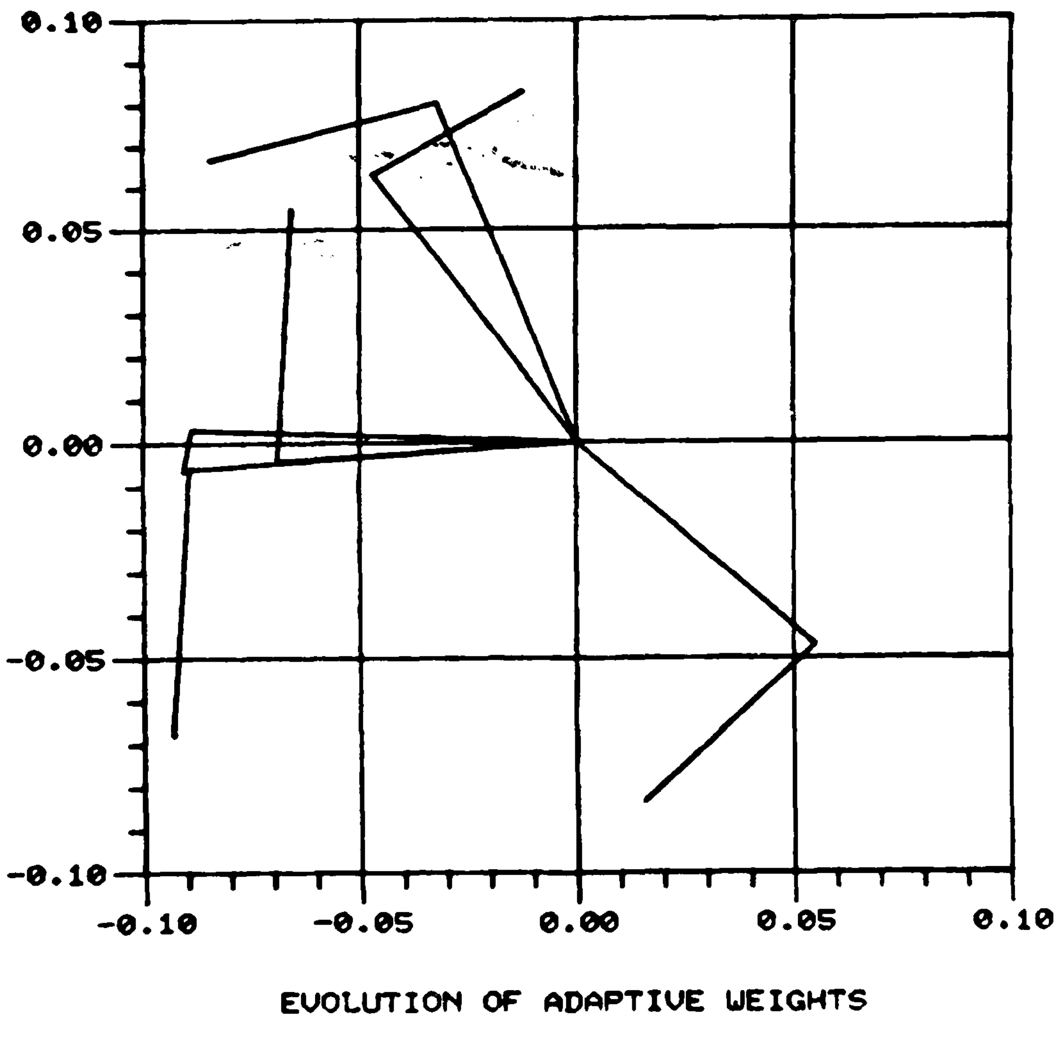


Figure 3-21 (Cont'd)

16:07 FEB 23, '81

Source Transfer Function Amplitude Responses Before and After BCR Adaptation

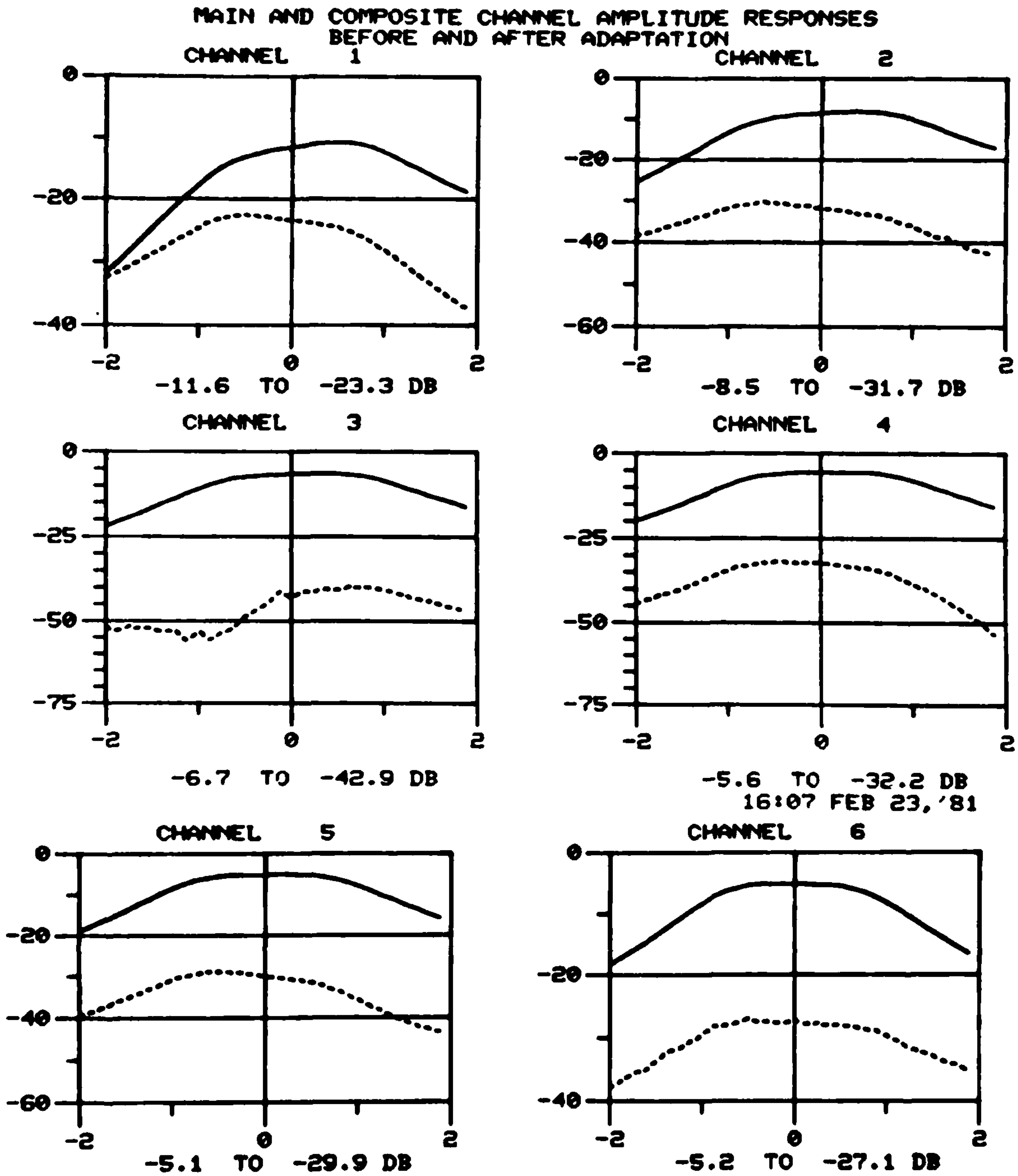
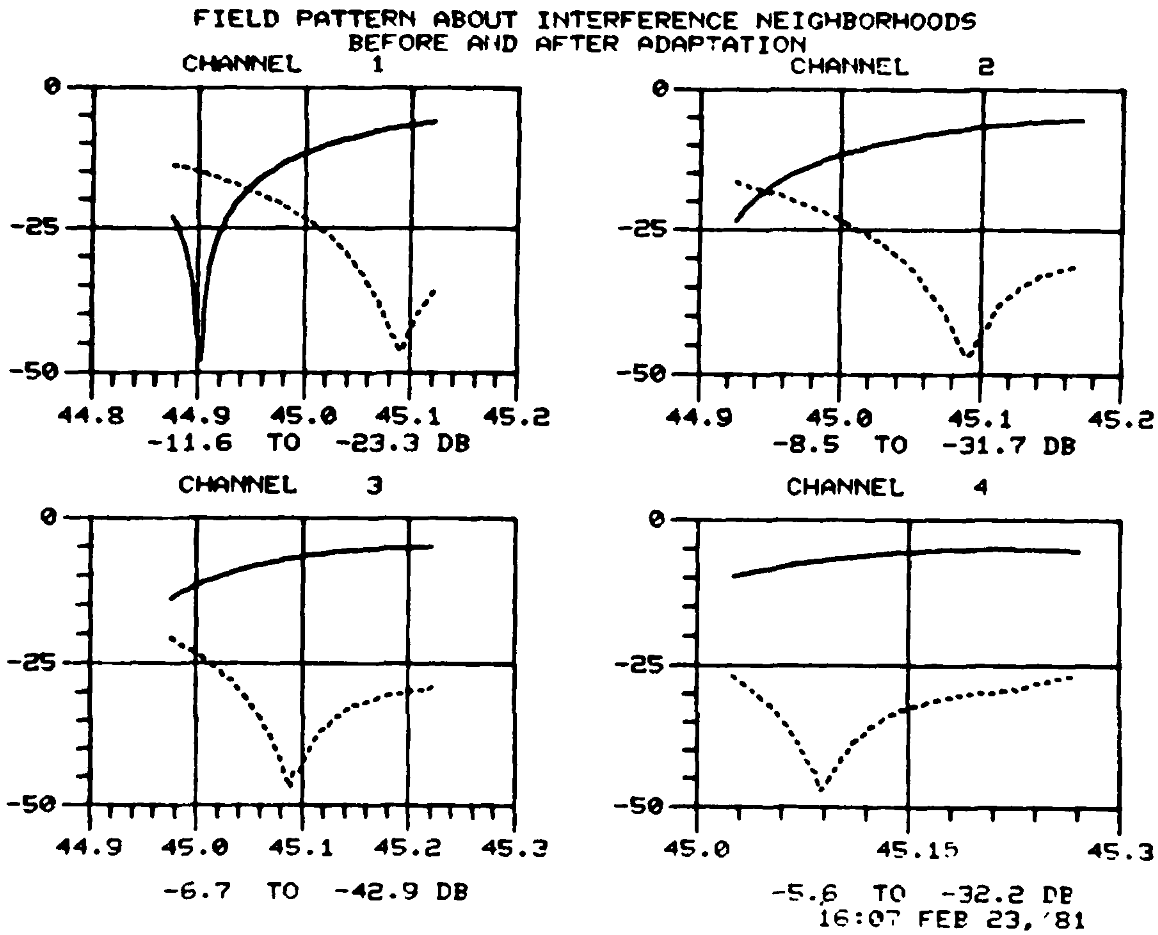


Figure 3-21 (Cont'd) 16:07 FEB 23,'81

Field Pattern Amplitudes Over 0.25 Neighborhoods About Source Angles of Arrival at RF Center



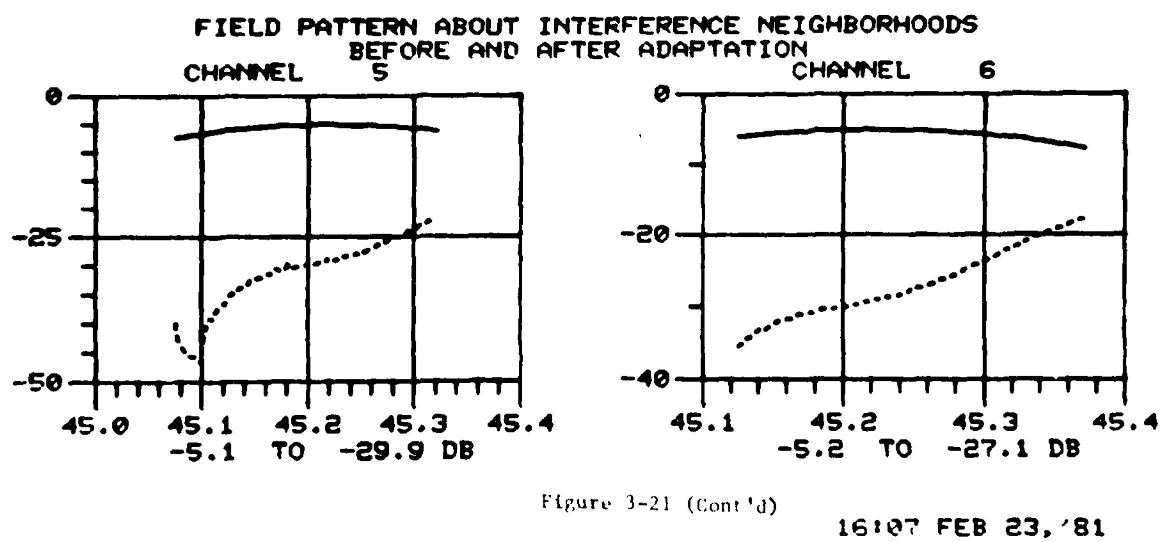


Figure 3-21 (Cont'd)

Main Beam Field Pattern Amplitude at Center RF Frequency, Before and After BCR Adaptation

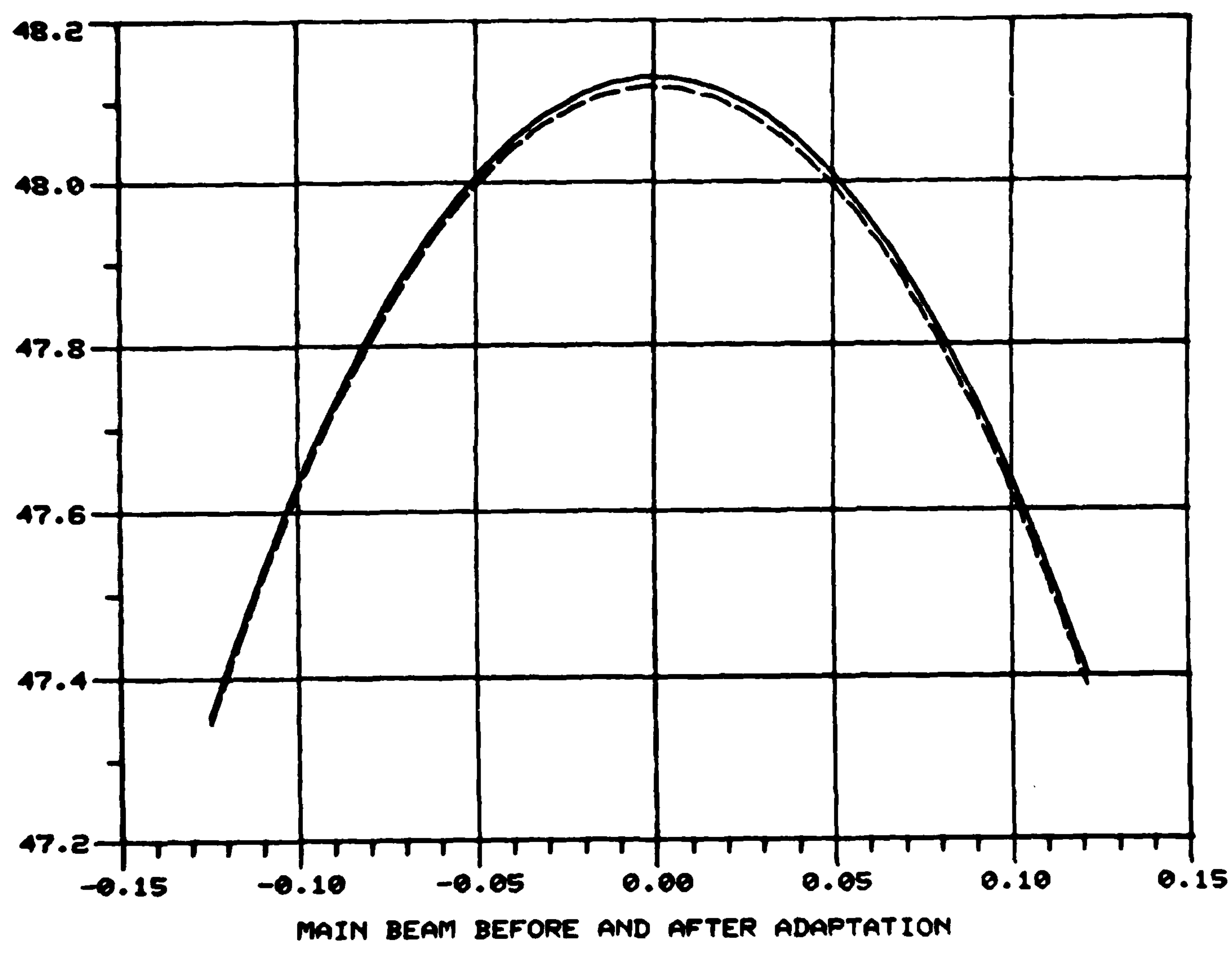


Figure 3-21 (Cont'd)

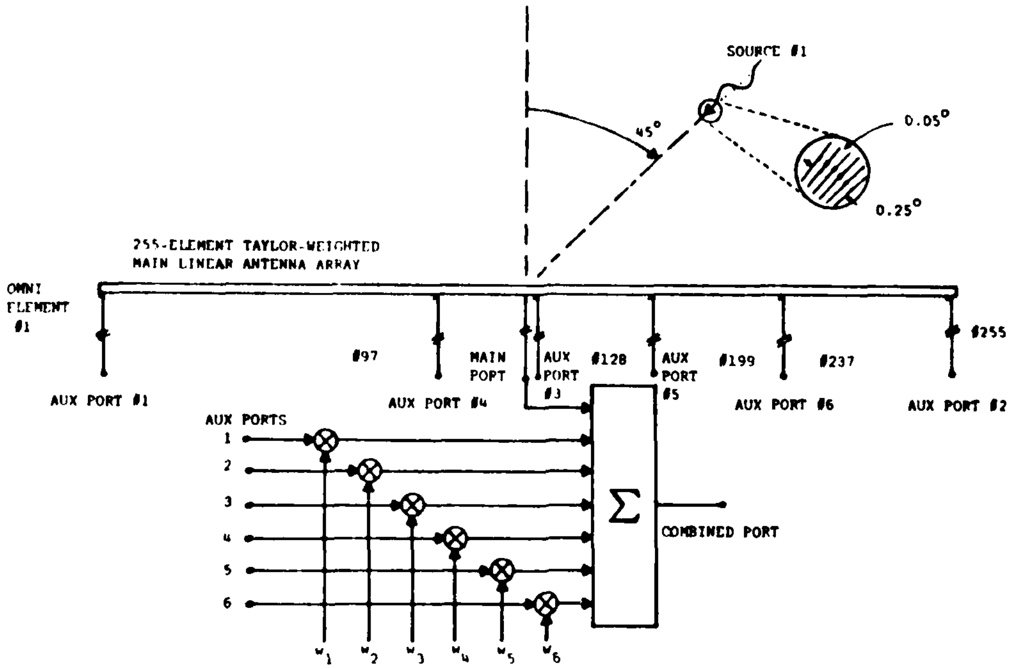


Figure 3-25. Scenario and System Description
Multipath Example
Uncorrelated Case
0.1% RF Bandwidth

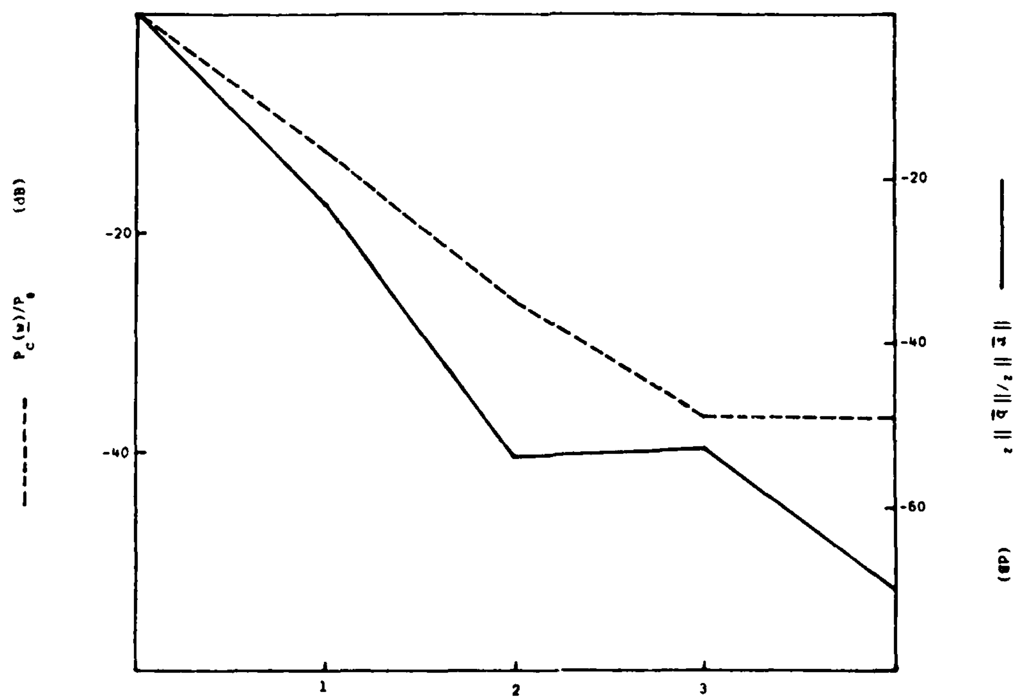


Figure 3-26. BCR Convergence Characteristics
Multipath Example
(See Figure 3-25)

Title Page

BCR ADAPTIVE PROCESSING

MULTIPATH EXAMPLE
(See Figure 3-25)

(UNCORRELATED MULTIPATH SIGNALS)

SOURCES	:	6 CONTINUOUS (45.00°, 45.05°, ..., 45.25°)
RF BANDWIDTH	:	0.1%
PORTS	:	6
TAP WEIGHTS	:	1/PORT

SIMULATION BY : S. M. DANIEL & I. KERTESZ
ADVANCED TECHNOLOGY AND SYSTEMS ANALYSIS
MOTOROLA, INC. - GED

12:08 MAR 01, '81

Figure 3-27

Relative Signal Amplitude Before and After BCR Adaptation

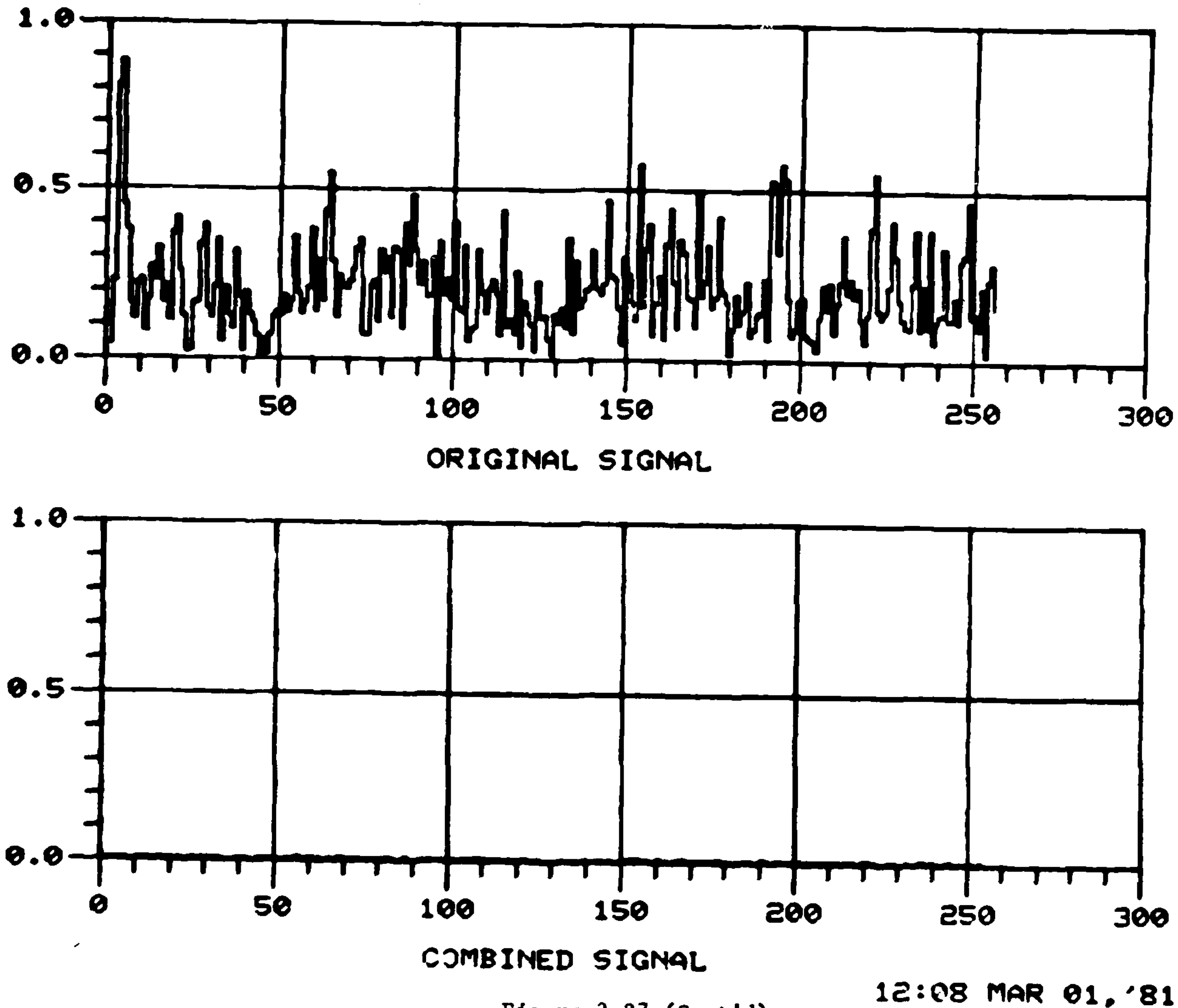


Figure 3-27 (Cont'd)

BCR Adaptive Weight Evolution in the Complex Plane

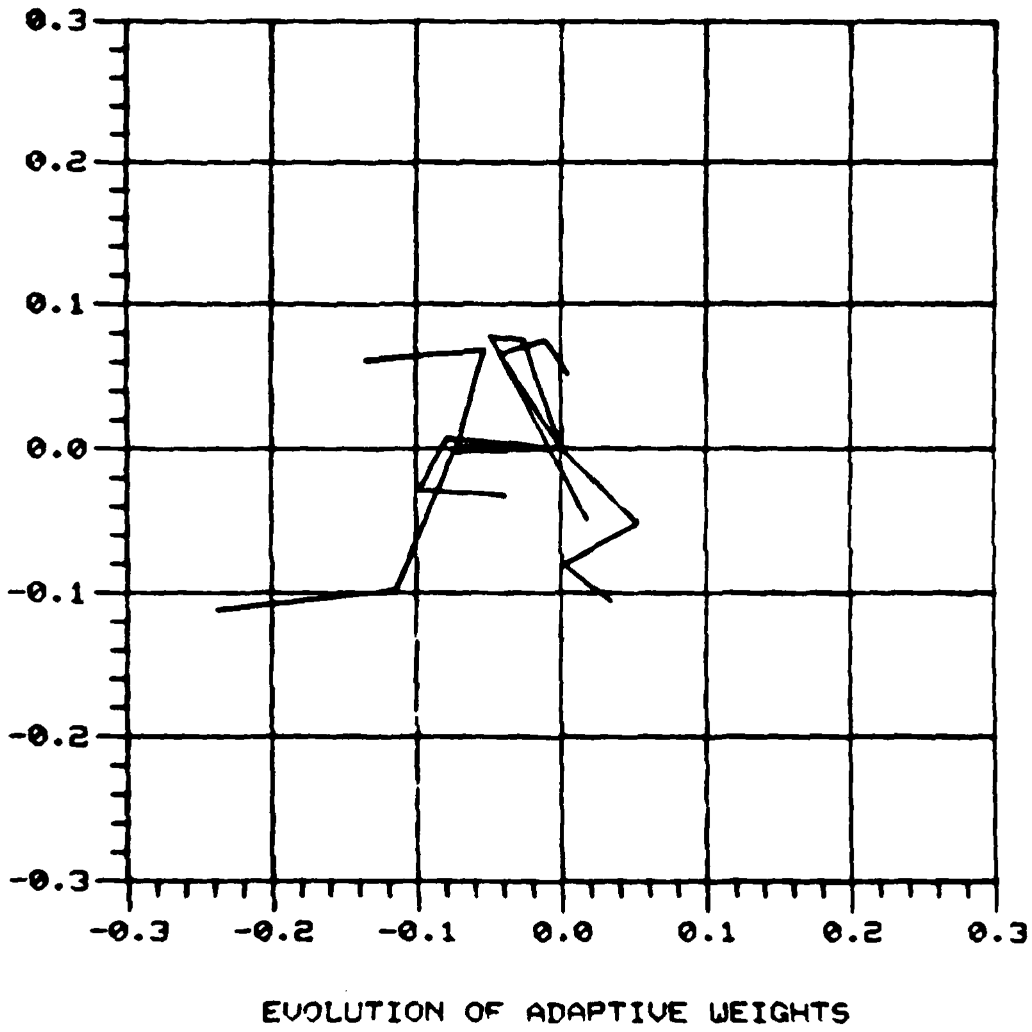


Figure 3-27 (Cont'd)

12:08 MAR 01, '81

Source Transfer Function Amplitude Responses Before and After BCR Adaptation

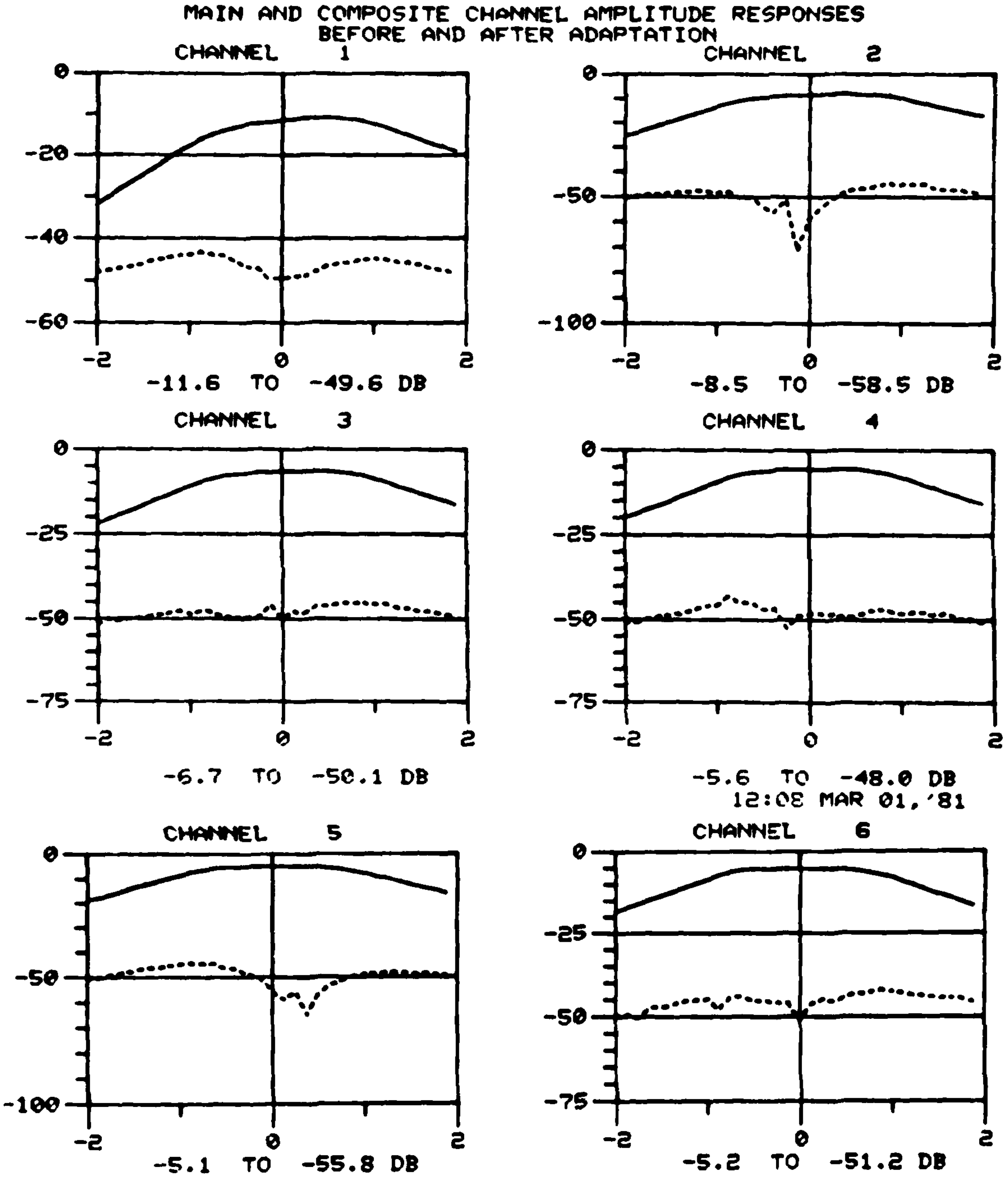


Figure 3-27 (Cont'd) 12:08 MAR 01,'81

Field Pattern Amplitudes Over 0.25 Neighborhoods About Source Angles of Arrival at RF Center

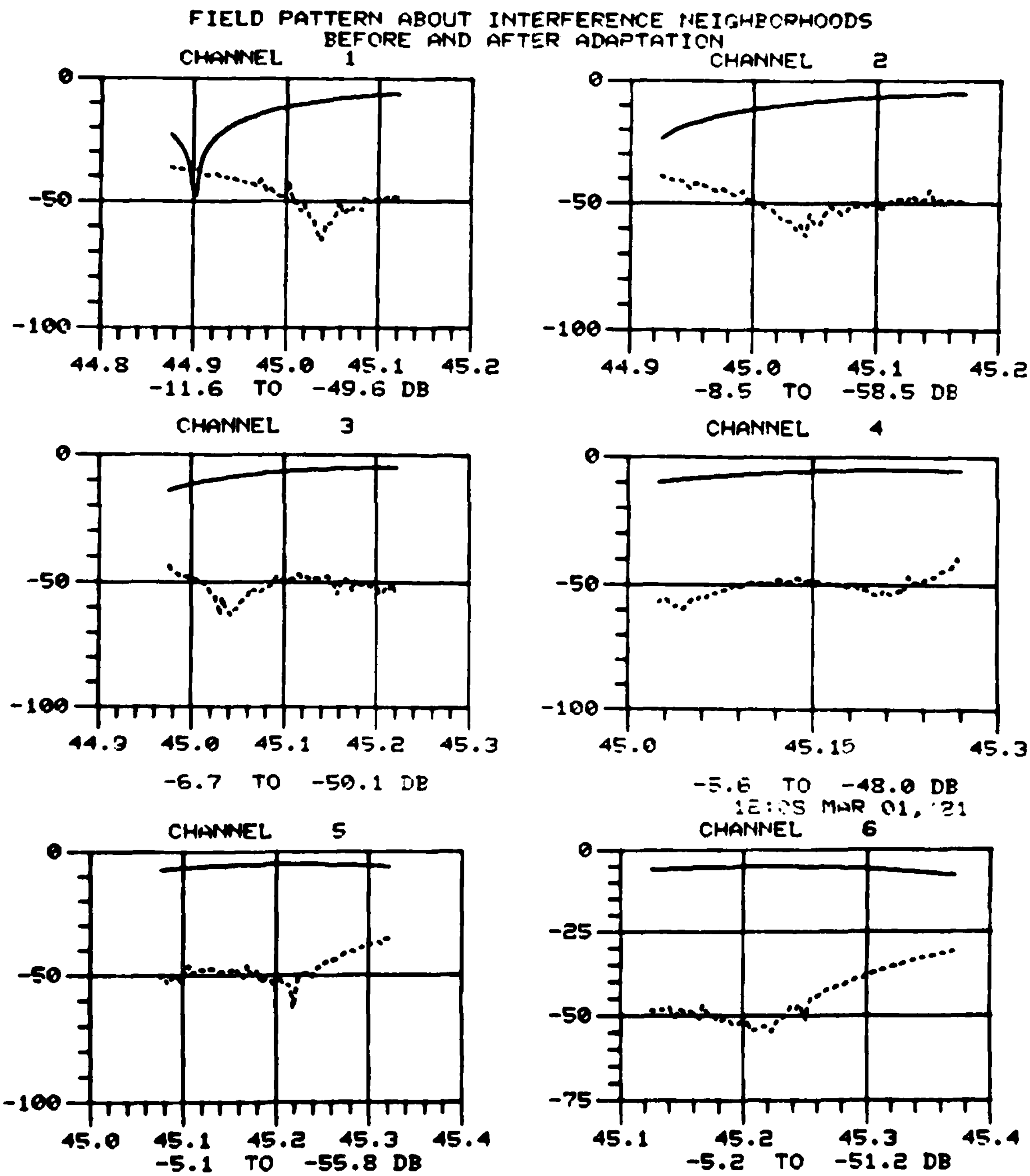


Figure 3-27 (Cont'd)

12:08 MAR 01, '81

Main Beam Field Pattern Amplitude at Center RF Frequency, Before and After BCR Adaptation

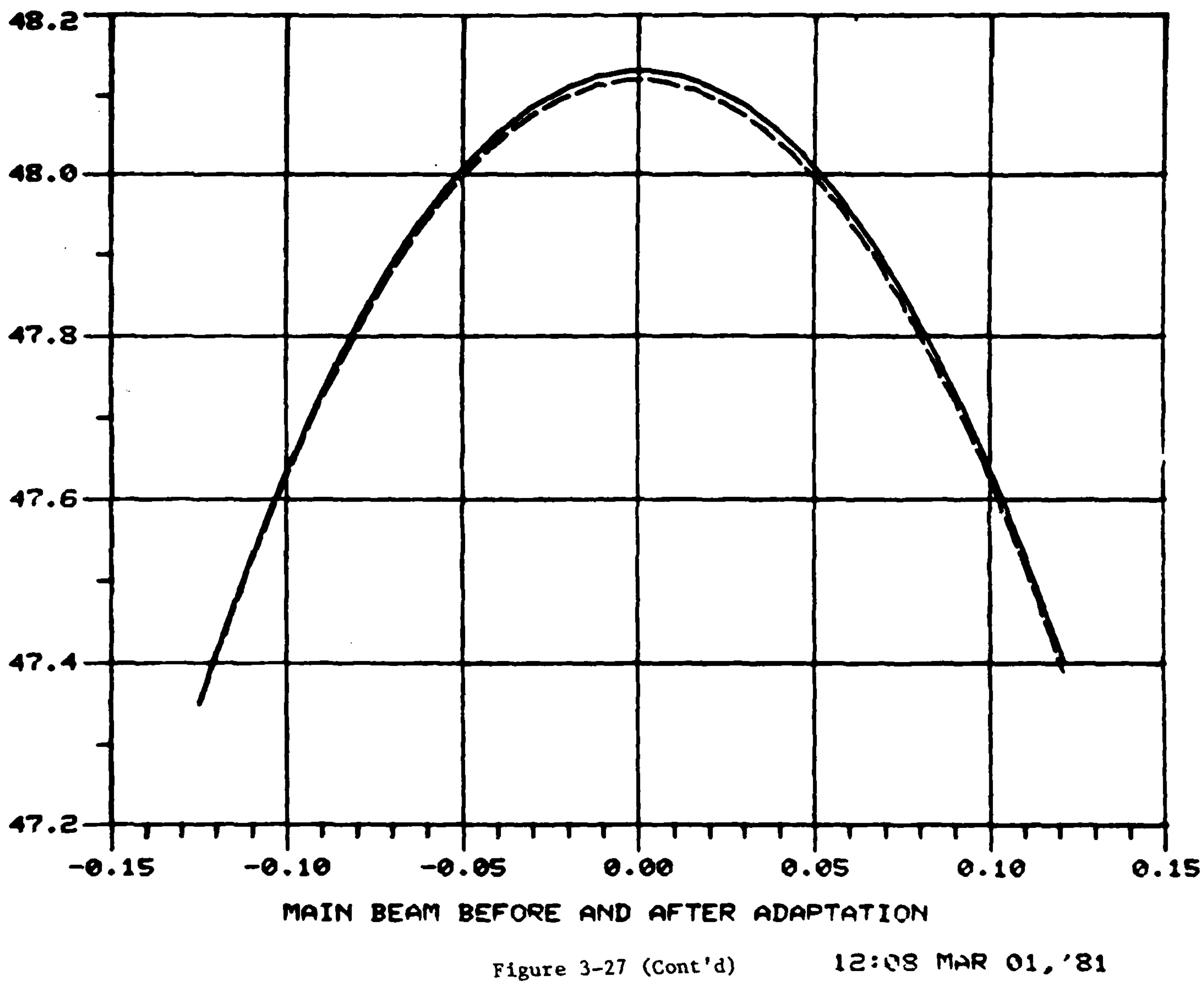


Figure 3-27 (Cont'd)

To be noted here is the relatively degraded nulling performance and the wide pattern nulls in comparison to the previous example. Since the multipath signals in latter example are uncorrelated, these observations are as expected.

3.2 General Results

The BCR simulation results presented in the previous section relate to specific single examples. In contrast, the general results included in this section deal with the BCR nulling performance as a function of various pertinent scenario and system parameters.

3.2.1 BCR Convergence Characteristics

Figure 3-28 attempts to demonstrate BCR convergence characteristics as a function of a number of incident sources. The scenario and system description is as shown in the Figure. Operating at a 0.1% RF bandwidth, the adaptive array system of a main linear antenna array and six omnidirectional auxiliaries is exposed to incident wideband noise sources ranging from one to six. Using a single adaptive weight per auxiliary port, Figure 3-28 gives the relative combined power and relative gradient metric at each BCR iteration for each choice of number of incident sources, indicated next to each set of two curves.

One overall prominent feature that may be observed in Figure 3-28 is the decrease in nulling performance with an increase in the number of sources. In fact, the relative combined power curves exhibit a monotonic upward trend with increased number of sources. The same behavior does not seem to predominate with respect to the relative gradient metric curves.

Two specific local features are of interest here. First, note that in every case at least one extra iteration was necessary to achieve convergence. This is due to the fact that even with the moderate 0.1% RF bandwidth, the rank of the covariance matrix over the six auxiliary port signals exceeded the actual number of incident sources by at least one. Recall that in the wideband examples discussed in the previous section, the apparent rank of the covariance matrix was well in excess of the one or two sources considered, as evidenced by the number of BCR iterations used.

The other prominent local feature is the nonmonatonic behavior of the relative gradient metric. In particular, note that, in the case involving five incident sources, there is a pronounced increase in relative gradient metric at the sixth iteration. This behavior is not unusual in the underlying CG algorithm employed. What is important here is that while the gradient metric has increased, the relative combined power has decreased.

3.2.2 RF Bandwidth Dependence

Figure 3-29 shows the dependence of BCR nulling performance for a 10-port 1-tap/port system as a function of % RF bandwidth for two cases of one and six incident sources.

In the case of six incident sources the degradation in nulling performance with increased %RF bandwidth is quite rapid. As expected, however, the nulling performance is rather respectable even up to 10% RF bandwidth. The rather irregular behavior over the 0-1% RF bandwidth region is not fully clear at this time. Perhaps it is related to the blind region phenomenon described next.

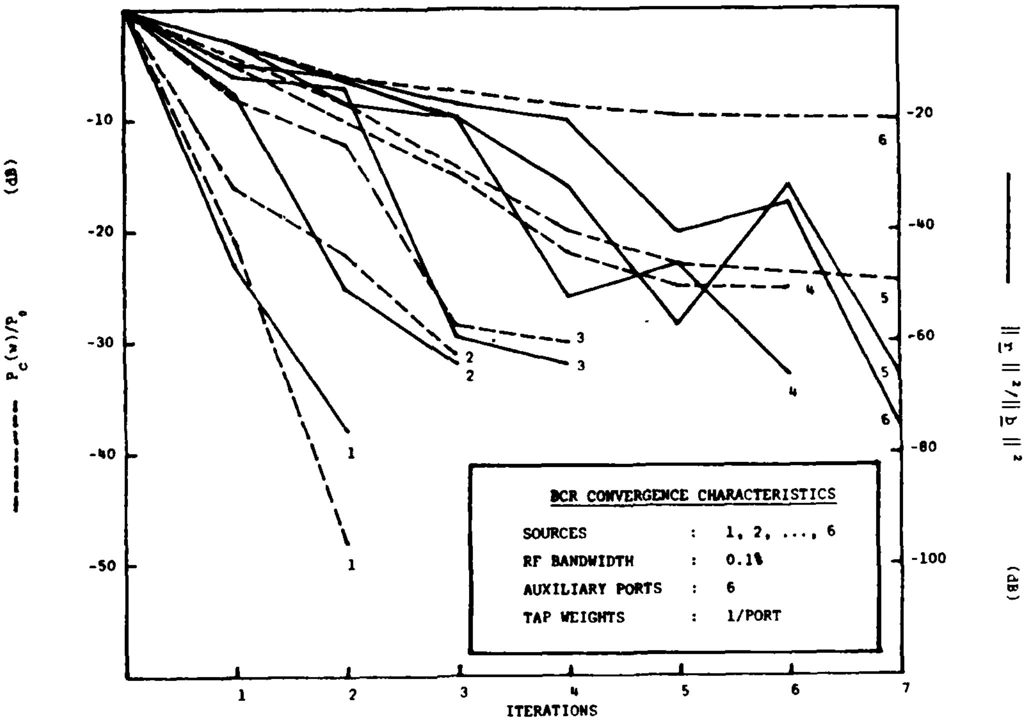


Figure 3-28. BCR Convergence Characteristics as a Function of Number of Incident Sources

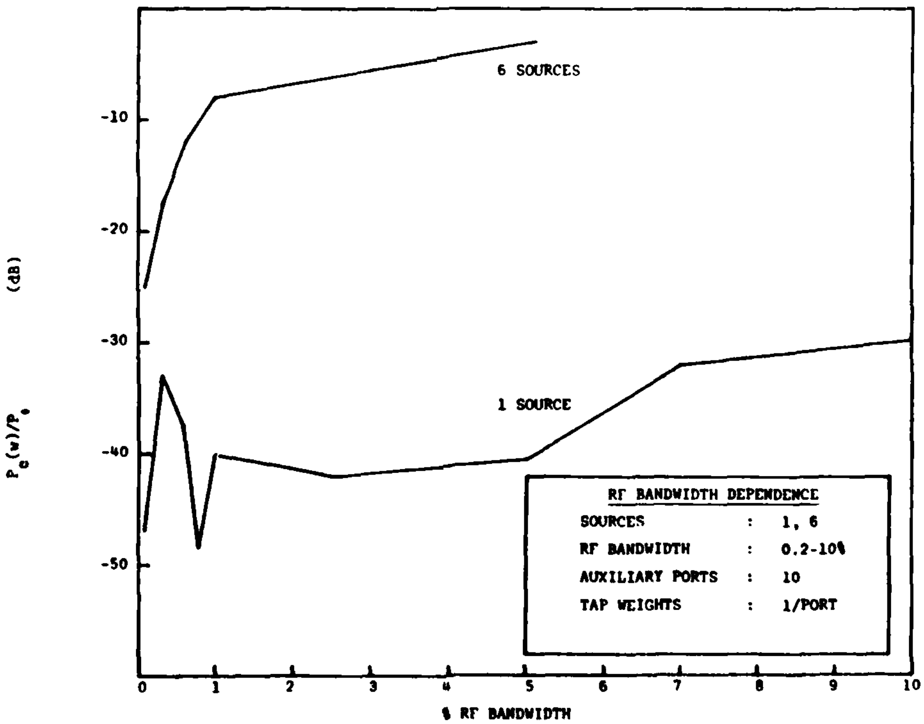


Figure 3-29. BCR Adaptive Nulling Performance as a Function of % RF Bandwidth

3.2.3 Blind Region Phenomenon

A phenomenon which often plagues adaptive nulling performance is the blind region phenomenon which refers to poor nulling performance over specific angular regions in the composite field pattern. As such, it may be encountered in adaptive E-scan systems. This phenomenon may be subdued substantially by adding more degrees of freedom, in the form of adaptive weights, either as additional taps in existing ports or single taps over additional corresponding ports.

Figure 3-30 demonstrates the blind region† phenomenon for an adaptive array system characterized by a 0.1% RF bandwidth and six 1-tap auxiliary ports for four scenarios involving 3, 4, 5 and 6 incident sources.

Of the four cases shown, the 6-source case gives rise to the most prominent blind region occurrence. Note, the relatively poor adaptive nulling performance over a nearly 0.8° "blind region" of the total 1.4° degrees scanned. The blind region in the 5-source case occupies a substantially smaller angular fraction. The indicated blind region in the 4-source case is rather shallow. No blind region is indicated in the 3-source case. Clearly, with additional single-tap ports, the blind region problem may be substantially minimized.

3.2.4 Multitap Weighting

Given an adaptive array system with a fixed number of auxiliary ports, its nulling performance may be gradually improved by appropriately introducing additional delaying tap weights at each port. The two examples included below demonstrate the nulling effectiveness of multitap weighting in a large bandwidth case and in a case involving a blind region.

The first example involves one incident source, a 10% RF bandwidth and one auxiliary port. Figure 3-31 shows the BCR adaptive nulling performance as a function of the number of tap weights, N_T, spaced uniformly over a delay equivalent to that of the full aperture of the main antenna array. To be noted here is an overall improvement trend with increased number of taps, although not monotonic. A satisfactory explanation of this behavior will have to await further specific investigation.

The second example considered involves three incident sources, a 0.1% bandwidth and three auxiliary ports. Figure 3-32 shows the BCR nulling performance as a function of aximuth scan shift for one and two tap weights per port. Note that the major blind region has remained practically intact in going from one to two taps. The obvious question at this point is whether additional taps could in fact diminish the blind region in any substantial way. An important observation here is that if the adaptive weights were applied over four different ports, then the blind region would, in fact, decrease substantially.

---

†Here, the blind region was arbitrarily defined to be that region over which the residual combined power is at least 6 dB higher than that of its minimum value over the scanned angular region. This, of course, is not a strict rule but it does permit a qualitative valuation in the present discussion.

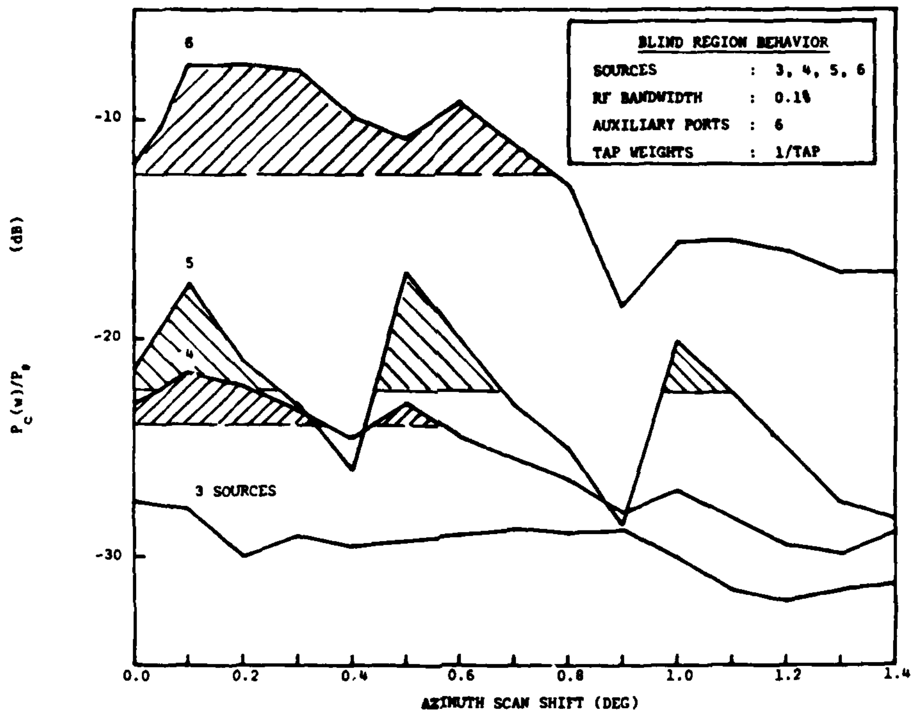


Figure 3-30. Blind Region Behavior as a Function of Number Of Sources

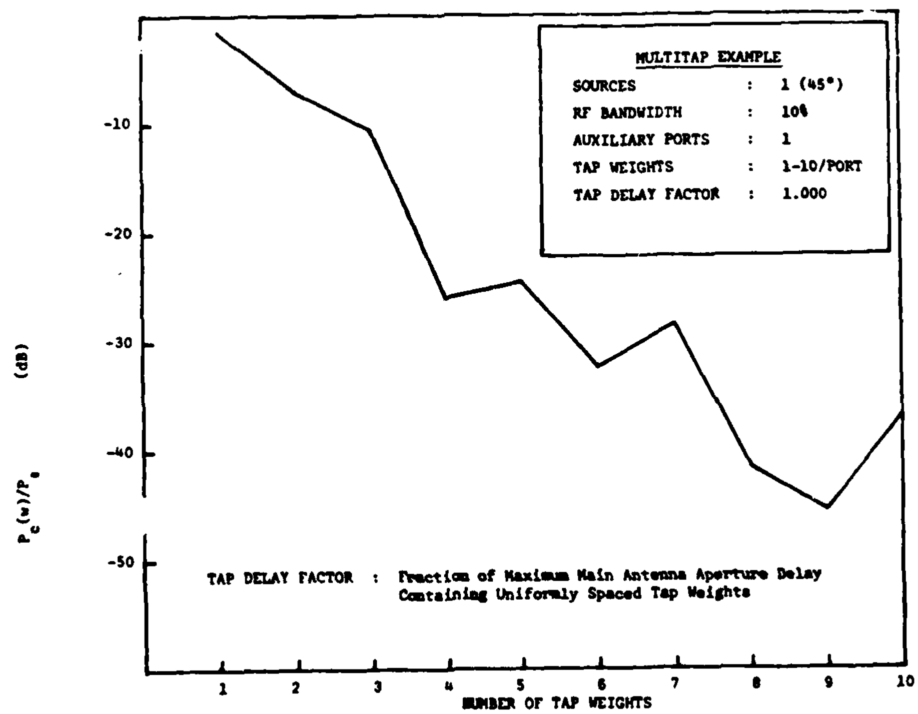


Figure 3-31. BCR Adaptive Nulling Performance as a Function of Number of Tap Weight

Still another indication that taps and ports are not interchangeable is seen in Table 3-4 which presents adaptive nulling performance of two single sources with two numbers of 1-tap ports, two numbers of taps over a single port with two associated tap delay factors.

To be noted immediately is the respectable performance of the two single-tap multiport cases. What is quite unexpected is the inconsistent nulling performance in the single-port multi-tap cases. For example, the 10-tap case with a tap delay factor of 1.000 yields a satisfactory performance in the presence of the 7° source but does rather poorly against the 45° source. Using a delay factor of 0.707, the performance quality is reversed for each of the separate sources. Increasing the number of taps to 20 and using a tap delay factor of 1.000 gave rise to a marginal performance against either source, each one worse than the previous best performance of -17.95 and -22.03. Clearly, increasing the number of taps did not improve the nulling performance. Using 20 taps with a tap delay factor of 0.707, however, has given the best nulling performance against each separate source.

Even though the last line in Table 3-4 of results might suggest an equivalence between taps and ports, the previous three lines provide evidence to the contrary. In fact, the additional results in Section 3.1.3.4 add to this evidence. Certainly, a more substantial investigation must be conducted before any general conclusions may be made in connection with the observation made here.

3.2.5 Receiver Mismatch

In all cases considered so far, the main and auxiliary receivers have been assumed to be perfectly matched. Specifically, the receiver transfer function has been assumed to be the baseband equivalent of a two-pole Butterworth 10% bandwidth IF filter. Since perfect matching among filters is not practically possible, it is desirable to determine the effect of their mismatch on nulling performance and thus establish design tolerances necessary for a given application.

Figure 3-33 demonstrates the dependence of nulling performance on receiver mismatch. The case considered involves three sources, a 0.1% RF bandwidth, three ports with two-pole Butterworth or six-pole Chebyshev IF filters. One and two-tap implementations are evaluated.

The particular mismatch on the receivers is in the form of a simultaneous percentage bandwidth reduction and center frequency offset. More specifically, each auxiliary receiver is assigned a mismatch level ranging from zero (perfect match with the main receiver) to a certain percent bandwidth tolerance level. In the present case, the first auxiliary receiver is matched to the main, the second is mismatched by half the tolerance level and the third is mismatched by the full percent bandwidth tolerance indicated in Figure 3-33.

As expected, the nulling performance is more sensitive in the case of the more selective six-pole Chebyshev filter than that of the two-pole Butterworth. Note that to achieve -20 dB of nulling, a 4% tolerance is required of the six-pole filter while as much a 15% is allowed for the two-pole.

Table 3-4. Multiport/Multitap Comparison

SCENARIO AND SYSTEM DESCRIPTION

SINGLE SOURCE AT	:	7° or 45°
RF BANDWIDTH	:	10%
NUMBER OF ONE-TAP PORTS	:	10 or 20
NUMBER OF TAPS ON SINGLE PORT	:	10 or 20

NUMBER OF PORTS	TAPS PER PORT	TAP DELAY FACTOR	RELATIVE COMBINED POWER LEVEL (dB) FOR EACH OF TWO SEPARATE SOURCES	
			AT 7°	AT 45°
10	1	-----	-36.34	-30.00
20	1	-----	------	-29.36
1	10	1.000	-17.35	- 2.25
1	10	0.707	- 1.38	-22.03
1	20	1.000	-10.12	-13.51
1	20	0.707	-47.01	-31.66

TAP DELAY FACTOR: Fraction of Maximum Main Antenna Aperture Delay Containing Uniformly Spaced Tap Weights.

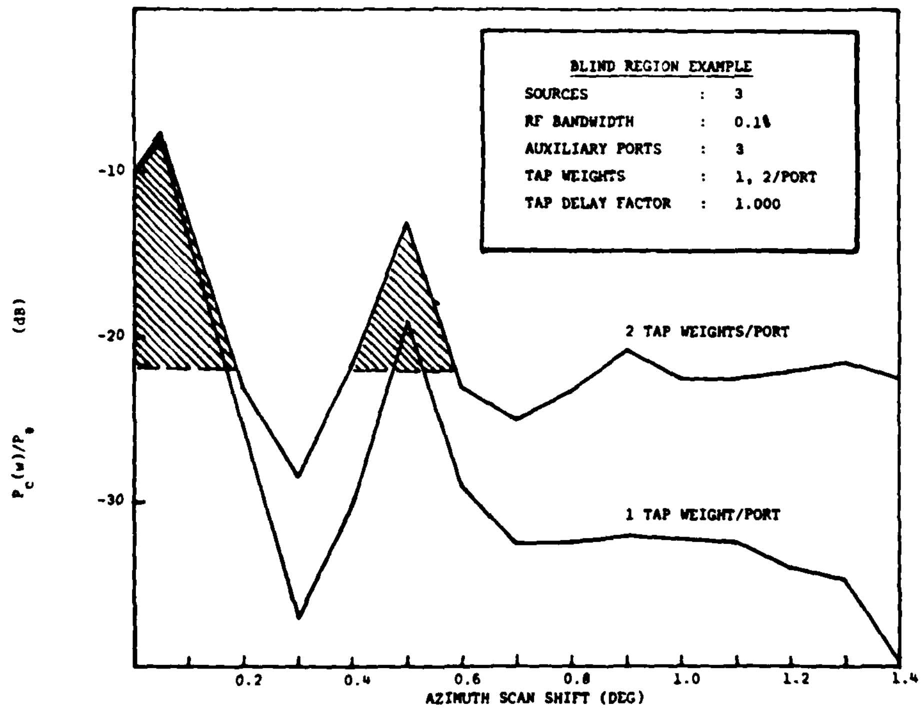


Figure 3-32. Blind Region Behavior as a Function of Number of Tap Weights

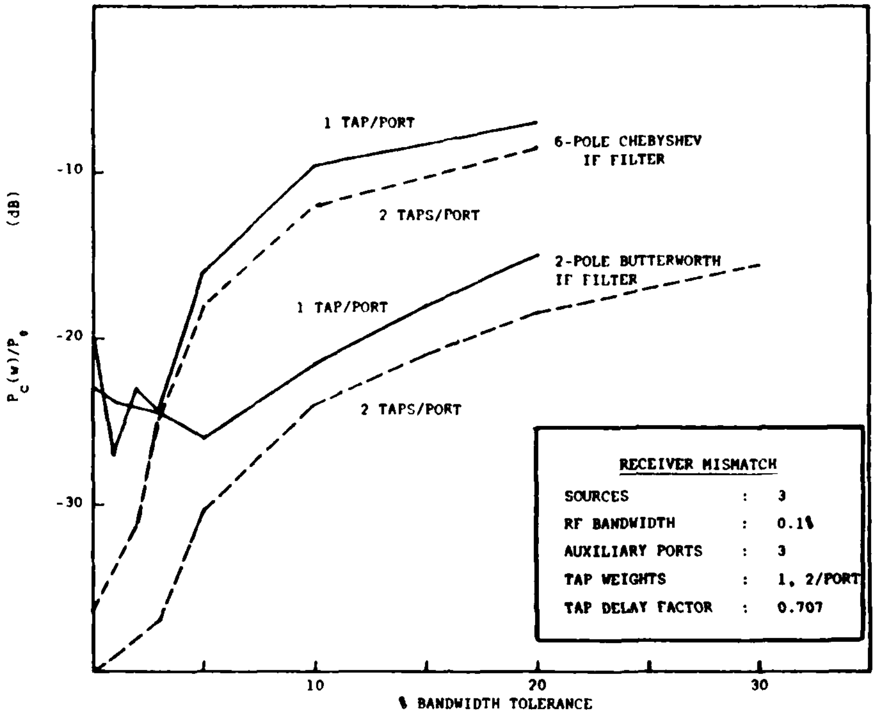


Figure 3-33. BCR Adaptive Nulling Performance as a Function of Bandwidth Tolerance for Two Different IF Filters

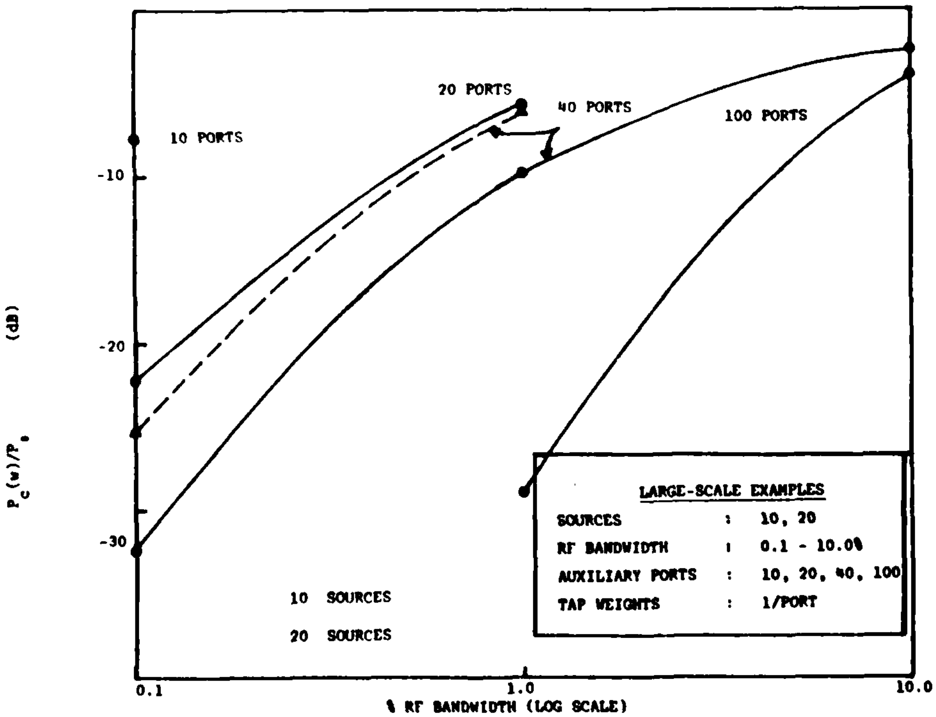


Figure 3-34. BCR Large-Scale Performance

Using 2 taps per port it is possible to achieve a certain amount of equalization between receivers which is manifested in improved nulling performance for both receiver designs. As such, by introducing two taps per port, the bandwidth tolerances of the 6-pole and 2-pole filter designs may be increased to 5% and 20%, respectively, in order to achieve -20 dB of nulling. Of course, design tolerances must be much lower than these if one is interested in minimizing the loss in potential nulling performance.

3.2.6 Large-Scale Performance

A number of large scale BCR simulation examples were run involving 10 and 20 sources. Relative combined power results after BCR adaptation have been combined into Figure 3-34 to demonstrate, graphically, general performance trends with various scenario and system parameters.

For the 10-source case, the adapted relative combined power level has been plotted against a log-scaled % RF bandwidth for 10, 20, 40 and 100 ports. For the 20-source case, only 40 ports were used. Even though the number of actual data points was small in each instance, curves have been drawn to enhance the visibility of the individual results. The nulling performance improvement against 10 sources in going from 10 to 20 to 40 and finally to 100 single-tap ports is clearly suggested.

It should be mentioned that the main antenna aperture used in these large-scale examples was 4-times that of all previous examples. In all previous examples a 255-element main antenna array was used; a 1001-element array was used here. As such, the sidelobe sensitivity to frequency over the specified bandwidth must be 4-times that encountered in the previous examples. For this reason, the results shown in Figure 3-34 would be substantially improved if the 255-element main antenna array were used.

3.3 Comments on BCR Variations

The BCR simulation program, BCRS, employed the conventional form of the CG algorithm presented in Theorem 3-9 of Project Memorandum 8512-03. For purposes of minimizing the roundoff error, the alternate CG form stated in Theorem 3-11 could have been used. During the early stages of program development, the latter formulation was compared to the former, but no significant numerical improvement was noted. A further investigation would certainly be needed before the preferred form of the CG algorithm could be identified for the present application. The numerical advantage of the alternate CG form could certainly be exploited in a reduced numerical resolution environment of a digital implementation.

The constrained BCR formulation developed in Project Memorandum 8512-03 was considered in the simulation of the adaptive array system in question. Based on the minimal effect observed on the main beam after constrained BCR adaptation, it did not make sense to switch to a constrained BCR simulation. Of course, the CBCR approach would be imperative for a fully adaptive array or adaptive beamforming system.

In an attempt to minimize the roundoff error in the BCR process, especially in large-scale examples, the CG algorithm may be repeated using the final estimate of the previous execution as the initial estimate of the second. This was discussed in Project Memorandum 8512-03. To minimize the total running time of this two-stage process it was decided to investigate a scheme for substantially decreasing the computational time during the first stage. The covariance matrix involved was approximated by a limited band about the main diagonal, while all entries beyond this region were assumed to be zero. By the very nature of the underlying CG algorithm, all computations involving the zero entries in the covariance matrix could be literally skipped.

When this Band-Diagonal BCR (BDBCR) approach was attempted with various choices of band-size, it was discovered that the resulting adaptive weight vector estimate was rather poor. Also, when this estimate was used as an initial estimate for the second BCR stage, it did not yield an estimate that was any better thar could be obtained via one normal BCR execution. Further, in cases where the rank of the covariance matrix was significantly less than the system dimensionality, the BDBCR approach took many more iterations to converge than the normal BCR process. As such, the anticipated computational saving was nearly eliminated in many cases. A more dedicated study would be necessary to establish the usefulness of the BDBCR approach for the current application. For the time being, it appears that the roundoff on a BCR estimate may be minimized by allowing a small number of additional iterations or limiting a second stage of processing to the same small number.

Another aspect related to BCR processing that would be of interest to the present application is the computation of the intrinsic inverse C^{x}, of the covariance matrix C. In Project Memorandum 8512-03 it has been conjectured that if the inverse, C^{-1}, exists, then $C^{-1} = C^{x}$. This conjecture was tested in several practical examples and proved to hold true. Hence, the motivation exists to carry out a formal proof of a theorem stating the claim of Conjecture 2-2 in Project Memorandum 8512-03.

4.0 CONCULDING REMARKS

Modular library-based FORTRAN programs, SIGGEN, BCRS and BCRP have been developed for the purpose of conducting an adaptive nulling performance analysis of an adaptive array processing system using the BCR adaptive process. These Signal Generation, BCR Simulation and BCR Performance and Plotting programs have been described in great detail. A variety of specific examples have been worked out and presented demonstrating the usefulness of these programs. General performance results were also obtained with these programs and presented here.

The variety of different examples considered are characterized by distinct system and scenario descriptions. These include a choice of %RF bandwidth, single or multiple tap weighting, receiver mismatch, continuous, blinking or multipath incident noise sources, etc. As such, it was possible to examine nulling performance as a function of bandwidth, multiport or multitap weighting, receiver mismatch and choice of source type.

A general observation that could be made from the results obtained is that with increased RF bandwidth the number of adaptive weights needed for acceptable nulling performance will increase accordingly well beyond the number of incident sources. This behavior was demonstrated in both moderate and large-scale examples. This trend was even more pronounced in the case of multitap weighting.

Although many additional examples could have been run, the ones included here suffice to prove the usefulness of the modular programs developed. More importantly, using the input data file SIGGEN:D as the primary menu, the user has the capability of analyzing any number of examples that he might be interested in.

Furthermore, one may examine the performance of related adaptive systems by making appropriate modifications to existing library modules or creating new ones, as needed. The facility for doing this is the inherent advantage of the modular program structure that has been developed.

BCR ADAPTIVE PROCESSING

Prepared by: Sam M. Daniel

S. M. Daniel
Project Leader
Advanced Technology and Systems Analysis

Imre Kertesz

I. Kertesz
Advanced Technology and Systems Analysis

Approved by: Guy V. Morris

G. V. Morris
Manager
Advanced Technology and Systems Analysis

D. L. Howell

D. L. Howell
Chief Engineer
Radar Systems Office
Radar Operations
Motorola Government Electronics Division
Tempe, Arizona 85283

TABLE OF CONTENTS

BCR ADAPTIVE PROCESSING IMPLEMENTATION
AND ITS PERFORMANCE EVALUATION VIA COMPUTER EMULATION

Prepared by: Sam M. Daniel
S. M. Daniel
Project Leader
Advanced Technology and Systems Analysis

I. Kertesz
Advanced Technology and Systems Analysis

Reviewed by: Cary R. Champlin
C. R. Champlin
Advanced Technology and Systems Analysis

Approved by: Guy V. Morris
G. V. Morris
Section Manager
Advanced Technology and Systems Analysis

D. L. Howell
D. L. Howell
Chief Engineer
Radar Systems Office
Radar Operations

1.0 INTRODUCTION

An effective digital implementation of the BCR process [1] should be characterized by an efficient architecture that guarantees reasonably high numerical precision. As an example, a floating-point implementation of BCR will provide the desired numerical accuracy, however, at the expense of a rather complex architecture. Consequently, a floating-point implementation of BCR is not effective. As another example, a fixed-point implementation of BCR may be architecturally efficient, however, it may not necessarily be numerically reliable. Hence, a standard fixed-point implementation may not be necessarily effective.

What constitutes an effective digital implementation of the BCR process is one employing fixed-point arithmetic equipped with a special form of adaptive scaling and overflow protection. Adaptive scaling enhances numerical resolution by inducing high-level bit-activity throughout the process while overflow protection insures against detrimental limit-cycle effects [2].

The present memorandum describes, in some detail, essential aspects of the adaptive fixed-point arithmetic underlying a particular digital implementation of a BCR processor.

A modular computer program, BCRM, is included which incorporates an exact model of the BCR processor. The benchtest example of Project Memorandum 8512-04 is used as a means of providing a detailed description of the program. The three examples included demonstrate the effective nulling performance of the BCR processor showing only a minor degradation with that of a floating-point simulation.

2.0 BCR PROCESSOR IMPLEMENTATION

As alre   described in Project Memorandum 8512-03, the BCR process applied to adaptive array processing consists of computing the covariance matrix C and a cross-correlation vector $\underline{b}$ from a batch of auxiliary and main signal samples, solving the system $C\underline{w} + \underline{b} = \underline{0}$ for the weight vector $\underline{w}$ via the CG algorithm and forming the minimum-interference combined signal, the weighted sum of the auxiliary signals combined with the main. Figure 2-1 shows these three stages of the BCR process.

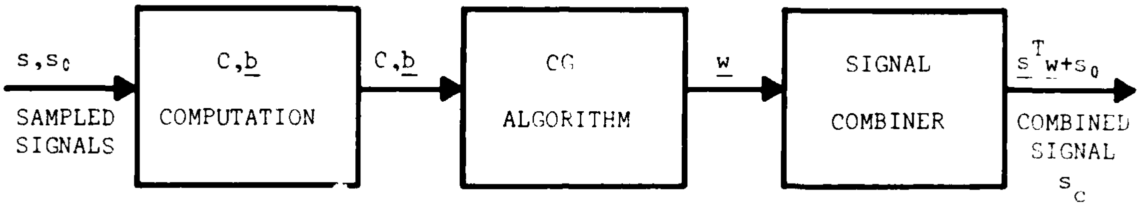


Figure 2-1. Three-Stage Representation of the BCR Adaptive Processor

The present section is primarily concerned with the description of the functional block diagram of the second stage of the BCR processor, since it is, by far, the most complex of the three. Included in the discussion is a qualitative explanation of the underlying adaptive fixed-point arithmetic employed. Pertinent discussion of the other two stages of computation are deferred to the next major section where the adaptive fixed-point arithmetic is formulated in great detail.

2.1 CG Algorithm

The particular form of the CG algorithm used to solve $C\underline{w} + \underline{b} = \underline{0}$ is the standard version already described in Project Memorandum 8512-03. This specific version is included below for convenient reference.

(i) Let the initial estimate for the weight vector $\underline{w}$ be $\underline{w}^0 = \underline{0}$ and define the initial residual and search vectors $\underline{r}$ and $\underline{p}$ by $\underline{p}^0 = \underline{r}^0 = \underline{b}$.

At iteration k + 1,

(ii) Compute the relaxation coefficient

$$\alpha_k = \frac{\|\underline{r}^k\|^2}{(\underline{p}^k, C\underline{p}^k)} \tag{2-1.1}$$

(iii) Update the weight vector by

$$\underline{w}^{k+1} = \underline{w}^k - \alpha_k \underline{p}^k \tag{2-1.2}$$

(iv) Update the residual vector by

$$\underline{r}^{k+1} = \underline{r}^k - \alpha_k C\underline{p}^k \tag{2-1.3}$$

(v) Computer the Gram-Schmidt coefficient

$$\beta_k = \frac{\|\underline{r}^{k+1}\|^2}{\|\underline{r}^k\|^2} \tag{2-1.4}$$

(vi) Update the relaxation vector by

$$\underline{p}^{k+1} = \underline{r}^{k+1} + \beta_k \underline{p}^k \tag{2-1.5}$$

(viii) If $k + 1 = N$ or $\|r^{k+1}\|^2 < \varepsilon$, a preassigned small number, the process is stopped. Otherwise, replace $k + 1$ by k and return to (ii).

Other versions of this algorithm exist [3], [4], however, this particular form is especially convenient from the implementation and operational point of view.

2.2 Elements of Adaptive Arithmetic

In mechanizing the CG algorithm via fixed-point arithmetic, it is important to realize that a significant loss in numerical precision may occur unless appropriate adaptive scaling measures are incorporated into the process. The intent in employing adaptive scaling is to maintain maximum numerical resolution by forcing high-level bit activity before going into an arithmetic operation that is followed by truncation. Doing this insures minimum roundoff error. The cumulative history of bit shifts that takes place while exercising adaptive scaling must be accounted for throughout the process in order to properly interpret and use the numerical quantities generated. Finally, the risk of potential overflow must be averted at critical points in the process.

2.2.1 Local and Global Scaling

The specific form of adaptive fixed-point arithmetic, envisioned in implementing the BCR process, involves scaling by variable, whether it be vector or scalar, real or complex. In general, the left justification of a complex vector variable involves upshifting all words that comprise it by the number of bits that will left-justify the maximum-magnitude word. The single number that is associated with the left-justification of a given BCR variable is referred to as the local scale, because it occurs locally within the process and is not explicitly dependent on any other relationship.

Considering the iterative nature of the BCR process, local scaling will be exercised repeatedly on each and every variable involved. As such, in order to keep track of the absolute numerical value of each variable, it is imperative to have a cumulative history of the local bit-shifts as they occur. Furthermore, recognizing that a given variable is an interaction of several others, it must be identified with a cumulative global shift which is functionally related to the global shifts of the interacting variables, including any local shift or truncation that the given variable may have experienced. By associating a given BCR variable with its global scale, its absolute value is defined with respect to a convenient reference.

2.2.2 Overflow Protection

In performing a summation function of two or more digital words having the same global scale, overflow protection is assured by providing all necessary bit-space for growth or by inducing single-bit downshifts at each stage of addition. The latter approach, although seemingly convenient, is numercially inferior in comparison to the former which is intended to minimize roundoff errors in the final truncated result.

When adding two left-justified digital words of varying global scales, scale equalization must be performed. Almost invariably, one of the words will have to be downshifted before the addition can take place. As such, if the downshift required is beyond the capability of the implementation, a system-interrupt command must intervene to stop the BCR process. Obviously, if the addition were allowed to occur without appropriate scale-equalization, the invalid result would most probably induce a limit-cycle into the BCR process in the form of uncontrolled overflows, rendering it inoperable. It should be pointed out that this special form of overflow protection would be useless if it were not for the fact that the existing variable w at the time of termination constitutes a valid estimate of the desired solution.

2.3 Functional Block Diagram

The mechanics of implementing adaptive fixed-point arithmetic in a hardware realization of the CG algorithm consists of judiciously locating special networks to perform necessary scaling functions at various stages of the CG process. Depending on their peculiar location, these scaling networks may be autonomous, externally-commanded or be capable of both options.

Figure 2-2 shows the functional block diagram corresponding to a 4-port 16-bit CG implementation. Major blocks in the diagram are clearly identified. Included are four distinct scaling networks, designed to implement the mechanics of adaptive arithmetic under the supervision of the "Global Scaling and Control" (GSC) block. The GSC block in combination with the block labeled "System Timing and Control" (STC) constitutes the Operating System of the BCR processor implementation.

2.3.1 Major Block Functions

In order to gain a clear understanding of the basic operation of the BCR process as implemented in the block diagram of Figure 2-2, it is important to define the operational characteristics of each of the major blocks that comprise it.

2.3.1.1 Input Block

In general, the input block consists of RAM for the purpose of accepting and storing the complex covariance matrix C and the complex forcing vector b derived from the preceeding C and b stage of computation. More specifically, before entering the input block, C and b are locally block-scaled independently and truncated to 16 bits. In a manner of speaking, these two variables have been AGC'd.

As shown in the block diagram, the C and b data are made available to other blocks in the system via the 128-bit Bus A. Accordingly, it is possible to access b in parallel and C serially, one row at a time.

2.3.1.2 Vector Storage

The Vector Storage block consists of sufficient RAM to store the BCR complex vectors w, r, p, and Cp with a 19-bit representation as entered via the 152-bit Bus C. These vectors are accessible, one at a time, through Scaler #1, a scaling network which produces a 16-bit properly vector-scaled representation, compatible with the capacity of Bus A. In this way, these vectors are made available to other blocks of the system as needed.

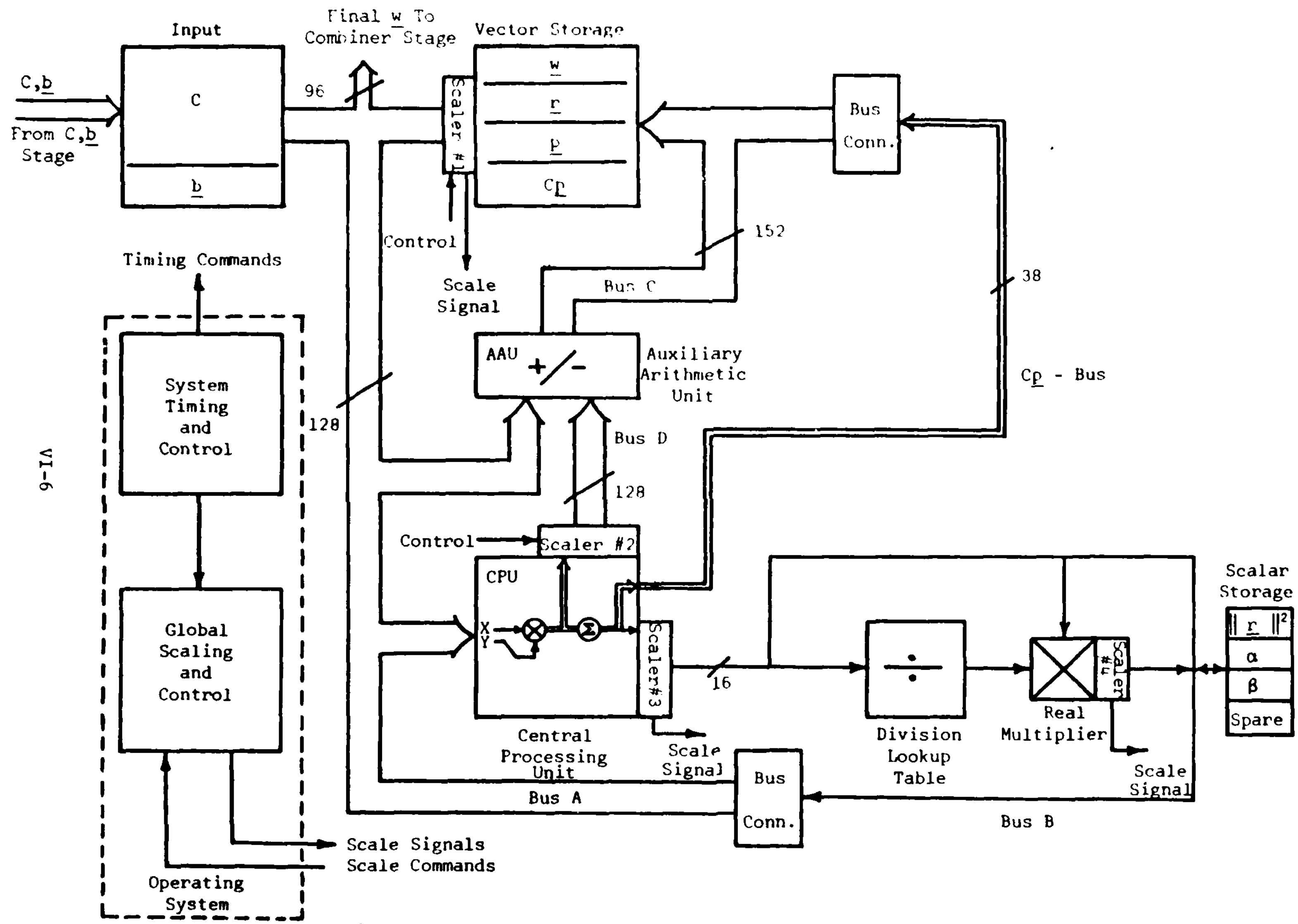


Figure 2-1. Functional Block Diagram of the BCK Processor Central Stage Mechanized for Adaptive Fixed-Point Arithmetic

2.3.1.3 Auxiliary Arithmetic Unit (AAU)

The AAU is designed to add or subtract two 16-bit 4-dimensional complex vectors. As such, the AAU is particularly useful in performing the $\underline{w}$, $\underline{r}$, and $\underline{p}$ updates according to relations 2-1.2, 2-1.3, and 2-1.5 of the BCR algorithm. In addition, this block is instrumental in setting up the initial conditions of the BCR process, namely, $\underline{w} = \underline{0}$ and $\underline{p} = \underline{r} = \underline{b}$. This is done by introducing the desired initial value in the Bus A input while setting the Bus D input to $\underline{0}$. The resulting output is routed through Bus C and loaded into the appropriate location in the Vector Storage RAM. Note that the + or the - option may be selected to perform the initial loading operation.

Since, in general, the 16-bit inputs to the AAU could give rise to a 17-bit left-justified output, Bus C must have a minimum capacity of 136 bits. The reason it is shown to have the larger capacity of 152 bits is that its $C\underline{p}$-branch requires it. This will be made clear later.

2.3.1.4 Central Processing Unit (CPU)

The CPU is designed to compute a dot product of two 4-dimensional 16-bit complex vectors. With variations of this basic operation, the CPU is capable of generating BCR quanitities $\|\underline{r}\|^2$, $C\underline{p}$, $(p,C\underline{p})$, $\alpha\underline{p}$, $\alpha C\underline{p}$ and $\beta\underline{p}$. Accordingly, the CPU is composed of four complex multipliers followed by an appropriate summation function.

Each complex multiplier is composed of a quad of 16-bit real multipliers operating with a selectable 16 or 15-bit truncation depending on the particular need for overflow protection. Note that when multiplying two maximum-magnitude negative 16-bit words, the positive result overflows into a 32nd bit, the overflow bit. To avoid this potential overflow, a 16-bit truncation is applied in general even though it is needed only in the instance indicated. As such, the resulting 16-bit output will generally have one sign-bit extension except in the case stated. However, the loss of this one bit of resolution need not be suffered when the multiplication is known to involve other than two negative numbers.

The summation function consists of three stages. The first stage is devoted to producing the four truncated complex componentwise products of the two input vectors. A given componentwise product results from the summation of appropriate pairs of 16-bit truncated real multiplier products which may be accommodated within 17 bits. The four complex componentwise products are subsequently summed pairwise over two more stages of addition to form a 19-bit complex scalar quantity that represents the dot product of the two 4-dimensional complex input vectors. It should be noted that rather than dropping a bit at each stage of summation and thus preserving a convenient 16-bit activity throughout, the preservation of the three bits over the three stages of summation limits the undesired propagation of roundoff error into subsequent operations.

An important feature of the CPU is that it can perform a dot-product of two 4-dimensional complex vectors one of which may be used directly or in its conjugate form via a selectable variation in structure.

The specific computation of $\|\underline{r}\|^2$ involves the dot product of $\underline{r}$ with $\underline{r}^*$. First, the $\underline{r}$ vector is accessed from the Vector Storage Ram, left-justified by Scaler #1, truncated to 16 bits, placed on Bus A and latched into the X-port of the CPU. Repeating this process, $\underline{r}$ is similarly latched into the Y-port. Selecting the conjugate option on Y, a dot product operation yields the desired 19-bit result, $\|\underline{r}\|^2$.

The $C\underline{p}$ computation proceeds as follows. First, the $\underline{p}$ vector is accessed from the Vector Storage RAM, left-justified by Scaler #1, truncated to 16-bits, placed on Bus A and latched into the X-port of the CPU. Then, the first row of C is taken from the Input Block, placed on Bus A and latched into the Y-port. A normal dot product operation by the CPU yields a 19-bit complex scalar that constitutes the first component of the $C\underline{p}$ vector. The 38-bit $C\underline{p}$-Bus takes this result and conveys it, via a Bus-Connect network, onto the appropriate partition of Bus C and loads it into the corresponding partition of the $C\underline{p}$ RAM location. Repeating this process with all subsequent rows of C completes the componentwise generation of the $C\underline{p}$ vector.

The computation of the theoretically positive real scalar quantity $(\underline{p},C\underline{p})$ is very similar to that of $\|\underline{r}\|^2$ already described. Note that, more explicitly, this inner product is equivalent to $\underline{p}^{*T}C\underline{p}$ or $\underline{p}^T(C\underline{p})^*$, since it is real. Consequently, the choice of conjugation is based on convenience of execution.

The remaining computation of $\alpha\underline{p}$, $\alpha C\underline{p}$, and $\beta\underline{p}$ are all accomplished in a similar way. It suffices to discuss only the $\alpha\underline{p}$ computation. Assume that $\underline{p}$ is already latched into the X-port of the CPU. The real number α is presented to a Bus Connect network and is fanned out into a uniform real-valued 4-dimensional vector, each component os which is equal to $\alpha + j0$. Placed this way as Bus A, this uniform vector is subsequently latched into the Y-port. The desired vector dilation is available at or prior to the first stage of summation.

In carrying out the computations of $\|\underline{r}\|^2$, $C\underline{p}$ and $(\underline{p},C\underline{p})$, the individual real multipliers comprising the CPU are overflow protected by means of one overflow-bit allowance. As mentioned before, this is necessary because of the possibility of multiplying two maximum-magnitude negative numbers which will yield a positive number with a single bit of overflow. The corresponding truncation at each multiplier output is noted to be 16 bits. In contrast, to the mentioned computations, quantities $\alpha\underline{p}$, $\alpha C\underline{p}$, and $\beta\underline{p}$ constitute products involving the known positive real parameters α and β. As such, the feared overflow condition cannot occur in any of these cases. For this reason, the overflow protection option is foregone here thus allowing an extra bit of resolution in each resulting quantities. The corresponding trunction at each multiplier output in the latter set of computations is 15 bits.

2.3.1.5 Real Scalar Section

The section of the BCR block diagram that is devoted to computing and storing the positive real scalar parameters α and β is appropriately referred to as the Real Scalar Section. This major BCR block involves a Division Lookup Table, a real multiplier and RAM storage for the positive real scalars $\|\underline{r}\|^2$, α and β.

The Division Lookup Table is designed to accept a 16-bit fully-justified positive real input and produce a 16-bit fully-justified output that represents the reciprocal of the input. An implicit local scaling coefficient is taken into account.

Computations of α and β are similar, as indicated by relations 2-1.1 and 2-1.4 of the BCR algorithm. It suffices to discuss the process for computing α. The positive real 19-bit scalar quantity $(\underline{p}, C\underline{p})$ is first locally-scaled by Scaler #3 to full significance and then truncated to a 16-bit fully-justified form. Presented this way to the Division Lookup Table yields the fully-justified 16-bit version of its reciprocal $1/(\underline{p}, C\underline{p})$. Using a 16-bit real multiplier, this reciprocal is multiplied by the RAM-accessed $\|\underline{r}\|^2$ quantity, yielding α. Note that the multiplier does not require overflow protection since only positive inputs are involved. It should also be mentioned that Scaler #4 at the real-multiplier output locally-scales α or β to full-justification prior to storing or proceeding to further computation.

2.3.2 Essential Operational Aspects

The previous section discussed briefly the functions performed by some major blocks of the central BCR processor stage. In actual operation these individual blocks must be capable of interacting appropriately in order to maintain numerical integrity in the execution of the BCR process. To a large extent, the overall operation of the BCR process is essentially evident upon careful examination of the block diagram and the description of the major block functions already offered. Discussed below are two essential operational aspects of the BCR implementation which should serve to give a more complete understanding of the overall system operation.

2.3.2.1 Vector Updating

At each iteration of the BCR process the vector variables $\underline{w}$, $\underline{r}$, and $\underline{p}$ are updated by means of incremental changes $\alpha\underline{p}$, $\alpha C\underline{p}$, and $\beta\underline{p}$ according to relations 2-1.2, 2-1.3, and 2-1.5, respectively. The updating process itself is accomplished by means of the AAU and with appropriate use of Scalers #1 and #2.

Consider, for example, the updating of vector $\underline{r}$. The current vector $\underline{r}$ resides in the Vector Storage RAM, the input to Scaler #1. The corresponding updating vector increment $\alpha C\underline{p}$ is available at the input of

Scaler #2. Because $\alpha C\underline{p}$ is essentially left-justified except for one sign-extended bit 75% of the time, Scaler #2 is provided only with the ability to downshift up to 15 bits, by external command. On the other hand, Scaler #1 is given the capability of autonomous or local shifting up to a limit of 8 bits. At the same time, it also has the capability of shifting down by as much as 7 bits via external command. With the combined capabilities of these two scaling networks, it is possible to combine the current $\underline{r}$ with its increment $\alpha C\underline{p}$ thus accommodating a sufficiently large dynamic range of numbers.

The actual updating process for $\underline{r}$ proceeds as follows. First, Scaler #1 senses the number of upshifts necessary to left-justify $\underline{r}$ without actually executing the local scaling. This number, which is limited to 8, is then incorporated into the global scale coefficient identified with $\underline{r}$. Comparing this global scale or $\underline{r}$ with that of $\alpha C\underline{p}$ determines which one of the two quantities must be shifted down from its fully-upshifted position before combining. Note that although the maximum downshift on $C\underline{p}$ is a constant 15 bits, that of $\underline{r}$ is a variable 7 bits plus the sensed upshift. In fact, if the sensed local upshift for $\underline{r}$ is k_r, a maximum downshift of $7 + k_r$ may be obtained by accessing $\underline{r}$ from RAM with an external shift-command corresponding to 7 downshifts. The downshift necessary is that which will allow the global scales of the two quantities to be equal before properly combining. This is global scale equalization, as mentioned previously.

2.3.2.2 Terminal Conditions and Stopping Criteria

When global scale equalization cannot be achieved within the implemented limits, the BCR process must be commanded to stop. The current value of $\underline{w}$ at the point of termination constitutes the best estimate of the solution within the numerical capability of the machine. It should be noted that the shifting range available appears to be sufficient to support a solution to an 8-bit accuracy. This has been substantiated in a number of cases investigated by means of computer emulation.

Besides the above, there is another natural terminal condition. This occurs in the Real Scalar Section at the input to the Division Lookup Table. If a 01 bit-activity is not detected there, the Division Table no longer applies and a termination condition occurs.

While the above termination conditions serve as unavoidable stopping criteria, there does exist at least one natural way to call for system-interrupt. In particular, when $\|\underline{r}\|^2/\|\underline{b}\|^2 < 2^{-15}$, BCR convergence has been achieved. Of course the default stopping option is the iteration count of four, the dimensionality of the system under consideration.

It should be pointed out that, with the termination conditions and stopping criteria available, the BCR implementation is completely overflow-protected under any circumstances.

3.0 BCR OPERATING SYSTEM

It has been mentioned previously that, strictly speaking, the Operating System for the BCR process consists of the composite of System Timing and Global Scaling and Control functions. Since a detailed definition of the BCR System Timing will be given in a future project memorandum, the emphasis in this section will be placed on developing the specifics of the Global Scaling and Control function, the very precise rules that govern the adaptive fixed-point arithmetic of the BCR implementation.

3.1 Prescaling and Postscaling

Given sequences $\{s_0(m)\}_{m=1}^{M}$ and $\{\underline{s}(m)\}_{m=1}^{M}$ representing main scalar and auxiliary vector M-sample complex baseband signal batches, it can be shown [1] that the covariance matrix is given by

$$C = (\underline{s}, \underline{s}^T)$$

$$= \sum_{m=1}^{M} \underline{s}^*(m)\underline{s}^T(m) \quad (3\text{-}1)$$

and the cross-correlation vector by

$$\underline{b} = (\underline{s}, s_0)$$

$$= \sum_{m=1}^{M} \underline{s}^*(m)\underline{s}_0(m) \quad (3\text{-}2)$$

Computed in this manner by means dedicated circuitry, C and $\underline{b}$ are subsequently locally scaled independently and truncated to 16 bits. More specifically, $\underline{b}$ is vector-scaled locally by upshifting all real and imaginary words involved in unison so that the maximum-magnitude word is left-justified. Subsequently, all words comprising $\underline{b}$ are truncated to 16 bits. Matrix C is locally scaled in a similar way. Note that the amount of upshift in this case is determined by examining the purely real diagonal elements, one of which happens to be the largest-magnitude word of all that comprise C.

Let us define the respective local scales on C and $\underline{b}$ by symbols KC and KB, the number of upward bit-shifts required for full justification.

Also, assume KT to be the common number of truncation bits required to limit C and $\underline{b}$ to 16 bits. Then, in terms of original C and $\underline{b}$ quantities, the equation to be solved is

$$2^{KC}C\underline{w} + 2^{KB}\underline{b} = \underline{0} \qquad (3\text{-}3)$$

whence, symbolically,

$$\underline{w}' = -2^{KWF}\underline{w}^{o} \qquad (3\text{-}4)$$

with KWF = KB - KC, the effective local shift on the final solution, w', with respect to the actual, $\underline{w}^{o} = C^{-1}\underline{b}$. In other words, whatever the correct solution of $\underline{w}$ is, the solution obtained via BCR with locally prescaled C and $\underline{b}$ quantities is in fact 2^{KWF} larger. This discrepancy must be taken into account with appropriate postscaling at the combiner stage, the last stage of the BCR processor.

3.2 Global Scaling Equations and Control

The adaptive fixed-point arithmetic underlying the BCR implementation is mechanized with the aid of a special set of equations that accounts for the bit-shift history of all the variables involved. Specifically, each variable is associated with a global bit-shift number that relates it to some convenient reference. This global bit-shift reflects the local-shift experienced by the associated variable, global shifts from interacting variables and any truncation that might be exercised.

What follows constitutes a detailed development of the global scaling equations associated with the BCR variables and accompanying commands that provide appropriate control of the process via the special scaling networks involved.

3.2.1 Initial Conditions

Referring to the functional block diagram of the BCR process given in Figure 2-1, assume that the prescaled C and $\underline{b}$ quantities are available in PROM or RAM at the Input block. In accordance to the algorithm, the BCR process is initialized by setting $\underline{w} = \underline{0}$ and $\underline{p} = \underline{r} = \underline{b}$. Specifically, this involves loading the zero vector, $\underline{0}$, into the $\underline{w}$-vector location of the Vector Storage RAM. Similarly, $\underline{b}$ is loaded into the $\underline{r}$ and $\underline{p}$ locations. As already mentioned, this initial loading process is accomplished through the use of the AAU.

Irrespective of the value of the prescaling differential shift for $\underline{w}$, namely, KWF, we may arbitrarily define the global bit-shift coefficients for $\underline{w}$, $\underline{r}$, and $\underline{p}$ to be zero. That is, we let

$$LW = LR = LP = 0 \tag{3-5}$$

Recall that in loading the Vector Storage Ram with the initial values of $\underline{w}$, $\underline{r}$ and $\underline{p}$ through the AAU, the Bus D input is set to $\underline{0}$ while the desired loading value is placed at the Bus A input. Choosing, arbitrarily, the + option, the initial loading value appears at the Bus C output of the AAU from where it is conveyed to its proper storage location. To prevent potential overflow during general operation, one overflow bit must be allocated to each and every word at the AAU output. For this reason Bus C must be at least 136 bits wide. As a consequence, the proper access of the initally loaded values of $\underline{r}$ and $\underline{p}$ from storage onto the 128-bit Bus A will require a 1-bit upshift, followed by a 1-bit truncation. Provisions for accomplishing this are available at scaling network #1.

In the process of initially loading $\underline{b}$ into the $\underline{r}$-location of the Vector Storage RAM, $\underline{b}$ may also be latched into the X-port of the CPU. While subsequently loading $\underline{b}$ into the $\underline{p}$ location of the RAM, $\underline{b}$ may simultaneously be latched into the Y-port of the CPU. Employing the conjugate-option on the Y-port and the 16-bit truncation-option on the individual real multipliers of the CPU, the real scalar quantity $\|\underline{r}\|^2$ is computed in three subsequent stages of addition, yielding a 19-bit real word. Specifically,

$$\|\underline{r}\|^2 = Re \sum_{i=1}^{4} r_i r_i^* \tag{3-6}$$

Note that the first stage of summation is understood to be associated with the individual complex products $r_i r_i^*$. As such, the individual complex products are accommodated in dual 17-bit words. The sum of the four products indicated in (3-6) accounts for the additional 2-bit growth.

The 19-bit real scalar quantity $\|\underline{r}\|^2$ is then presented to scaling network #3 which senses and executes the needed local shift for left-justification. Limited to 15 bits of upward shift capability, this scaling network subsequently truncates the bottom three bits and transmits a fully-justified 16-bit version of $\|\underline{r}\|^2$ to the Scalar Storage RAM via Bus B. Letting

KRRH = the hardware local shift on $\|\underline{r}\|^2$ provided by scaling network #3, limited to 15 bits of upshift

KRRO = the effective 3-bit truncation of $\|\underline{r}\|^2$ from 19 bits to 16 bits, after scaling

KRR = KRRH - KRRO

= the effective local shift of $\|\underline{r}\|^2$ referred to 16-bit arithmetic

then, the global scale of the 16-bit representation of $\|\underline{r}\|^2$ as stored in RAM is given by

$$LRR = LR + LR + KRR - KM \tag{3-7}$$

where

LR = the global scale of the 16-bit version of the initial value of $\underline{r}$ as is available at Bus A or at the X and Y ports of the CPU

KM = the 16-bit truncation at each individual real multiplier of the CPU

Clearly, LR is involved twice in (3-7) since $\underline{r}$ is essentially multiplied by itself.

The time devoted to computing the intial value of $\|\underline{r}\|^2$ is referred to as the initialization period and must be distinguished from the rest of a given iteration. Accordingly, upon completion of an iteration, the process returns to a computing cycle immediately following the initialization period. Note that the computation of $\|\underline{r}\|^2$ within the initialization period is dictated naturally by the fact that subsequent $\|\underline{r}\|^2$ values are already being computed for the sake of evaluating β.

3.2.2 Complete BCR Iteration

The repeating portion of the BCR iteration may logically start with the computation of $C\underline{p}$. First, $\underline{p}$ is accessed from Ram with maximum upshift and truncated to 16 bits with the aid of Scaler #1, placed on Bus A and latched into the X-port of the CPU. Considering that Scaler #1 has a limited upshift capability of 8 bits, $\underline{p}$ might not be completely left-justified at the X-port. In any case, letting

KPH = the hardware local scale on $\underline{p}$ provided by Scaler #1, limited to 8 bits of upshift.

KPO = the effective 1-bit truncation of $\underline{p}$ from 17 to 16 bits, after scaling

KP = KPH - KPO

= the effective local scale of $\underline{p}$ referred to 16-bit arithmetic

then, given that the global scale of $\underline{p}$ in RAM is LP´, its global scale at the X-port of the CPU becomes

$$LP = LP' + KP \tag{3-8}$$

The C$\underline{P}$-computation is then executed as follows. Successive rows of C are accessed from the Input Block and latched into the Y-port of the CPU. Performing normal dot products yields C$\underline{p}$, one component at a time, according to

$$(C\underline{p})_i = \sum_{j=1}^{4} C_{ij} P_j \tag{3-9}$$

with i = 1, 2, 3, 4. As in the case of $\|\underline{r}\|^2$, the sequentially generated complex components of C$\underline{p}$ are represented by dual 19-bit words. Accordingly, a 38-bit C$\underline{p}$-Bus is used to convey each component onto the 156-bit Bus C via a specially-designed Bus-Connect network and storing it appropriately within the C$\underline{p}$-location of the Vector Storage RAM. Keeping in mind the multiplier truncation within the CPU, the global scale of the C$\underline{p}$ vector in RAM happens to be

$$LCP' = LP - KM \tag{3-10}$$

where,

LP = the global scale of vector $\underline{p}$ at the X-port of the CPU

KM = the 16-bit truncation of the individual real multipliers of the CPU

With $C\underline{p}$ in RAM, the computation of the real scalar quantity $(p, C\underline{p})$ follows. Specifically, this quantity is given by

$$(\underline{p}, C\underline{p}) = Re \sum_{i=1}^{4} p_i^{*}(C\underline{p})_i$$

$$= Re \sum_{i=1}^{4} p_i(C\underline{p})_i^{*} \qquad (3\text{-}11)$$

Since $\underline{p}$ is already available at the X-port of the CPU, $C\underline{p}$ is accessed from RAM with maximum upshift and truncated to 16 bits with the aid of Scaler #1, placed on Bus A and latched into the Y-port. Accordingly, letting

KCPH = the hardware local shift on $C\underline{p}$ provided by Scaler #1, limited to 8 bits of upshift

KCPO = the effective 3-bit truncation of $C\underline{p}$ from 19 bits to 16 bits, after scaling

KCP = KCPH - KCPO

= the effective local scale of $C\underline{p}$ referred to 16-bit arithmetic

then, the global scale of $C\underline{p}$ at the Y-port of the CPU is given by

$$LCP = LCP' + KCP \qquad (3\text{-}12)$$

Employing the conjugate-option on the Y-port and the 16-bit truncation option on the individual real multipliers of the CPU, the desired real scalar quantity $(\underline{p}, C\underline{p})$ is computed in the very similar way as $\|\underline{r}\|^2$. Letting

KPCPH = the hardware local scale on $(\underline{p}, C\underline{p})$ provided by Scaler #3, limited to 15 bits of upshift

KPCPO = the effective 3-bit truncation of $(\underline{p}, C\underline{p})$ from 19 bits to 16 bits, after scaling

KPCP = KPCPH - KPCPO

= the effective local scale of $(\underline{p}, C\underline{p})$ referred to 16-bit arithmetic

KM = the 16-bit truncation of the real multipliers of the CPU

then, the global scale of the 16-bit left-justified version of $(\underline{p}, C\underline{p})$ as presented into the input of the Division Lookup Table is given by

$$\text{LPCP} = \text{LP} + \text{LCP} + \text{KPCP} - \text{KM} \tag{3-13}$$

The Division Lookup Table is designed to accept a positive real left-justified 16-bit word and yield a 16-bit left justified version of its reciprocal. Consequently, the output is understood to have an implicit 29-bit local shift. Letting

KD = the 20-bit implicit local shift in the Division Lookup Table

then, the global shift of the reciprocal $1/(\underline{p}, C\underline{p})$ at the input to the real multiplier is simply

$$\text{LPCPI} = \text{KD} - \text{LPCP} \tag{3-14}$$

The computation of α follows immediately. Accessing $\|\underline{r}\|^2$ directly from RAM, it is presented to the other input of the real multiplier. The product of $\|\underline{r}\|^2$ and $1/(\underline{p}, C\underline{p})$ yields α. Since both of these quantities are positive, the 15-bit truncation-option is used to obtain a 16-bit output which is subsequently left-justified. Letting

KAL = the hardware or effective local scale on α, limited to 2 bits

KMO = the 15-bit truncation at the real multiplier

then, the global scale for α is given by

$$LAL = LRR + LPCPI + KAL - KMO \tag{3-15}$$

At the same time α is being stored in RAM, it is also routed via Bus B, fanned out uniformly on Bus A via the special Bus-Connect network at the juncture and latched into the Y-port of the CPU. With $\underline{p}$ already available at the X-port, the real dilation $\alpha\underline{p}$ is obtained by employing the partial dot-product option in the CPU. Then, at the input of Scaler #2, the global scale of $\underline{p}$ is given by

$$LAP = LAL + LP - KMO \tag{3-16}$$

Note here that the truncation at each real multiplier is 15 bits since α is a positive real scalar. Also, since $\underline{p}$ and α are left-justified going into the CPU, $\alpha\underline{p}$ is left-justified with a probability of 0.25 and is at most one bit away from full justification. Accordingly, for purposes of general simplification, Scaler #2 is not given the capability of upshifting. In fact, Scaler #2 is designed to downshift only up to a limit of 15 bits.

With $\alpha\underline{p}$ available at the input to Scaler #2 and the current $\underline{w}$ in the Vector Storage RAM proceeding Scaler #1, the updated 17-bit version of $\underline{w}$, $\underline{w} - \alpha\underline{p}$, may now be obtained via the AAU following scale equalization. Specifically, the global scales of $\underline{w}$ and $\alpha\underline{p}$ must be adjusted to a common level such that one of the two quantities is placed at the highest possible significance. Note that if $\underline{w}$ were to be viewed at maximum upshift and considering that $\alpha\underline{p}$ is already at maximum possible significance, scale equalization will be achieved by shifting one of the two quantities downward by a number of bits within an associated limit. Of course, $\underline{w}$ must not be explicitly upshifted by Scaler #1 because there is no provision for subsequently downshifting, if it is deemed necessary. Rather, we may sense the maximum hardware shift that may be provided by Scaler #1 without actually applying it. As such, the global scale of $\underline{w}$ incorporating the maximum upshift would correspond to a virtual maximum upshifted version of $\underline{w}$. A downshift from this virtual state of $\underline{w}$ is possible with an appropriate external command to Scaler #1. This argument may be made more precise in quantitative terms. Let

KWH = the hardware local scale on $\underline{w}$ as sensed by Scaler #1, limited to 8 bits of upshift

KWO = the effective 1-bit truncation of w from 17 to 16 bits, after scaling

KW = the effective local scale of w, KWH - KWO, referred to 16-bit arithmetic

Then, the global scale of a maximally-upshifted virtual w is given by

$$LW = LW' + KW \tag{3-17}$$

where LW´ is the global scale of w in RAM.

Suppose that, upon comparing (3-16) with (3-17), we have that LAP = LW. Then, the desired w-update involves accessing w from RAM at the maximum-upshift provided by Scaler #1, passing αp through Scaler #2 at zero-downshift and subsequently performing the appropriate subtraction at the AAU. In this special case, the global scale of the updated 17-bit w is given by

$$LW' = LW \tag{3-18}$$

In the case when LAP $\geq$ LW, αp must be downshifted by a number of bits equal to the differential shift LAP - LW, while w is upshifted fully at Scaler #1. Note that this includes the previous special case and that the global scale of the updated w is as given in (3-18). Also, it should be mentioned that the maximum positive differential shift that may be accommodated is 15 bits.

When LAP < LW, the scale equalization procedure is a little more involved. Keeping in mind that w may be downshifted by as much as 7 bits, the maximum negative differential shift LAP - LW that could be tolerated is -(KWH + 7). In the specific case where KWH = 0, a maximum downshift of 7 bits can be provided by Scaler #1 with αp remaining as is, w is accessed through Scaler #1 and downshifted by LW - LAP from its fully upshifted position.† This is done by an appropriate command, namely a shift of KWH + LAP - LW. The global scale of the update 17-bit w in this case is

$$LW' = LAP \tag{3-19}$$

†As an example, let KWH = 5 and LAP - LW = -3. Then, the command to Scaler #1 in accessing w from RAM is such that it effects an upshift of KWH + LAP - LW = 2.

It should be mentioned here that, since the initial value of $\underline{w}$ has been chosen to be $\underline{0}$, the updated $\underline{w}$ at the first iteration is simply $-\alpha\underline{p}$. The scale equalization logic described above for properly performing the $\underline{w}$-update may thus be circumvented at the first iteration. As far as the global scale of the updated $\underline{w}$ is concerned, it is given by (3-19).

The computation of $\alpha C\underline{p}$ may be started even before the $\underline{w}$-update is over. Since α is already available as a uniform vector at the Y-port of the CPU, the $C\underline{p}$-vector may be accessed from RAM with maximum upshift through Scaler #1, placed on Bus A and latched into the X-port of the CPU. A partial dot-product operation leads to the dilation $\alpha C\underline{p}$ at the input to Scaler #2. Letting

KCPH = the hardware local scale on $\underline{w}$ as provided by Scaler #1, limited to 8 bits of upshift

KCPO = the effective 3-bit truncation of $C\underline{p}$ from 19 to 16 bits after scaling

KCP = the effective local scale of $C\underline{p}$, KCPH - KCPO, referred to 16-bit arithmetic

Then, the global scale of the maximally upshifted 16-bit version of $C\underline{p}$, as presented to the X-port of the CPU, happens to be

$$LCP = LCP' + KCP \tag{3-20}$$

where LCP´ is the global scale of $C\underline{p}$ in RAM. The global scale of $\alpha C\underline{p}$ at the input to Scaler #2 is given by

$$LACP = LAL + LCP - KMO \tag{3-21}$$

where, of course, KMO is the optimal 15-bit truncation applied at each of the individual multipliers in the CPU.

The updating process of $\underline{r}$ may now take place in a manner similar to that of updating $\underline{w}$. Letting

KRH = the hardware local scale on $\underline{r}$ as sensed by Scaler #1, limited to 8 bits

KRO = the effective 1-bit truncation of r from 17 to 16 bits, after scaling

KR = KRH - KRO

= the effective local scale of r referred to 16-bit arithmetic

then, the global scale of a maximally upshifted virtual r is given by

$$LR = LR' + KR \tag{3-22}$$

where LR′ is the global scale of r in RAM.

Consider the case where LACP - LR < 0. Then, r is accessed on Bus A at maximum upshift through Scaler #1 and αCp is downshifted by a number of bits equal to the differential shift, LACP - LR, but not exceeding the full 15-bit downshift capability of Scaler #2. Upon thus achieving scale equalization, the updated r-vector is obtained by forming the difference r - Cp via the AAU. The global scale of the 17-bit updated r-vector is then given by

$$LR' = LR \tag{3-23}$$

When, on the other hand, LACP - LR < 0, then αCp is left alone while the r-vector is downshifted from its virtual fully upshifted state within the capability of Scaler #1. The command to Scaler #1 is, in fact, one corresponding to an absolute shift of KRH + LACP - LR. Of course, as mentioned before, the maximum negative differential shift that can be accommodated is -(KRH - 7) = -(KR + 8), since KRO = 1. In this case, the global scale of the updated 17-bit r-vector is simply

$$LR' = LACP \tag{3-24}$$

The updated r in RAM may now be accessed at maximum upshift through Scaler #1, placed on Bus A and latched into the X-port of the CPU. Accessed again, it is latched into the Y-port. Letting

KRH = the hardware local scale on r as provided by Scaler #1, limited to 8 bits of upshift

KRO = the effective 1-bit truncation of $\underline{r}$ from 17 to 16 bits, after scaling

KR = KRH - KRO

= the effective local scale of $\underline{r}$ referred to 16-bit arithmetic

Then, the global scale of the maximally upshifted 15-bit $\underline{r}$ vector at the X and Y ports of the CPU happens to be

$$LR = LR + KR \tag{3-25}$$

Employing the conjugate-option on the Y-port, the CPU yields the 19-bit real scalar $\|\underline{r}\|^2$ at the input to Scaler #3. Locally scaling to full significance and truncating by 3 bits yields the left-justified 16-bit version of $\|\underline{r}\|^2$ whose global scale is given by

$$LRR = LR + LR + KRR - KM \tag{3-26}$$

the identical expression used in the initialization period involving the first $\|\underline{r}\|^2$ computation.

Prior to storing the newest $\|\underline{r}\|^2$ into the Scalar Storage RAM, the previous $\|\underline{r}\|^2$ value is accessed, presented to the Division Lookup Table and its 16-bit left-justified reciprocal made available at one input of the real multiplier. During this exact time, the newest $\|\underline{r}\|^2$ on its way to RAM is also presented to the other input to the multiplier. The locally scaled 16-bit product represents β which is sent simultaneously to RAM and around Bus B for further processing. Letting

LRR′ = the global scale of the previous $\|\underline{r}\|^2$

KD = the 29-bit implied upshift at the Division Lookup Table

then, the global scale of the reciprocal of the previous $\|\underline{r}\|^2$ is given by

$$LRRI = KD - LRR' \tag{3-27}$$

whence, in view of (3-25), the global scale of β becomes

$$LBET = LRR + LRRI + KBET - KMO$$

where KBET is the local scale at the real-multiplier output and KMO is the optional 15-bit truncation prior to scaling.

With β at hand, the computation of $\beta\underline{p}$ follows easily. Prior to completing the computation of β, $\underline{p}$ is accessed from RAM with maximum upshift as provided by Scaler #1, placed on Bus A and latched into the X-port of the CPU. Letting

KPH = the hardware local scale on $\underline{p}$ as provided by Scaler #1, limited to 8 bits of upshift

KPO = the effective 1-bit truncation of $\underline{r}$ from 17 to 16 bits, after scaling

KP = KPH - KPO

= the effective local scale of $\underline{p}$ referred to 16-bit arithmetic

then, the global scale of the maximally upshifted 16-bit p-vector at the X-port of the CPU is given by

$$LP = LP' + KP \tag{3-28}$$

where LP′ is its global scale of $\underline{p}$ in RAM.

As soon as β becomes available at the Scaler #4 output, it is conveyed along Bus B, fanned out into a uniform real vector via the available Bus-Connect network, placed on Bus A and latched into the Y-port of the CPU. A partial dot product subsequently yields $\beta\underline{p}$ which is presented to the input of Scaler #2 with a global scale that is given by

$$LBP = LBET + LP - KMO \tag{3-29}$$

where KMO is the optional 15-bit truncation at the individual real multipliers of the CFU.

The final step in the repeating portion of the BCR iteration involves the updating of $\underline{p}$. Keeping in mind that the updated $\underline{p}$ is given by $\underline{r} + \beta\underline{p}$, scale-equalization of $\underline{r}$ and $\underline{p}$ must first be attained before performing the required summation via the AAU. Letting

KRH = the hardware local scale on $\underline{r}$ as sensed by Scaler #1, limited to 8 bits

KRO = the effective 1-bit truncation of $\underline{r}$ from 17 to 16 bits, after scaling

KR = KRH - KRO

= the effective local scale of $\underline{r}$ referred to 16-bit arithmetic

then, the global scale of a maximally upshifted virtual $\underline{r}$ is given by

$$LR = LR' + KR \qquad (3\text{-}30)$$

where LR′ is the global scale of $\underline{r}$ in RAM.

When LBP - LR $\geq$ 0, $\underline{r}$ is accessed on Bus A at maximum upshift via Scaler #1 and $\beta\underline{p}$ is downshifted by LBP - LR bits prior to entering the AAU. The updated 17-bit $\underline{p}$-vector which is subsequently entered into RAM has a global scale given by

$$LP' = LR \qquad (3\text{-}31)$$

When LBP - LR < 0, $\beta\underline{p}$ is left alone. In contrast, $\underline{r}$ is accessed from RAM through Scaler #1 with a shift of KRH + LBP - LR, provided it is within the variable lower bound of -(KRH + 7). In this case, the global scale of the updated 17-bit $\underline{p}$-vector sent to RAM is obviously

$$LP' = LBP \qquad (3\text{-}32)$$

3.2.3 Stopping Criteria

According to the CG algorithm as given in Section 2.1, the process may be terminated either when the iteration number is equal to the dimensionality of the system $C\underline{w} + \underline{b} = \underline{0}$, or when $\|\underline{r}\|^2$ has become smaller than a preassigned small positive real number ε.

Since $\underline{r}^0 = \underline{b}$, a logical choice for ε is $2^{-15}\|\underline{b}\|^2$. More specifically, the BCR process can be terminated prior to or during its last iteration if

$$\|\underline{r}\| < 2^{-15}\|\underline{b}\|^2 \tag{3-33}$$

which is consistent with the fact that the 16-bit arithmetic employed cannot possibly provide a resolution more reliable than one part in 2^{15}. For the sake of simplicity, the practical mechanization of (3-33) need not involve the actual numerical quantities $\|\underline{r}\|^2$ and $\|\underline{b}\|^2$ but, rather, an essentially equivalent statement in terms of their respective global scales. Letting LRRO represent the global scale of $\|\underline{b}\|^2$, or $\|\underline{r}^0\|^2$, then, an easily implementable version of stopping condition (3-33) is

$$\text{LRR} - \text{LRRO} > 15 \tag{3-34}$$

Of course, this would require that LRRO, the initially computed global scale of $\|\underline{r}\|^2$, be stored and made available for comparison with subsequent global scales of $\|\underline{r}\|^2$ in accordance to (3-34).

Although the above two mentioned stopping conditions are necessary and sufficient in theory, in the context of a finite-bit machine they do not suffice. As an example, consider the operation of the Division Lookup Table. If Scaler #3 cannot left-justify $(\underline{p},C\underline{p})$ or $\|\underline{r}\|^2$ prior to addressing the table, a system-interrupt must be induced, terminating the BCR process. Assuming that there exist no system malfunctions, this situation is indicative of the fact that Scaler #3 has exhausted all 15 upshifts without succeeding to left-justify the quantity in question. This, in turn, implies that the quantity involved is sufficiently small, indicating BCR convergence. The weighting vector $\underline{w}$ in RAM may then be taken as the best estimate of the desired solution, within the numerical capability of the machine. It should be mentioned that when $\|\underline{r}\|^2$ is so small that it cannot be left-justified by Scaler #3, then the system $C\underline{w} + \underline{b} = \underline{0}$ is satisfied within the accuracy of the 16-bit arithmetic employed. Noting that $(\underline{p},C\underline{p})$ is essentially an eigenvalue of C, when $(\underline{p},C\underline{p})$ cannot itself be left-justified by Scaler #3, then we have clearly exhausted the rank of C and the process must stop.

Another pertinent stopping condition is related to the nature of the updating process. Recall that, in connection with updating $\underline{w}$, $\underline{r}$, $\underline{p}$, Scaler #1 is limited to 8 bits of downshift.† As such, if the incremental change on any one of these quantities is greater than 2^8 of that in RAM, then, scale-equalization cannot be effected and updating cannot take place any more accurately than 1 part in 2^8.

This scaling limitation may or may not prove to be detrimental to the operation of the BCR process. It might be argued that an error of 1 part in 2^8 may be tolerated considering the the signal samples that were used to form the 16-bit covariance matrix were themselves 8-bit samples. On the other hand, subsequent iterations may, in fact, magnify this error so much so as to give rise to numerical instability manifested, possibly, in the form of a limit-cycle. However, for theoretical reasons, it is desirable to terminate the process when scale equalization may not be accommodated. Very briefly, the phenomenon described above is indicative of the fact that the rank of the matrix has been exhausted at the iteration where it occurred and convergence has consequently been achieved. Of course, the first update of $\underline{w}$ from its initial value of $\underline{0}$ is excepted from this argument.

With the last two stopping conditions mechanized in the BCR implementation, the system is essentially overflow-protected. The set of four stopping conditions discussed constitute the stopping criteria that render the system numerically fail-safe.

3.2.4 Construction of the Combined Signal

At the termination of execution in the CG algorithm stage of the BCR process, the weight vector $\underline{w}$ is accessible locally-scaled by Scaler #1 at a 16-bit or some lower desired resolution, say NBW. In Figure 2-2, the indicated weight vector resolution to the signal combiner is 12 bits. Keeping in mind the implied scale discrepancy on $\underline{w}$ of KWF at the (C,b)-stage of the BCR process, its global scale at the combiner is given by

$$LW = LW + KWF + NBW - NBP$$

where,

LW = the global scale of $\underline{w}$ at the termination of the CG algorithm

KWF = the prescale discrepancy in the value of $\underline{w}$

†Note that 7 bits of downshift are available through hardware; an additional 1 bit is due to bit-alignment in the case of $\underline{r}$, $\underline{p}$ and $\underline{w}$.

NBW = the desired bit-resolution of $\underline{w}$ at the signal combiner. In the present case this value is 12.

NBP = the processor word length, which is 16 in the present case

As a consequence, the combined signal becomes

$$s_c = 2^{-LW'} \underline{s}^T \underline{w} + s_0$$

which involves post-scaling the weighted sum of auxiliary signals, $\underline{s}^T \underline{w}$, by mechanizing a downshift of LW′ bits.

3.2.5 Summary Table

Global Scaling and associated control have been discussed to some detail in this section. For clarity and convenience, the entire set of global scaling equations including initial and terminal conditions have been summarized in the form of Table 3-1. Shown there is a set of numerical specifications indciating auxiliary and main signal word lengths, a 16-bit processor word length, 16 x 16-bit multiplications with optional 16 and 15-bit truncation, implicit local scaling of the 16-bit Division Lookup Table and effective truncations due to bit-alignments in bussing junctures.

The table is subdivided into three columns, entitled "Operation", "Global or Local Scale Values and Equations" and "Remarks". Included are pertinent descriptions of the three BCR processor stages. Following the brief description of the C, $\underline{b}$ stage is the CG-stage. This second BCR stage is subdivided into two major portions, the "Initialization" followed by the "Iterative" portion. Included under "Operation", are initial conditions, equations, quantities to be computed and scaled. Listed under "Global or Local Scale Values and Equations" are local and global scale value or equations corresponding to quantities in the first column. Finally, under "Remarks" are found brief explanations regarding the origin of the scale information, indications of possible external commands and potential stopping conditions imposed numerically or by the particular implementation. The final BCR processor stage deals with related information about the actual signal combining, the stage where the combined signal is produced.

Table 3-1. Summary of Global Scaling Equations and Associated Commands For an Adaptive Fixed-Point Implementation of the BCK Process.

Numerical Specifications	
Main Port Signal Word Length	: NBO = 10
Auxiliary Port Signal Word Length	: NB1 = 8
Processor Word Length (including C,b)	: NBP = 16
16 x 16-bit Multiplier Truncation #1	: KM = 16
16 x 16-bit Multiplier Truncation #2	: KMO = 15
Implicit Local S-ale by 16-bit Division Lookup Table	: KD = 29
Effective Truncation at Scaler #1 KRO, KPO, KWO	: KT1 = 1
Effective Truncation at Scaler #1 and #3 KCPO, KRRO, KPCPO	: KT3 = 3
Desired Word Length to Combiner	: NBW = 12

Operation	Global or Local Scale Values and Equations	Remarks
	C,b Stage Prescaling	
C,b computation	KC	Local scale for C.
	KB	Local scale for b.

Table 3-1. Continued -

	KWF = KB - KC	Prescale discrepancy on final $\underline{w}$. This quantity indicates the number of additional bits of upshift included in the value of $\underline{w}$ produced in the CG-stage that follows.
	CG-Algorithm Stage — Initialization	
$\underline{w} = \underline{0}$	LW	The initial indeterminate scale for $\underline{w} = \underline{0}$.
$\underline{p} = \underline{r} = \underline{b}$	LP = LR = 0	The initial global scales for $\underline{r}$ and $\underline{p}$.
$\lVert\underline{r}\rVert^2$	LRR = LR + LR + KRR - KM	The global scale of $\lVert\underline{r}\rVert^2$ at the out of Scaler #3. Here, the effective local scale is KRR = KRRH - KRRO, where KRRH is the hardware scale limited to 15 bits of upshift and KRRO = KR3 is the effective truncation from 19 to 16 bits. No external command required.
	Repeating Portion of Iteration	
$\underline{p}$	LP = LP′ + KP	The global scale of $\underline{p}$ after it is accessed from RAM through Scaler #1. Here, LP′ is the global scale of $\underline{p}'$ as stored in RAM and KP = KPH - KPO is the effective local scale, where KPH is the hardware scale limited to 8 bits of upshift and KPO = KT1 is the effective truncation from 17 to 16 bits. No external command required.

$C\underline{p}$	$LCP' = LP - KM$	The global scale of $C\underline{p}$ as stored in RAM with 19-bit resolution.
	$LCP = LCP' + KCP$	The global scale of $C\underline{p}$ as accessed from RAM through Scaler #1. Here, the effective local scale is KCP = KCPH - KCPO, where KCPH is the hardware scale limited to 8 bits of upshift and KCPO = KT3 is the effective truncation from 19 to 16 bits. No external command required.
$(\underline{p},C\underline{p})$	$LPCP = LP + LCP + KPCP - KM$	The global scale of $(\underline{p},C\underline{p})$ at the output of Scaler #3. Here, the local scale is KPCP = KPCPH - KPCPO, where KPCPH is the hardware shift limited to 8 bits of upshift and KPCPO = KT3 is the effective truncation from 19 to 16 bits. No external command required.
$1/(\underline{p},C\underline{p})$	$LPCPI = KD - LPCP$	The global scale of the 16-bit reciprocal of $(\underline{p},C\underline{p})$ at the output of the Division Loopup Table. Note that if Scaler #3 cannot provide left-justification of $(\underline{p},C\underline{p})$ going into the Division Lookup Table, the <u>process is terminated</u>.
$\alpha = \frac{\lVert\underline{r}\rVert^2}{(\underline{p},C\underline{p})}$	$LAL = LRR + LPCPI + KAL - KMO$	The global scale of α at the output of Scaler #4. Here, the effective local scale KAL is, in fact, equal to the hardware scale. Note that the multiplier truncation used is KMO = 15. No external command required.
$\alpha\underline{p}$	$LAP = LAL + LP - KMO$	The global scale of $\alpha\underline{p}$ at the input to Scaler #2. Note that no local scaling is required here since α and $\underline{p}$ are already left-justified. Also, the multiplier truncation KMO = 15 is used since α is a positive real scalar.

Table 3-1. Continued -

$\underline{w}$	LW = LW´ + KW	The global scale of a virtual $\underline{w}$ as if it were accessed from RAM through Scaler #1. Here, LW´ is the global scale of $\underline{w}$ as stored in RAM and KW = KWH - KP0 is the effective local scale, where KWH is the sensed hardware scale limited to 8 bits of upshift and KW0 = KT1 is the effective 1-bit truncation from 17 to 16 bits. An inhibit command is required.
$\underline{w}' = \underline{w} - \alpha\underline{p}$	LAP $\geq$ LW LW´ = LW	The global scale of the updated 17-bit $\underline{w}$ is identical to that of the virtual maximally upshifted version. Following this decision, $\underline{w}'$ is first locally scaled by Scaler #1 to $\underline{w}$ and $\underline{p}$ is downshifted by LAP - LW before combining at the AAU.
	LAP < LW LW´ = LAP	Vector $\alpha\underline{p}$ is left alone. Considering that $\underline{w}'$ in RAM could be shifted up by KWH, and that it can independently be shifted down by 7 bits, Scaler #1 is commanded to shift by KWH + LAP - LW. If downshift exceeds KWH + 7 the process is terminated. Note that at iteration #1 when $\underline{w}' = \underline{0}$, LW´ = LAP, arbitrarily. Also, if the process goes into the last iteration, the computation of $\underline{w}$ is an appropriate exit point.
$C\underline{p}$	LCP = LCP´ + KCP	As previously described.
$\alpha C\underline{p}$	LACP = LAL + LCP - KM0	Similar to $\alpha\underline{p}$.
$1/\Vert\underline{r}\Vert^2$	LRRI = KD - LRR	Similar to $1/(\underline{p},C\underline{p})$.

Table 3-1. Continued -

$\underline{r}$	LR = LR´ + KR	The global scale of a virtual $\underline{r}$ as if it were accessed from RAM with maximum upshift through Scaler #1. Here, LR´ is the global scale of $\underline{r}'$ in RAM and KR = KRH - KRO is the effective local scale, where KRH is the sensed hardware scale limited to 8 bits of upshift and KRO = KT1 is the effective 1-bit truncation from 17 to 15 bits. An inhibit command is required.
$\underline{r}' = \underline{r} - \alpha C\underline{p}$	LACP $\geq$ LR LR´ = LR	Similar to $\underline{w}$-update. First, $\underline{r}'$ is locally scaled by Scaler #1 and αCp shifted down by LACP - LR before combining at the AAU.
	LACP < LR LR´ = LACP	Similar to $\underline{w}$-update with corresponding stopping condition. Leaving $\alpha C\underline{p}$ alone, r' is shifted by KWH + LACP - LR before combining at the AAU. If downshift exceeds KWH + 7, the <u>process is terminated</u>.
$\underline{r}$	LR = LR´ + KR	As described previously.
$\|\underline{r}\|^2$	LRR = LR + LR + KRR - KM	As described previously. Note that this quantity is computed simultaneously with the reciprocal of the previous $\|\underline{r}\|^2$ so as to compute β by means of the real multiplier.
β	LBET = LRR + LRRI + KBET - KMO	Similar to α.
$\underline{p}$	LP = LP´ + KP	As described previously.
$\beta\underline{p}$	LBP = LBET + LP - KMO	Similar to $\alpha\underline{p}$.

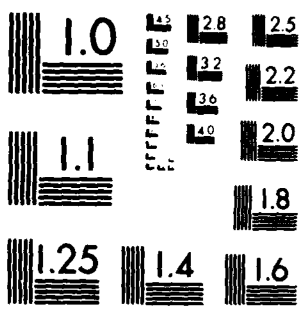
Table 3-1. Continued -

$\underline{r}$	LR = LR′ + KR	Global scale of maximally-upshifted virtual $\underline{r}$.
$\underline{p}' = \underline{r} + \beta\underline{p}$	LBP $\geq$ LR LP′ = LR	Similar to $\underline{w}$-update. First, $\underline{r}$ is locally scaled by Scaler #1 and $\beta\underline{p}$ is shifted down by LBP - LR before combining at the AAU.
	LBP < LR LR′ = LBP	Similar to $\underline{w}$-update with corresponding stopping condition. Leaving $\beta\underline{p}$ alone, $\underline{r}$ is shifted by KRH + LBP - LR before combining at the AAU. If downshift exceeds KWH + 7, the process is terminated.
	Signal Combiner Stage — Postscaling	
$\underline{s}^T\underline{w}$	LSW = LW′ = LW + KWF + NBW - NBP	Weighted sum of auxiliary signals. Since the global scales for the auxiliary signal vector, $\underline{s}$, and that of the main signal, s_0, may be arbitrarily taken to be zero, the global scale of $\underline{s}^T\underline{w}$ is that of $\underline{w}$; namely, LW′. Here, NBW is the desired bit-resolution for $\underline{w}$ at the combiner stage; NBP is the processor word length.
$(\underline{s}^T\underline{w})' = 2^{-LW'}\underline{s}^T\underline{w}$	LSW = 0	Postscaled weighted sum of auxiliary signals prior to direct addition to the main signal.
$s_c = (\underline{s}^T\underline{w})' + s_0$	LSC = 0	Combined signal.

AD-A109 927 MOTOROLA INC TEMPE AZ GOVERNMENT ELECTRONICS DIV F/G 20/14
BATCH COVARIANCE RELAXATION (BCR) ADAPTIVE PROCESSING.(U)
AUG 81 S M DANIEL, I KERTESZ F30602-80-C-0031
UNCLASSIFIED 8512-F RADC-TR-81-212 NL

7 of 7

END
DATE FILMED
3 82
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

4.0 COMPUTER EMULATION

A FORTRAN IV program has been written for the purpose of evaluating the adaptive fixed-point implementation of the BCR process via a precise emulation. To aid the explanation of the computer emulation program, BCRM, a numerical example is included which illustrates the BCR arithmetic activity in full detail. Finally, the fixed-point performance is compared with that of a 32-bit single-precision floating-point, which consistitutes a reference.

4.1 BCR Emulation Program - BCRM

The BCRM program is of modular construction similar to that of programs SIGGEN, BCRS and BCRP already described in Project Memorandum 8512-04. Being closely related to the floating-point simulation program, BCRS, the present BCRM program uses output data generated by SIGGEN, in a similar way. Essential aspects of BCRM are discussed below. The benchtest example used in Project Memorandum 8512-04 provides a means of describing BCRM in some detail. Results from other examples are included.

4.1.1 Input Data File - BCRM:D

The input data file, BCRM:D, for the BCR emulation program, BCRM, is listed in Table 4-1. It consists of a header file, BCRM:DO, which bears a resemblance to the header file, BCRS:DO, of BCRS:D. In fact, it differs only in that it contains a number of emulation parameters that pertain to the present program. The remaining portion of BCRM:D is the signal generation output file, SIGGEN:O, with all optional output suppressed.

As described in Project Memorandum 8512-04 for the case of BCRS:D, the BCRM:DO header of BCRM:D will require appropriate editing according to the SIGGEN:O used.

4.1.2 BCRM Program Structure

Just as the previous programs described, BCRM has a linear structure when considered on a major modular level. Physically, this implies that any module can be replaced without affecting any other module, as long as the interface is maintained. Procedurally, this implies that each branch of a program is executed before proceeding to the next in a top-down fashion, without backtracking and recursion. Such a structure is shown by the tree diagram of Figure 4-1.

In the tree diagram, the named blocks correspond to subprograms or modules by the same name, and the lines connecting them show their relationship. These lines also show the order of execution of the program, but not in the sense of a flow diagram, since no arrows are assigned to show the direction of flow. Starting at the top, the main (sub) program is seen to reference the executive subprogram, BCRMSET. It, in turn, uses the general

Table 4-1. Input Data File for BCR Emulation Program, BCRM:D
Benchtest Example (See Figure 2-2, PM 8512-04)

```
--------------------------------------------------------------------------
                    DIGITAL ADAPTIVE ARRAY PROCESSING
                    ---------------------------------
                                  USING
                                  -----
                       BATCH COVARIANCE RELAXATION
                       ---------------------------

--------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWSO    -  OPTIONAL OUTPUT OF  S0                  :         1
  IWS1    -  OPTIONAL OUTPUT OF  S1                  :         0
  IWSOF   -  OPTIONAL OUTPUT OF  FIXED-POINT S0      :         1
  IWS1F   -  OPTIONAL OUTPUT OF  FIXED-POINT S1      :         1
  IWCB    -  OPTIONAL OUTPUT OF  C  AND  B           :         1

BCR PARAMETERS
--------------
  NSX     -  MAXIMUM NUMBER OF SIGNAL SAMPLES        :       256
  IWO     -  SIZE OF BAND DIAGONAL                   :         0
  NWX     -  MAXIMUM NUMBER OF WEIGHTS               :        20
  ISW     -  WEIGHT SELECTOR ARRAY                   :    1 1 1 1 0 0 0 0 0 0
                                                     :    0 0 0 0 0 0 0 0 0 0
  ITERX   -  MAXIMUM NUMBER OF ITERATIONS            :         4
--------------------------------------------------------------------------
  ITER    -  ACTUAL NUMBER OF ITERATIONS             :         X
--------------------------------------------------------------------------
EMULATION PARAMETERS
--------------------
  NB0     -  NUMBER OF BITS IN A WORD OF S0          :        10
  NB1     -  NUMBER OF BITS IN A WORD OF S1          :         8
  NBP     -  NUMBER OF BITS IN THE PROCESSOR WORD    :        16
  NBW     -  NUMBER OF BITS IN A WORD OF W OUTPUT    :        12
  KM      -  MULTIPLIER TRUNCATION WITH OVERFLOW     :        16
  KM0     -  MULTIPLIER TRUNCATION WITHOUT OVERFLOW  :        15
  KD      -  DIVISION TABLE BIT-SHIFT     (2*NBP-3)  :        29
  KT1     -  1 - BIT TRUNCATIONS KR0,KP0,KW0         :         1
  KT3     -  3 - BIT TRUNCATIONS KRR0,KCP0,KPCP0     :         3
  LIM1    -  LOW  BIT-SHIFT LIMIT AT SCALER #1       :        -7
  LIM2    -  HIGH BIT-SHIFT LIMIT AT SCALER #1       :         8
  LPRT    -  ALLOWED GLOBAL SCALE REDUCTION OF LRR   :        18
--------------------------------------------------------------------------
```

```
--------------------------------------------------------------------------
                    SPECIFICATION OF SYSTEM PARAMETERS
                    ----------------------------------

--------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWANT   -  MAIN ANTENNA ARRAY WEIGHTING                :        0
  IWRAP   -  RECEIVER AMPLITUDE AND PHASE                :        0
  IWRI    -  RECEIVER IMPULSE RESPONSE                   :        0
  IWCAP   -  CHANNEL AMPLITUDE AND PHASE                 :        0
  IWCI    -  CHANNEL IMPULSE RESPONSE                    :        0
  IWSC    -  INDIVIDUAL CHANNEL SIGNALS                  :        0

FILTER PARAMETERS
-----------------
  NPOL    -  NUMBER OF LOWPASS PROTOTYPE POLES           :        2
             POL(1)                                      :  -.70700   -.70700
             POL(2)                                      :  -.70700    .70700
  FBIF    -  FRACTIONAL BANDWIDTH AT FINAL IF            :   .10000
  FBRF    -  FRACTIONAL BANDWIDTH AT RF                  :   .00100
  RADRNG  -  NORMALIZED RADIAN FREQUENCY RANGE           :  4.00000
  RADIN   -  NORMALIZED INITIAL RADIAN FREQUENCY         : -2.00000
  NF      -  NUMBER OF FREQUENCY SAMPLES                 :       32
  LPBP    -  LOWPASS/BANDPASS OPTION                     :        1
             0  :    LOWPASS
             1  :    BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS  -  NUMBER OF CHANNELS PER PORT                 :       20
  ISC     -  CHANNEL SELECTOR ARRAY                      :  1 1 0 0 0 0 0 0 0 0
                                                         :  0 0 0 0 0 0 0 0 0 0

CHANNEL  1:
  AN      -  AMPLITUDE OF NOISE SOURCE                   :  1.00000
  IX0     -  INITIAL RANDU SETTING                       :        1
  N1      -  FIRST-TIME-ON SAMPLE NUMBER                 :        1
  NB      -  BLINK DURATION IN SAMPLES                   :    10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)           : 45.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS          :   .00000
```

```
CHANNEL  2:
  AN      -  AMPLITUDE OF NOISE SOURCE                 :     1.00000
  IX0     -  INITIAL RANDU SETTING                     :          11
  N1      -  FIRST-TIME-ON SAMPLE NUMBER               :           1
  NB      -  BLINK DURATION IN SAMPLES                 :       10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)         :    10.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS        :      .10000

PORT PARAMETERS
---------------
  D0      -  ANTENNA-ELEMENT SEPARATION FACTOR         :     1.00000
  NS      -  NUMBER  F SIGNAL SAMPLES                  :         128
  NPORTS  -  NUMBER OF PORTS                           :          21
  ISP     -  PORT SELECTOR ARRAY                       :     1 1 1 1 0 0 0 0 0 0
                                                       :     0 0 0 0 0 0 0 0 0 0

PORT     0:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :         255
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :      .00000

PORT     1:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :      .00000

PORT     2:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :           1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :      .10000

PORT     3:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                :           1
  LOC1    -  LOCATION OF THE FIRST ELEMENT             :         255
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                :     1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                   :      .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY        :      .00000
```

```
PORT     4:
  NEL      -  NUMBER OF ANTENNA ELEMENTS                    :          1
  LOC1     -  LOCATION OF THE FIRST ELEMENT                 :        255
  BWFCTR   -  BANDWIDTH TOLERANCE FACTOR                    :    1.00000
  AWOFF    -  BANDWIDTH OFFSET FACTOR                       :     .00000
  PDEL     -  FRACTION OF MAXIMUM APERTURE DELAY            :     .10000
------------------------------------------------------------------------------
```

PORT 0

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .359902E+00

.05464	.03818	-.27935	-.18506	-.80850	-.80400	-.96805	-1.00000
-.89117	-.93809	-.86581	-.91809	-.51389	-.71681	-.01191	-.37046
.51795	.07067	.69954	.48863	.33405	.37157	-.72323	-.56527
-.48070	-.23752	-.28359	.13504	-.05666	.10339	.17505	.00267
-.06350	-.11284	-.27573	-.25835	-.11408	.03863	.07952	.27616
-.00055	-.11476	.45160	.31795	-.25093	-.05661	-.53158	-.23006
.10718	-.05535	.29687	.03986	-.22163	-.13246	-.32173	-.22894
-.37581	.00988	-.03880	.09207	.23237	.18048	-.03938	.26499
-.60312	-.16596	-.22783	-.27798	.15059	.04556	.49784	.37388
.05924	.20007	-.67014	-.26716	-.51796	-.17555	.41518	.42498
.46477	.55190	-.24890	.00095	-.22332	-.22121	-.17883	-.20944
-.14506	-.08280	-.48174	-.33351	-.15840	-.23864	.03017	.12233
-.24552	.07159	.07142	.10982	.79052	.48894	.42138	.42662
-.56041	-.25491	-.64554	-.51348	.04592	.16574	.45397	.33689
-.15339	-.20463	-.60183	-.43150	-.23293	.15092	-.05293	.03788
.70177	.26965	.66569	.35406	.76548	.50369	.33144	.11207
.34198	.16070	-.22887	.04206	-.10226	.16876	.08978	.24863
.07344	-.10328	.46115	.36154	.51861	.34287	.17532	.33440
-.01579	.20464	.13438	.29128	-.17331	-.08766	.00878	.12494
-.21701	-.41771	.06459	-.23581	-.57297	-.59389	-.25658	-.38524
-.71101	-.39013	-.17867	-.31976	.16877	.45003	-.15970	.15444
.40744	.45485	.68224	.67182	.68668	.85575	-.08121	.25855
.30583	.30440	.19326	.26502	-.49589	-.22002	-.67978	-.41590
-.30489	-.53854	.35300	.05499	-.29986	.01299	-.08097	-.02180
.53392	.59253	-.08945	.13345	-.56345	-.37763	-.77421	-.36072
-.67038	-.15848	-.09812	.07619	-.07994	.11141	-.19309	-.00092
-.09846	-.05803	-.32363	-.11191	-.70204	-.51644	-.42421	-.15956
.35976	.29671	.55861	.31601	.34769	-.05135	.29570	.17155
-.02046	.16898	-.40331	-.40163	-.33261	-.31393	.02263	.05234
-.29163	-.15834	.20011	.26848	.21790	.33672	.14524	.09034
.12943	.14092	-.11299	.08524	.11257	.27772	.18866	.44657
.08273	.17843	.09462	.15594	-.23630	-.08516	.11599	-.09951

PORT 1

RECEIVED BASEBAND PORT SIGNAL

NORMALIZATION CONSTANT : .610091E+00

.03302	.02712	-.09132	-.17755	-.18820	-.83060	-.29433	-1.00000
-.21299	-.98208	-.24348	-.93500	.04583	-.82961	.37524	-.58496
.69815	-.25163	.44716	.32677	.25227	.27871	-.37896	-.48880
-.63976	.03568	-.61052	.42171	-.26588	.28932	.34864	-.15242
.05773	-.16416	-.08271	-.27327	-.37270	.21433	-.13248	.38092
.09187	-.14752	.32176	.21072	-.18946	-.02783	-.67211	.07588
.21314	-.11392	.48308	-.20447	-.13321	-.11077	-.26331	-.15314
-.66599	.31100	-.21420	.24147	.14301	.13614	-.34743	.42636
-.71764	.15784	-.08099	-.23739	.13595	-.00945	.29930	.28787
-.02352	.19411	-.72186	.01725	-.74155	.16904	.00674	.52368
.18736	.50627	-.36548	.17965	-.07136	-.19976	-.04465	-.21834
-.09107	-.06771	-.31924	-.23553	-.04164	-.23201	-.08992	.14816
-.48834	.30318	-.12398	.22782	.61549	.29267	.28063	.30504
-.47300	-.08395	-.49339	-.34048	-.27158	.33822	.38861	.21674
.17691	-.35199	-.53938	-.24903	-.61482	.44777	-.16926	.17435
.76491	-.02179	.61187	.10926	.60848	.28806	.45776	-.09097
.31446	.02369	-.35141	.16868	-.53609	.46701	-.04419	.31528
.17626	-.17624	.25267	.29613	.40248	.20129	-.07790	.37641
-.37641	.43022	-.07058	.36290	-.20013	.00489	-.11260	.20098
.22704	-.54907	.41214	-.47726	-.07624	-.65830	-.02222	-.44125
-.53181	-.22691	-.10767	-.27365	-.22590	.56261	-.51941	.44397
.01805	.53026	.17133	.70150	.10187	.89872	-.42416	.49915
.00668	.38858	.11727	.21994	-.48694	-.03700	-.53686	-.20884
.10807	-.61208	.48876	-.19993	-.42084	.16529	-.26779	.13871
.10717	.61734	-.15058	.20583	-.41848	-.23829	-.80477	-.06036
-.97474	.28751	-.27616	.24423	-.24091	.22343	-.30947	.16292
-.08795	-.02728	-.29755	.00077	-.45309	-.39201	-.63687	.13347
.15087	.30367	.57666	.09911	.65897	-.36033	.20237	.06469
-.16686	.26395	-.06628	-.42963	-.25135	-.22688	.01462	.03858
-.28190	-.05992	-.13892	.38777	.01027	.37187	.09397	.07921
.05936	.11497	-.31679	.23686	-.24357	.43610	-.24705	.62380
-.07275	.25836	-.03243	.18642	-.22215	.01502	.20369	-.18164

PORT 2

RECEIVED BASEBAND PORT SIGNAL

NORMALIZATION CONSTANT : .610608E+00

-.02491	.03355	.16263	-.08818	.81748	-.18453	1.00000	-.29458
.98087	-.21286	.93707	-.24589	.83185	.03664	.59328	.36582
.26113	.69540	-.31089	.45527	-.29870	.25877	.48329	-.35991
-.01577	-.64259	-.42033	-.60788	-.29770	-.28708	.14425	.34693
.16590	.06461	.27090	-.07696	-.19333	-.36938	-.39666	-.14192
.14848	.08694	-.20708	.31965	.02145	-.16564	-.07124	-.67903
.10375	.18729	.20995	.49534	.10654	-.12410	.16072	-.25671
-.30212	-.66093	-.24916	-.23378	-.13129	.14652	-.42227	-.33021
-.17230	-.72136	.23572	-.09746	.01897	.13590	-.28427	.29409
-.19967	-.00354	-.01921	-.71096	-.15962	-.74948	-.51749	-.01636
-.51148	.20221	-.18992	-.36335	.19277	-.07815	.22331	-.04254
.06712	-.08752	.23033	-.31654	.23836	-.05146	-.13878	-.07756
-.30424	-.48622	-.22803	-.14235	-.29011	.60396	-.30762	.30404
.06820	-.46383	.34942	-.49497	-.32462	-.28447	-.23160	.37586
.34197	.19756	.26476	-.53024	-.43914	-.61555	-.18685	-.19341
.02313	.75344	-.10330	.61841	-.28966	.60655	.08191	.46258
-.01371	.32165	-.16907	-.33224	-.45422	-.54541	-.33628	-.05353
.18200	.17323	-.29086	.24971	-.20186	.40144	-.37266	-.05740
-.42744	-.38361	-.37236	-.07101	-.00402	-.19901	-.20547	-.11646
.53097	.21369	.48674	.42032	.64545	-.07112	.46042	-.01821
.21547	-.52287	.29255	-.12527	-.55336	-.20650	-.44781	-.52980
-.52516	.01045	-.69464	.16639	-.89979	.11444	-.50962	-.42002
-.38674	-.01006	-.22910	.12950	.03470	-.47597	.19819	-.54335
.60872	.08529	.21755	.50033	-.16688	-.40291	-.13090	-.28446
-.61051	.10815	-.22541	-.14262	.23738	-.41077	.06817	-.79295
-.28067	-.98174	-.24945	-.29299	-.21914	-.23018	-.17132	-.31848
.03016	-.08478	-.00962	-.29434	.39153	-.44412	-.11776	-.64398
-.30695	.12837	-.10783	.57398	.35383	.65887	-.04413	.22125
-.27802	-.16981	.42009	-.06146	.23955	-.25383	-.04047	.01365
.06365	-.27440	-.37577	-.14819	-.38166	.01140	-.08118	.08872
-.11255	.06829	-.23277	-.31185	-.42853	-.24660	-.62527	-.24683
-.26828	-.07985	-.18350	-.02597	-.02687	-.22517	.18985	.19357

PORT 3

RECEIVED BASEBAND PORT SIGNAL

NORMALIZATION CONSTANT : .779343E+00

.03553	.03825	-.23307	-.25001	-.74354	-.87491	-.86642	-1.00000
-.84147	-.96906	-.79776	-.91628	-.62521	-.77081	-.31204	-.45886
.05329	-.07371	.44374	.46365	.20395	.22604	-.48456	-.53220
-.09156	.00943	.13003	.30762	.10785	.17217	-.04291	-.13119
-.10665	-.13830	-.23519	-.26839	.13020	.23388	.17714	.26432
-.06153	-.12531	.25730	.25908	-.09900	-.06139	-.12132	-.02551
-.06191	-.13662	-.00140	-.09031	-.14191	-.13168	-.15835	-.14524
.06242	.22468	.09327	.14617	.16672	.16929	.22804	.37299
-.14449	-.03258	-.20566	-.26463	.06339	.05043	.32116	.33779
.11695	.17053	-.20065	-.10117	-.05091	.07172	.42112	.50242
.41170	.49108	-.00008	.06620	-.19522	-.24028	-.16501	-.19502
-.09367	-.07952	-.29634	-.29418	-.16157	-.20566	.10885	.17313
.08267	.20812	.14721	.18026	.41812	.37577	.29959	.33705
-.25749	-.21473	-.35798	-.36526	.21317	.31954	.21523	.19404
-.25396	-.33762	-.27480	-.24231	.17093	.35754	.04293	.06770
.21649	.09150	.29874	.23991	.39797	.36847	.05161	-.03802
.15369	.12049	.02681	.12367	.25114	.40101	.13566	.18775
-.04054	-.11894	.31294	.34029	.28543	.26222	.27731	.37342
.24385	.35734	.21012	.28148	-.03584	-.02366	.08202	.13720
-.40433	-.56414	-.22577	-.35089	-.57194	-.67218	-.27968	-.33856
-.39289	-.33852	-.15000	-.20281	.36619	.54276	.19071	.32112
.43369	.51311	.62765	.71943	.70947	.86662	.23836	.36384
.31599	.35595	.15954	.18996	-.17788	-.11882	-.36966	-.34193
-.44087	-.59280	.04145	-.02978	-.01252	.10422	.07955	.11803
.51118	.62030	.03943	.09123	-.30200	-.29875	-.26235	-.14836
-.05444	.13452	.07886	.14670	.10642	.18723	.00257	.06106
-.03515	-.03093	-.13717	-.08959	-.42822	-.43797	-.03624	.08101
.26072	.28348	.20613	.13930	-.08240	-.25361	.18838	.19791
.09070	.16655	-.37476	-.45706	-.20103	-.20717	.01002	.03062
-.10695	-.07620	.29852	.38727	.24513	.31122	.09329	.07568
.11462	.13322	.10823	.19735	.29595	.40479	.39115	.53786
.15041	.18361	.14103	.17837	-.09906	-.07480	-.03096	-.10842

PORT 4

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .780274E+00

-.03678	.03492	.23329	-.21909	.86342	-.73378	1.00000	-.86610
.96786	-.84023	.91825	-.80002	.77471	-.63001	.46831	-.32138
.08469	.04396	-.45162	.43556	-.24909	.22376	.52855	-.47955
.00875	-.10555	-.30803	.13039	-.17855	.10913	.12626	-.03892
.13781	-.10485	.26768	-.23405	-.21584	.11663	-.28070	.18853
.13078	-.06747	-.25776	.25729	.05354	-.08949	.02853	-.12514
.13186	-.06230	.09489	-.00227	.12508	-.13571	.15364	-.16373
-.21955	.05893	-.15048	.09321	-.16307	.16264	-.37372	.23197
.01814	-.13456	.26797	-.21060	-.04088	.05573	-.33448	.31802
-.17906	.12709	.09810	-.19682	-.06110	-.06102	-.49374	.41082
-.49855	.42044	-.07721	.00836	.23598	-.19245	.20013	-.16858
.07728	-.09084	.28895	-.29244	.21472	-.17014	-.16661	.10609
-.21063	.08431	-.17727	.14133	-.37081	.41300	-.34407	.30951
.19766	-.24299	.37754	-.36810	-.30777	.20201	-.20702	.22364
.32705	-.24216	.25794	-.28607	-.35328	.16801	-.07539	.04373
-.08686	.21189	-.23436	.29481	-.37201	.40113	.03077	.05726
-.11226	.14949	-.12633	.03116	-.38935	.24063	-.20771	.14987
.12903	-.04933	-.33737	.31059	-.26064	.28383	-.37398	.28123
-.35380	.23941	-.29183	.21894	.02728	-.03921	-.14539	.08896
.55285	-.39874	.35714	-.22750	.66054	-.56345	.35608	-.29137
.32431	-.38152	.22754	-.17236	-.54062	.36826	-.32227	.18842
-.50683	.42798	-.71107	.61984	-.87153	.71631	-.37348	.24541
-.35185	.31092	-.20023	.17013	.11499	-.17318	.33096	-.36196
.59618	-.44777	.04543	.03237	-.11078	-.00558	-.10588	.06664
-.61690	.50981	-.11093	.05577	.29847	-.30015	.15461	-.26520
-.12893	-.06044	-.14835	.07754	-.18465	.10630	-.06845	.00663
.03464	-.03712	.07825	-.12736	.43991	-.42871	-.06727	-.04842
-.28342	.25692	-.14837	.21305	.25127	-.08094	-.18200	.17960
-.18334	.10331	.45062	-.36908	.21937	-.21103	-.03344	.01272
.08065	-.11038	-.37613	.28883	-.32143	.25304	-.07556	.09229
-.13175	.11534	-.19421	.10610	-.39669	.28910	-.54147	.39412
-.19048	.15429	-.17685	.14169	.06323	-.09104	.12127	-.04268

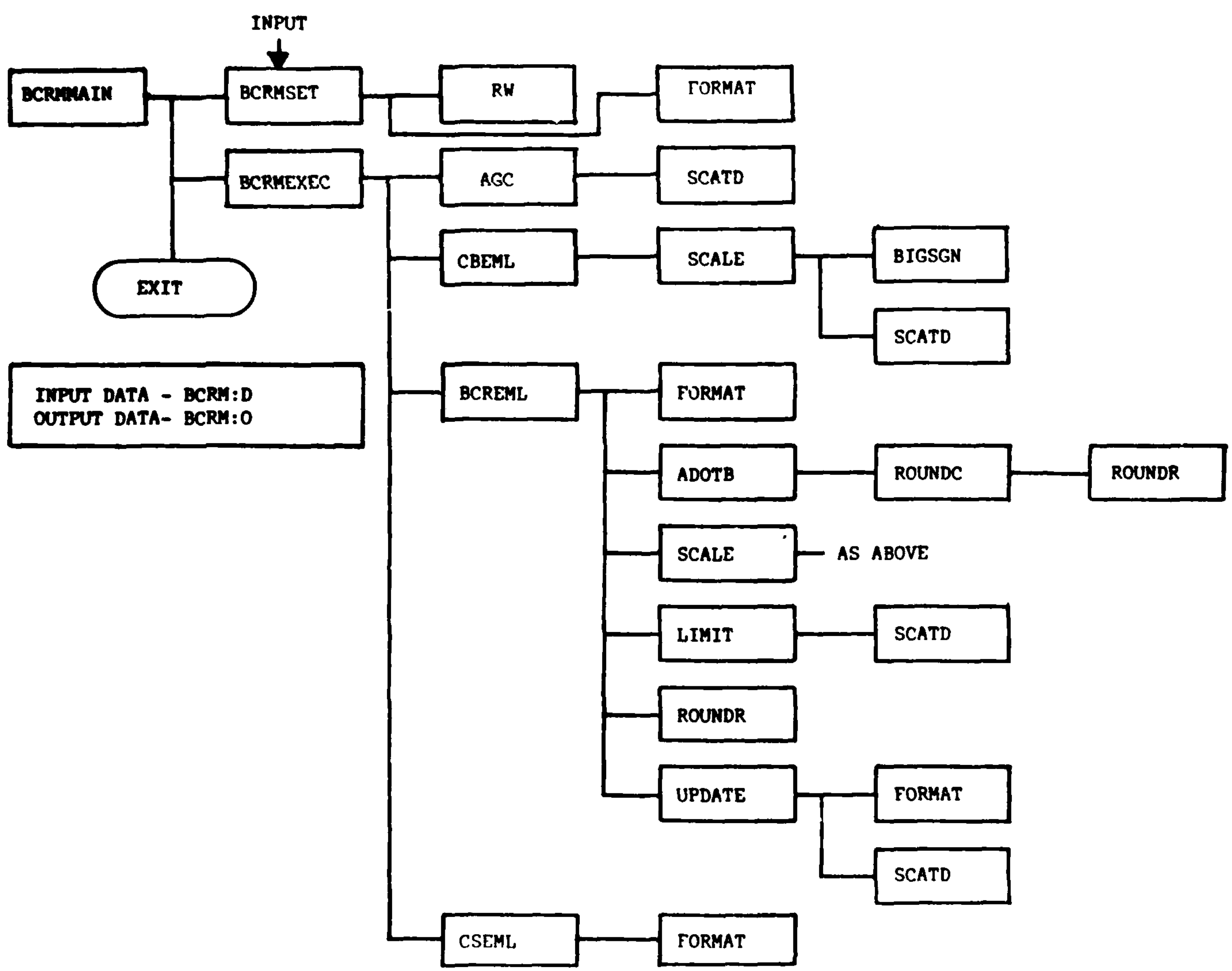


Figure 4-1. Tree Diagram of the BCR Emulation Program, BCRM

subprogram, RW, to input data.† Several calls are made to RW before all the input is completed, and then control returns to BCRMMAIN via BCRMSET. This completes one major branch of the structure.

The next major branch is accessed by calling the other executive subprogram, BCRMEXEC, which actually directs the BCR emulation via four dedicated subprograms. The first subprogram, AGC, provides the proper gain needed for sampling the main and auxiliary signals at desired bit levels with maximum resolution. The second subprogram, CBEML, heads the branch which is dedicated to computing and scaling the covariance matrix and forcing vector, C and $\underline{b}$, respectively. Local bit-shifting and truncations are provided here to yield quantities $\bar{C}$ and $\underline{\bar{b}}$ which are block-scaled to the resolution designated for the BCR processor. The third branch headed by BCREML actually executes the BCR emulation. The adaptive weight vector produced by BCREML is used in the final branch headed by CSEML where the optimal combined signal is computed. The evaluation of the power suppression achieved also takes place here.

After this last branch is complete, control again returns to BCRMMAIN, and the execution terminates via a call to EXIT. Functional descriptions of each module comprising BCRM is given in Table 4-2.

4.1.3 Source Modules

The BCRM source program consists of all the subprograms whose names are shown in the tree diagram of Figure 4-1. A complete list of these subprograms appears in Table 4-3 in the same order in which they are called. Each subprogram has a name which is identical to the subroutines or function name for which it stands. A short functional description of the program-module, dates of origin and revision, and the author's names precede the definition of the interface. It starts with input and output data identification, is followed by name and description of entry points and ends with the list of down-stream references to subroutines, functions and entries.

All the executive subprograms share data through a set of common statements. The major dedicated subprograms are conveniently accessed through a minimum set of common statements. In contrast to this, the minor dedicated subprograms and the general subprograms are accessed through a minimum set of arguments of a call statement. The purpose in each case was to enhance clarity, efficiency, and versatility.

4.1.4 Binary Modules

The individual source modules comprising BCRM are usually compiled separately as a matter of convenience. For every source module a corresponding

---

†Note that the module RW used here is much more general than that used in Project Memorandum 8512-04 for programs SIGGEN, BCRS and BCRP. In fact, it accommodates calls previously made to RWIRC and does much more as should be evident upon a close examination. What is important is the fact that calls made to previous RW may also be made validly to its present version.

Figure 4-2. Functional Description of Program Modules Comprising BCRM

FUNCTIONAL DESCRIPTION OF BCRM PROGRAM MODULES

BCRMMAIN	-	MAIN EXECUTIVE PROGRAM. IT CALLS OTHER EXECUTIVE SUBPROGRAMS WHICH TOGETHER PERFORM THE BCR EMULATION.
BCRMSET	-	EXECUTIVE SUBPROGRAM. IT IS DESIGNED TO READ FROM THE INPUT DATA FILE, BCRMID, USING THE GENERAL INPUT AND OUTPUT SUBROUTINE, RW, IN ACCORDANCE TO PRESET FORMAT. ALL DATA ARE MADE AVAILABLE TO APPROPRIATE SUBROUTINES THROUGH COMMON BLOCKS.
RW	-	GENERAL SUBPROGRAM. THIS INPUT/OUTPUT SUBPROGRAM IS DESIGNED TO READ, WRITE OR SKIP LINES OF TEXT.
BCRMEXEC	-	EXECUTIVE SUBPROGRAM. IT DIRECTS THE BCR EMULATION PROCESS BY CALLING FOUR DEDICATED SUBPROGRAMS IN THE PROPER SEQUENCE, MAKING DATA AVAILABLE TO EACH AS NECESSARY.
AGC	-	DEDICATED SUBPROGRAM. IT IS DESIGNED TO DENORMALIZE THE MAIN AND AUXILIARY PORT SIGNALS USING THE SUPPLIED CONSTANT, AND TO SUBSEQUENTLY SCALE THEM IN SUCH A WAY THAT THEIR LARGEST ELEMENT IS A LEFT-JUSTIFIED WORD OF A GIVEN NUMBER OF BITS.
SCATD	-	GENERAL SUBPROGRAM. IT IS DESIGNED TO SCALE A REAL OR COMPLEX VECTOR USING THE SUPPLIED BIT-SHIFT, THEN TO TRUNCATE THE RESULT ACCORDING TO 2'S COMPLEMENT ARITHMETIC.
CREML	-	DEDICATED SUBPROGRAM. IT IS DESIGNED TO COMPUTE THE COVARIANCE OF A BATCH OF AUXILIARY PORT SIGNALS, AND THE CROSS CORRELATION VECTOR OF THE AUXILIARY AND THE MAIN PORT SIGNALS. THE COVARIANCE MATRIX AND THE CROSS CORRELATION OR FORCING VECTOR ARE THEN INDIVIDUALLY SCALED TO A SIZE DETERMINED BY THE PROCESSOR WORD. THE VECTORS AND THEIR SCALE FACTORS ARE RETURNED.
SCALE	-	GENERAL SUBPROGRAM. IT IS DESIGNED TO SCALE REAL OR COMPLEX VECTORS BY SHIFTING ALL COMPONENT WORDS BY A NUMBER OF BITS THAT WILL LEFT-JUSTIFY THE LARGEST-MAGNITUDE WORD.

BIGSGN	-	GENERAL SUBPROGRAM. IT IS DESIGNED TO SEARCH A REAL OR COMPLEX VECTOR TO FIND THE ELEMENT WHICH HAS THE LARGEST ABSOLUTE VALUE. THIS VALUE AND ITS SIGN ARE RETURNED.
BCREML	-	DEDICATED SUBPROGRAM. IT IS DESIGNED TO EMULATE THE ARITHMETIC ACTIVITY OF THE PROCESSOR WHICH PERFORMS THE ACTUAL BCR PROCESS. THIS TASK IS ACCOMPLISHED WITH THE AID OF OTHER DEDICATED AND GENERAL SUBPROGRAMS. THE OUTPUT OF THE SUBPROGRAM IS A SCALED VALUE OF THE ADAPTIVE WEIGHT VECTOR.
FORMAT	-	GENERAL SUBPROGRAM. IT IS DESIGNED TO OUTPUT DATA IN A VERSATILE AND STANDARDIZED FORMAT.
ADOTR	-	GENERAL SUBPROGRAM. IT IS DESIGNED TO PERFORM THE DOT PRODUCT OF TWO COMPLEX VECTORS WITH THE OPTION OF USING THE CONJUGATE OF THE FIRST VECTOR. EACH INDIVIDUAL MULTIPLICATION RESULT IS ROUNDED AND SUBSEQUENTLY TRUNCATED ACCORDING TO THE ACTUAL OPERATION OF THE MULTIPLIER CHIP USED.
ROUNDC	-	GENERAL SUBPROGRAM. IT IS DESIGNED TO MULTIPLY TWO COMPLEX NUMBERS, AND TO ROUND OFF THE RESULT.
ROUNDR	-	GENERAL SUBPROGRAM. IT IS DESIGNED TO MULTIPLY TWO REAL NUMBERS, AND TO ROUND OFF THE RESULT.
LIMIT	-	DEDICATED SUBPROGRAM. IT IS DESIGNED TO PROVIDE THE ALGORITHM WHICH REPRESENTS HARDWARE LIMITS. THESE LIMITS DEFINE THE SHIFTING RANGE.
UPDATE	-	DEDICATED SUBPROGRAM. IT IS DESIGNED TO CARRY OUT THE UPDATING OF W, R, AND P VECTORS WHILE IT EXERCISES SCALE EQUALIZATION WITHIN HARDWARE LIMITS.
CSEML	-	DEDICATED SUBPROGRAM. IT IS DESIGNED TO PRODUCE THE COMBINED SIGNAL, AND THE WEIGHTED SUM OF THE AUXILIARY PORT SIGNALS ADDED TO THE MAIN. THE COMBINED POWER SUPPRESSION IS ALSO COMPUTED HERE.

Table 4-3. FORTRAN Listing of Source Modules Comprising the BCR Emulation Program, BCRM

```
C  ======================================================================
C                      BCR EMULATION PROGRAM :  BCRM:S
C                      ------------------------------
C
C                  DIGITAL ADAPTIVE ARRAY PROCESSING
C                                USING
C                     BATCH COVARIANCE RELAXATION
C
C           PROGRAM       :  BCRMMAIN:S
C           ORIGINAL      :  AUGUST 15, 1979
C           REVISION      :  MARCH  11, 1981
C
C           PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                            RADAR SYSTEMS ANALYSIS GROUP
C                            MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                            TEMPE, ARIZONA  85282
C  ======================================================================
      CALL BCRMSET
      CALL BCRMEXEC
      CALL EXIT
      END
```

```
C ======================================================================
      SUBROUTINE BCRMSET
C ======================================================================
C                     SPECIFICATION OF SYSTEM PARAMETERS
C                 MAIN-PORT AND AUXILIARY BASEBAND SIGNALS
C
C               PROGRAM       :  BCRMSET:S
C               ORIGINAL      :  JULY   23, 1980
C               REVISION      :  MARCH  25, 1981
C
C               PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C       INPUT  :    DATA FILE BCRM:D
C       OUTPUT :    DATA FILE BCRM:O
C ----------------------------------------------------------------------
C       ENTRY POINTS          :  BCRMSET
C
C       PROGRAMS CALLED       :  SUBROUTINE   FUNCTION     ENTRY
C                                ----------   --------     -----
C                                RW              -         RW
C                                                -         RWI
C                                                -         WI
C                                                -         RWR
C                                                -         WR
C                                                -         RWC
C                                                -         WC
C                                                -         RWIV
C                                                -         RWRV
C                                                -         WRV
C                                FORMAT          -         FORMAT
C ======================================================================
      COMPLEX S1,S0
      COMMON /BCR0/ IWS0,IWS1,IWS0F,IWS1F,IWCB,IDP(20)
      COMMON /BCR1/ S0(256),S1(256,20),NSX,NS,NBITS0,NBITS1,S0MAX,S1MAX
      COMMON /BCR2/ ISW(20),NWX,IW0,NPM1,ITERX,NBITSP,NBITSW
     -,KM,KM0,KD,KT1,KT3,LIM1,LIM2,LRRT
      DIMENSION ISC(20),ISP(20),WARNIN(20)
      DATA WARNIN/
     -'****  SELECTED WEIGHTS ARE NOT COMPATIBLE WITH DESIGNATED PORTS
     -****    '/
```

```
C ----------------------------------------------------------------
C     READ PRINT OPTIONS
C ----------------------------------------------------------------
      CALL RW(12)
      CALL RWI(IWS0)
      CALL RWI(IWS1)
      CALL RWI(IWS0F)
      CALL RWI(IWS1F)
      CALL RWI(IWCR)
C ----------------------------------------------------------------
C     READ BCR PARAMETERS
C ----------------------------------------------------------------
      CALL RW(3)
      CALL RWI(NSX)
      CALL RWI(IW0)
      CALL RWI(NWX)
      CALL RWIV(NWX,ISW)
      CALL RWI(ITERX)
C ----------------------------------------------------------------
C     READ AND ADJUST EMULATION PARAMETERS
C ----------------------------------------------------------------
      CALL RW(5)
      CALL RWI(NB0)
      CALL RWI(NB1)
      CALL RWI(NBP)
      CALL RWI(NBW)
      NBITS0=NB0-1
      NBITS1=NB1-1
      NBITSP=NBP-1
      NBITSW=NBW-1
      CALL RWI(KM)
      CALL RWI(KM0)
      CALL RWI(KD)
      CALL RWI(KT1)
      CALL RWI(KT3)
      CALL RWI(LIM1)
      CALL RWI(LIM2)
      CALL RWI(LRRT)
C ---------------------------------------------  ----------------
C     READ SYSTEM DESCRIPTION PARAMETERS
C ----------------------------------------------------------------
      CALL RW(18)
      CALL RWI(NPOL)
```

```
      CALL RW(NPOL+11)
C ----------------------------------------------------------------------
C     READ CHANNEL SELECTOR ARRAY AND ASSOCIATED CHANNEL DESCRIPTIONS
C ----------------------------------------------------------------------
      CALL RWI(NCHNLS)
      CALL RWIV(NCHNLS,ISC)
      ICSUM=0
      DO 10 IC=1,NCHNLS
   10 ICSUM=ICSUM+ISC(IC)
      CALL RW(4+8*ICSUM)
      CALL RWI(NS)
      NS2=NS*2
C ----------------------------------------------------------------------
C     READ THE PORT SELECTOR ARRAY AND ASSOCIATED PORT DESCRIPTIONS
C ----------------------------------------------------------------------
      CALL RWI(NPORTS)
      NPM1=NPORTS-1
      CALL RWIV(NPM1,ISP)
      IPSUM=0
      DO 20 IP=1,NPM1
      IPIW=ISP(IP)*ISW(IP)
      IF(IPIW.EQ.ISW(IP)) GOTO 20
      CALL FORMAT(WARNIN,80)
      STOP
   20 IPSUM=IPSUM+ISP(IP)
      CALL RW(7*(IPSUM+1))
C ----------------------------------------------------------------------
C     READ MAIN AND AUXILIARY BASEBAND SIGNALS
C ----------------------------------------------------------------------
      MULT=1
      IF(IWS0.EQ.0) MULT=-1
C ----------------------------------------------------------------------
C     READ S0 AND WRITE IT ONLY IF IWS0=1
C ----------------------------------------------------------------------
      CALL RW(7*MULT)
      CALL RWRV(NS2*MULT,S0,SMAX)
C ----------------------------------------------------------------------
C     DENORMALIZE S0
C ----------------------------------------------------------------------
      DO 30 IS=1,NS
   30 S0(IS)=SMAX*S0(IS)
      S0MAX=SMAX
C ----------------------------------------------------------------------
```

```
C     READ ONLY THOSE OF THE DESIGNATED PORTS WHICH ARE SELECTED BY ISW
C ---------------------------------------------------------------------
      MULT=1
      IF(IWS1.EQ.0) MULT=-1
      IIP=0
      NROWS=(NS-1)/4+1
      S1MAX=0
      DO 50 IP=1,NPM1
      IF(ISP(IP).EQ.0) GOTO 50
      IF(ISW(IP).EQ.1) GOTO 1000
      CALL RW(-9-NROWS)
      GOTO 50
1000  IIP=IIP+1
      IDP(IIP)=IP
C ---------------------------------------------------------------------
C     READ S1 AND WRITE IT ONLY IF IWS1=1
C ---------------------------------------------------------------------
      CALL RW(7*MULT)
      CALL RWRV(NS2*MULT,S1(1,IIP),SMAX)
C ---------------------------------------------------------------------
C     DENORMALIZE S1
C ---------------------------------------------------------------------
      DO 40 IS=1,NS
40    S1(IS,IIP)=SMAX*S1(IS,IIP)
      IF(SMAX.GT.S1MAX) S1MAX=SMAX
50    CONTINUE
      NPM1=IIP
      CALL RW(1)
C ---------------------------------------------------------------------
      RETURN
      END
```

```
C =====================================================================
      SUBROUTINE RW(NV,IV,RV,AR14)
C =====================================================================
C                          GENERAL READ/WRITE ROUTINE
C                                     FOR
C                 SKIPPING SPECIFIED NUMBER OF LINES OF INPUT
C                                     AND
C                             READING AND WRITTING
C                            80-CHARACTER COMMENTS
C  56-CHARACTER DEFINITION OF AN INTEGER, REAL OR COMPLEX SCALAR
C   56-CHARACTER DEFINITION OF AN INTEGER SELECTOR ARRAY VECTOR
C    48-CHARACTER COMMENT WITH REAL SCALAR FOLLOWED BY A REAL
C
C            PROGRAM        :  RW:S
C            ORIGINAL       :  APRIL  15, 1978
C            REVISION       :  MARCH  17, 1981
C
C            PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                              RADAR SYSTEMS ANALYSIS GROUP
C                              MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                              TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C    INPUT  :    NV         -  NUMBER OF COMMENT LINES TO READ AND
C                              WRITE, THE NUMBER OF LINES TO SKIP,
C                              VECTOR DIMENSIONALITY
C
C    INPUT AND OUTPUT :
C                CX         -  COMPLEX SCALAR
C                RX         -  REAL SCALAR
C                IX         -  INTEGER SCALAR
C                IV         -  INTEGER VECTOR
C                RV         -  REAL OR COMPLEX VECTOR
C                RX         -  REAL SCALAR EXCLUSIVE OF CX & IX
C                IX         -  INTEGER SCALAR EXCLUSIVE OF CX & RX
C                AR14       -  56 - CHARACTER COMMENT
C ---------------------------------------------------------------------
C    DUMMY ARRAYS              : IV, RV, AR14
C    LOCAL CONSTANT ARRAYS     : LC12, LC20
C    LOCAL VARIABLE ARRAYS     : LR12, LR12, LR20
C ---------------------------------------------------------------------
C    ENTRY POINTS           :  RW    - READ AND WRITE NV COMMENT LINES
C                                      IF NV<0, -NV LINES ARE SKIPPED
C                              RWI   - READ AND WRITE INTEGER SCALAR
```

```
C                                 WI   - WRITE INTEGER SCALAR
C                                 RWR  - READ AND WRITE REAL SCALAR
C                                 WR   - WRITE REAL SCALAR
C                                 RWC  - READ AND WRITE COMPLEX SCALAR
C                                 WC   - WRITE COMPLEX SCALAR
C                                 RWIV - READ AND WRITE INTEGER VECTOR
C                                 RWRV - READ AND/OR WRITE REAL OR
C                                        COMPLEX VECTOR
C                                 WRV  - WRITE REAL OR COMPLEX VECTOR
C
C     SUBROUTINES CALLED    :  NONE
C   ==================================================================
      COMPLEX CX
      DIMENSION IV(1),RV(1),AR14(14)
      DIMENSION LC12(12),LC20(20),LR12(12),LR14(14),LR20(20)
      DATA LC12/4*'    ','   NORMALIZATION CONSTANT    :  '/
      DATA LC20/' ---',19*'----'/
100   FORMAT(14A4,I12)
200   FORMAT(14A4,F12.5)
300   FORMAT(14A4,2X,2(F10.5))
400   FORMAT(20A4)
500   FORMAT(14A4,4X,10I2)
600   FORMAT(12A4,E13.6)
700   FORMAT(8F10.5)
800   FORMAT(1X)
C   ------------------------------------------------------------------
C     READ AND WRITE NV COMMENTS.  IF NV<0,  SKIP -NV LINES: ENTRY RW
C   ------------------------------------------------------------------
      IF(NV.LE.0) GOTO 1000
      DO 10 I=1,NV
      READ(105,400) LR20
10    WRITE(108,400) LR20
      RETURN
1000  NVP=-NV
      DO 20 I=1,NVP
20    READ (105,800)
      RETURN
C   ------------------------------------------------------------------
C     READ AND WRITE INTEGER DATA :                           ENTRY RWI
C   ------------------------------------------------------------------
      ENTRY RWI(IX)
      READ(105,100) LR14,IX
      WRITE(108,100) LR14,IX
```

```
      RETURN
C ----------------------------------------------------------------------
C     WRITE INTEGER DATA :                                      ENTRY WI
C ----------------------------------------------------------------------
      ENTRY WI(AR14,IX)
      WRITE(108,100) AR14,IX
      RETURN
C ----------------------------------------------------------------------
C     READ AND WRITE REAL DATA :                                ENTRY RWR
C ----------------------------------------------------------------------
      ENTRY RWR(RX)
      READ(105,200) LR14,RX
      WRITE(108,200) LR14,RX
      RETURN
C ----------------------------------------------------------------------
C     WRITE REAL DATA :                                         ENTRY WR
C ----------------------------------------------------------------------
      ENTRY WR(AR14,RX)
      WRITE(108,200) AR14,RX
      RETURN
C ----------------------------------------------------------------------
C     READ AND WRITE COMPLEX DATA :                             ENTRY RWC
C ----------------------------------------------------------------------
      ENTRY RWC(CX)
      READ(105,300) LR14,CX
      WRITE(108,300) LR14,CX
      RETURN
C ----------------------------------------------------------------------
C     WRITE COMPLEX DATA :                                      ENTRY WC
C ----------------------------------------------------------------------
      ENTRY WC(AR14,CX)
      WRITE(108,300) AR14,CX
      RETURN
C ----------------------------------------------------------------------
C     READ AND WRITE INTEGER VECTOR :                           ENTRY RWIV
C ----------------------------------------------------------------------
      ENTRY RWIV(NV,IV)
      N1=1
      N2=10
2000  READ (105,500) LR14,(IV(I),I=N1,N2)
      WRITE(108,500) LR14,(IV(I),I=N1,N2)
      N1=N1+10
      N2=N2+10
```

```
      IF(N1.GT.NV) RETURN
      IF(N2.GE.NV) N2=NV
      GOTO 2000
C ---------------------------------------------------------------------
C     READ AND WRITE(ONLY IF NV.GE.0) REAL VECTOR :                RWRV
C ---------------------------------------------------------------------
      ENTRY RWRV(NV,RV,RMAX)
      ANV=IABS(NV)
      READ (105,600) LR12,RMAX
      READ (105,400) LR20
      READ (105,700) (RV(I),I=1,ANV)
      IF(NV.LE.0) RETURN
C ---------------------------------------------------------------------
C     WRITE REAL VECTOR:                                       ENTRY WRV
C ---------------------------------------------------------------------
      ENTRY WRV(NV,RV,RMAX)
      ANV=IABS(NV)
      WRITE(108,600) LC12,RMAX
      WRITE(108,400) LC20
      WRITE(108,700) (RV(I),I=1,ANV)
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE FORMAT(LABEL,NX,CX,RX,IX,IFIELD)
C ======================================================================
C             STANDARDIZED OUTPUT PROGRAM FOR FULL-LINE COMMENTS
C                     AND INTEGER, REAL, AND COMPLEX ARRAYS
C
C              PROGRAM       :  FORMAT:S
C              ORIGINAL      :  APRI    15, 1978
C              REVISION      :  MARCH   17, 1981
C
C              PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                               RADAR SYSTEMS ANALYSIS GROUP
C                               MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                               TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     INPUT  :    LABEL      -  IDENTIFYING LABEL OR FULL LINE COMMENT
C                 CX         -  COMPLEX-VALUED ARRAY
C                 RX         -  REAL-VALUED ARRAY
C                 IX         -  INTEGER-VALUED ARRAY
C                 NX         -  ARRAY DIMENSIONALITY
C                 IFIELD     -  FIELD OPTION FOR FORMR AND FORMC
C                                 5  : PRINT WITH F10.5 FORMAT
C                                 1  : PRINT WITH F10.1 FORMAT
C ----------------------------------------------------------------------
C     ENTRY POINTS           :  FORMAT   -  FOR COMMENT LINE OF 0-120
C                                           CHARACTERS
C                               FORMI    -  FOR INTEGER TYPE
C                               FORMR    -  FOR REAL TYPE
C                               FORMC    -  FOR COMPLEX TYPE
C                               FORMG    -  FOR GENERAL TYPE WHERE
C                                           EXPONENTIAL NOTATION MAY
C                                           BE REQUIRED
C
C     SUBROUTINES CALLED     :  NONE
C
C ======================================================================
      COMPLEX CX
      DIMENSION LABEL(30),CX(1),RX(1),IX(1)
  100 FORMAT(30A4)
  200 FORMAT(7A4,8I10/(28X,8I10))
  300 FORMAT(/7A4/)
  400 FORMAT(7A4,8F10.5/(28X,8F10.5))
  500 FORMAT(7A4,8F10.1/(28X,8F10.1))
```

```
600   FORMAT(5A4,8G11.5/(20X,8G11.5))
C ------------------------------------------------------------------
C     PRINT A LINE OF COMMENT OF 0 TO 120 CHARACTERS :     ENTRY FORMAT
C ------------------------------------------------------------------
      NB=NX/4
      WRITE(108,100) (LABEL(I),I=1,NB)
      RETURN
C ------------------------------------------------------------------
C     PRINT INTEGER ARRAY :                                 ENTRY FORMI
C ------------------------------------------------------------------
      ENTPY FORMI(LABEL,IX,NX,ISKIP)
      WRITE(108,200) (LABEL(I),I=1,7),(IX(I),I=1,NX)
      RETURN
C ------------------------------------------------------------------
C     PRINT REAL ARRAY :                                    ENTRY FORMR
C ------------------------------------------------------------------
      ENTRY FORMR(LABEL,RX,NX,IFIELD)
      IF(IFIELD.EQ.1) WRITE(108,500) (LABEL(I),I=1,7),(RX(I),I=1,NX)
      IF(IFIELD.NE.1) WRITE(108,400) (LABEL(I),I=1,7),(RX(I),I=1,NX)
      RETURN
C ------------------------------------------------------------------
C     PRINT COMPLEX ARRAY :                                 ENTRY FORMC
C ------------------------------------------------------------------
      ENTRY FORMC(LABEL,CX,NX,IFIELD)
      IF(IFIELD.EQ.1) WRITE(108,500) (LABEL(I),I=1,7),(CX(I),I=1,NX)
      IF(IFIELD.NE.1) WRITE(108,400) (LABEL(I),I=1,7),(CX(I),I=1,NX)
      RETURN
C ------------------------------------------------------------------
C     PRINT ANY DATA TYPE WHEN EXPONENTIAL NOTATION MAY BE REQUIRED.
C     FOR COMPLEX DATA USE NX=2*NX :                        ENTRY FORMG
C ------------------------------------------------------------------
      ENTRY FORMG(LABEL,RX,NX,IFIELD)
      WRITE(108,600) (LABEL(I),I=1,5),(RX(I),I=1,NX)
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE BCRMEXEC
C ======================================================================
C                         BCR EMULATION EXECUTION PROGRAM
C
C                PROGRAM        :  BCRMEXEC:S
C                ORIGINAL       :  JANUARY 15, 1981
C                REVISION       :  MARCH   19, 1981
C
C                PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                  RADAR SYSTEMS ANALYSIS GROUP
C                                  MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                  TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C       ENTRY POINTS            :  BCRMEXEC
C
C       PROGRAMS CALLED         :  SUBROUTINE   FUNCTION     ENTRY
C                                  ----------   --------     -----
C                                  AGC             -         AGC
C                                  CBEML           -         CBEML
C                                  BCREML          -         BCREML
C                                  CSEML           -         CSEML
C ======================================================================
      CALL AGC
      CALL CBEML
      CALL BCREML
      CALL CSEML
      RETURN
      END
```

```
C ==================================================================
      SUBROUTINE AGC
C ==================================================================
C           AGC OF MAIN AND AUXILIARY SIGNALS BEFORE A/D CONVERSION
C
C                 PROGRAM         :  AGCIS
C                 ORIGINAL        :  FEBRUARY  2, 1981
C                 REVISION        :  MARCH    25, 1981
C
C                 PREPARED  BY    :  S. M. DANIEL  &  I. KERTESZ
C                                    RADAR SYSTEMS ANALYSIS GROUP
C                                    MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                    TEMPE, ARIZONA  85282
C ------------------------------------------------------------------
C     INPUT  :      NBITS0     -  NUMBER OF BITS IN S0
C                   NBITS1     -  NUMBER OF BITS IN S1
C                   NS         -  NUMBER OF SAMPLES
C                   NSX        -  MAXIMUM NUMBER OF SAMPLES IN S0
C                   S0         -  MAIN PORT SIGNAL VECTOR
C                   S1         -  AUXILIARY PORT SIGNAL VECTORS
C                   S0MAX      -  LARGEST ELEMENT OF S0
C                   S1MAX      -  LARGEST ELEMENT OF S1
C                   IWS0F      -  PRINT OPTION FOR S0 (1 FOR PRINT)
C                   IWS1F      -  PRINT OPTION FOR S1 (1 FOR PRINT)
C
C     OUTPUT :                 -  SCALED VERSIONS OF S0 AND S1
C                                 MAIN AND AUXILIARY SIGNALS
C ------------------------------------------------------------------
C      ENTRY POINTS            :  AGC
C
C      PROGRAMS CALLED         :  SUBROUTINE   FUNCTION     ENTRY
C                                 ----------   --------     -----
C                                 SCATD        -            SCATU
C                                 FORMAT       -            FORMAT
C                                              -            FORMI
C                                              -            FORMR
C                                              -            FORMC
C ==================================================================
      COMPLEX S0,S1
      COMMON /BCR0/ IWS0,IWS1,IWS0F,IWS1F,IWCB,IDP(20)
      COMMON /BCR1/ S0(256),S1(5120),NSX,NS,NBITS0,NBITS1,S0MAX,S1MAX
      COMMON /BCR2/ ISW(20),NWX,IW0,NPM1,ITERX1,NBITSP,NBITSW
     -,KM,KM0,KD,KT1,KT3,LIM1,LIM2,LRRT
```

```
      DIMENSION PAGE(30),LINE(30)
      DATA LINE/' ---',29*'----'/,PAGE/'1---',29*'----'/
C ---------------------------------------------------------------
C     DENORMALIZE S0 AND S1, THEN SCALE
C ---------------------------------------------------------------
      NS2=NS*2
      DO 10 IS=1,NS
10    S0(IS)=S0(IS)/S0MAX
      CALL SCATU(NS2,NBITS0,S0)
      IF(IWS0F.NE.1) GOTO 1000
      CALL FORMAT(PAGE,108)
      CALL FORMAT('  MAIN PORT SIGNAL            ',28)
      CALL FORMAT(LINE,108)
      CALL FORMAT('   PORT                  :          0  ',40)
      CALL FORMR('   S0MAX                :',S0MAX,1,5)
      CALL FORMI('   NBS0                 :',NBITS0+1,1,0)
      CALL FORMC('   S0                   :',S0,NS,1)
      CALL FORMAT(LINE,108)
1000  NC=1
      DO 30 IP=1,NPM1
      NCP=NC+NSX-1
      DO 20 IS=NC,NCP
20    S1(IS)=S1(IS)/S1MAX
      CALL SCATU(NS2,NBITS1,S1(NC))
      IF(IWS1F.NE.1) GOTO 30
      CALL FORMAT(PAGE,108)
      CALL FORMAT('  AUXILIARY PORT SIGNAL       ',28)
      CALL FORMAT(LINE,108)
      CALL FORMI('   PORT                 :',IDP(IP),1,0)
      CALL FORMR('   S1MAX                :',S1MAX,1,5)
      CALL FORMI('   NBS1                 :',NBITS1+1,1,0)
      CALL FORMC('   S1(IP)               :',S1(NC),NS,1)
      CALL FORMAT(LINE,108)
30    NC=NCP+1
      RETURN
      END
```

```
C =====================================================================
      SUBROUTINE SCATD(NX,NS,X)
C =====================================================================
C           BIT-SHIFT AND TRUNCATION OF REAL OR COMPLEX VECTOR
C
C              PROGRAM          :  SCATDIS
C              ORIGINAL         :  AUGUST  15  1979
C              REVISION         :  MARCH   17, 1981
C
C              PREPARED  BY     :  S. M. DANIEL  &  I. KERTESZ
C                                  RADAR SYSTEMS ANALYSIS GROUP
C                                  MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                  TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C     INPUT  :    X            -   INPUT AND OUTPUT VECTOR TO BE SHIFTED
C                                  AND TRUNCATED
C                 NX           -   DIMENSIONALITY OF X
C                 NS           -   NUMBER OF BIT SHIFTS
C ---------------------------------------------------------------------
C      ENTRY POINTS            :   SCATD
C                                  SCATU
C
C      PROGRAMS CALLED         :   NONE
C =====================================================================
      DIMENSION X(1)
C ---------------------------------------------------------------------
C     PROVIDE STRICT DOWN-SHIFT AND TRUNCATION:              ENTRY SCATU
C ---------------------------------------------------------------------
      IDN=2**(-NS)
      DO 10 I=1,NX
      IX=X(I)
      IXD=X(I)/IDN
      IF(IXD.GE.0) GOTO 10
      IXS=IXD*IDN
      IF(IXS.EQ.IX) GOTO 10
      IXD=IXD-1
   10 X(I)=IXD
      RETURN
C ---------------------------------------------------------------------
C     PROVIDE UP-SHIFT AND TRUNCATION:                       ENTRY SCATU
C ---------------------------------------------------------------------
      ENTRY SCATU(NX,NS,X)
      IUP=2**NS
```

```
      DO 20 I=1,NX
      IX=X(I)*IUP
   20 X(I)=IX
      RETURN
      END
```

```
C =====================================================================
      SUBROUTINE CBEML
C =====================================================================
C                         COMPUTATION AND SCALING
C                                   OF
C           BATCH COVARIANCE MATRIX AND CROSS-CORRELATION VECTOR
C
C             PROGRAM        :  CBEMLIS
C             ORIGINAL       :  AUGUST 15, 1979
C             REVISION       :  MARCH  25, 1981
C
C             PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                               RADAR SYSTEMS ANALYSIS GROUP
C                               MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                               TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C       INPUT    :   S0        -  MAIN PORT SIGNAL VECTOR
C                    S1        -  ARRAY OF AUXILIARY PORT SIGNALS
C                    NS        -  NUMBER OF SIGNAL SAMPLES
C                    NPM1      -  NUMBER OF AUXILIARY PORTS
C                    NBITSP    -  NUMBER OF BITS IN THE PROCESSOR WORD
C                    IWCB      -  PRINT OPTION FOR C AND B (1 TO PRINT)
C
C       OUTPUT   :   C         -  BATCH COVARIANCE OF THE AUXILIARY PORT
C                                 SIGNALS
C                    B         -  FORCING VECTOR, THE CROSS CORRELATION
C                                 VECTOR OF MAIN PORT SIGNAL AND THE AUX-
C                                 ILIARY PORT SIGNAL VECTORS
C                    KC        -  NUMBER OF BITS BY WHICH C WAS SHIFTED
C                    KB        -  NUMBER OF BITS BY WHICH B WAS SHIFTED
C ---------------------------------------------------------------------
C       ENTRY POINTS           :  CBEML
C
C       PROGRAMS CALLED        :  SUBROUTINE   FUNCTION     ENTRY
C                                 ----------   --------     -----
C                                 SCALE           -         SCALE
C                                 FORMAT          -         FORMAT
C                                                 -         FORMI
C                                                 -         FORMC
C =====================================================================
      COMPLEX S0,S1,C,B,R,P,CP,W,X,ACP,AP,BP,CS1
      COMMON /BCR0/ IWS0,IWS1,IWS0F,IWS1F,IWCB,IDP(20)
      COMMON /BCR1/ S0(256),S1(256,20),NSX,NS,NBITS0,NBITS1,S0MAX,S1MAX
```

```
      COMMON /BCR2/ ISW(20),NWX,IWO,NPM1,ITERX,NBITSP,NBITSW
     -,KM,KMO,KD,KT1,KT3,LIM1,LIM2,LRRT
      COMMON /BCR3/ C(20,20),B(20),R(20),P(20),CP(20),X(400)
     -,W(20),ACP(20),AP(20),BP(20),KC,KB,LW
      DIMENSION LINE(30),PAGE(30)
      DATA LINE/' ---',29*'----'/,PAGE/'1---',29*'----'/
C ---------------------------------------------------------------------
C     INITIALIZATION
C ---------------------------------------------------------------------
      DO 20 I=1,NPM1
      B(I)=0
      DO 20 J=1,NPM1
20    C(I,J)=0
C ---------------------------------------------------------------------
C     COMPUTE C AND B
C ---------------------------------------------------------------------
      DO 40 IS=1,NS
      DO 40 IP=1,NPM1
      CS1=CONJG(S1(IS,IP))
      B(IP)=B(IP)+CS1*S0(IS)
      DO 40 JP=1,NPM1
      C(IP,JP)=C(IP,JP)+CS1*S1(IS,JP)
40    CONTINUE
      CALL SCALE(B,NPM1*2,NBITSP,KB)
C ---------------------------------------------------------------------
C     SCALE C AND B
C ---------------------------------------------------------------------
      NX=0
      DO 50 I=1,NPM1
      DO 50 J=1,NPM1
      NX=NX+1
50    X(NX)=C(I,J)
      CALL SCALE(X,NX*2,NBITSP,KC)
      NX=0
      DO 60 I=1,NPM1
      DO 60 J=1,NPM1
      NX=NX+1
60    C(I,J)=X(NX)
C ---------------------------------------------------------------------
C     PRINT C AND B IF INDICATED
C ---------------------------------------------------------------------
      IF(IWCB.NE.1) RETURN
      CALL FORMAT(PAGE,108)
```

```
      CALL FORMAT('  BATCH COVARIANCE MATRIX    ',28)
      CALL FORMAT(LINE,108)
      CALL FORMI('   NBITSP                    !',NBITSP+1,1,0)
      CALL FORMI('   KC                        !',KC,1,0)
      CALL FORMC('   C                         !',X,NX,1)
      CALL FORMAT(LINE,108)
C -----------------------------------------------------------------
      CALL FORMAT('  FORCING VECTOR             ',28)
      CALL FORMAT(LINE,108)
      CALL FORMI('   NBITSP                    !',NBITSP+1,1,0)
      CALL FORMI('   KB                        !',KB,1,0)
      CALL FORMC('   B                         !',B,NPM1,1)
      CALL FORMAT(LINE,108)
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE SCALE(X,NX,NB,NS)
C ======================================================================
C           SCALING AND TRUNCATION OF REAL OR COMPLEX VECTOR
C                         TO GIVEN WORD-LENGTH
C                         WITH COMMON BIT-SHIFT
C                               SO THAT
C              LARGEST-MAGNITUDE WORD IS LEFT-JUSTIFIED
C
C             PROGRAM         :  SCALE:S
C             ORIGINAL        :  AUGUST  15, 1979
C             REVISION        :  MARCH   17, 1981
C
C             PREPARED  BY    :  S. M. DANIEL  &  I. KERTESZ
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     INPUT  :     X          -  VECTOR TO BE SCALED
C                  NX         -  NUMBER OF ELEMENTS IN X
C                  NB         -  NUMBER OF BITS IN THE INTEGER WORD
C                                WHICH THE OUTPUT SIMULATES
C                  .
C     OUTPUT :     X          -  X SCALED AND TRUNCATED
C                  NS         -  EXPONENT OF TWO USED IN SCALING
C ----------------------------------------------------------------------
C     ENTRY POINTS            :  SCALE
C
C     PROGRAMS CALLED         :  SUBROUTINE   FUNCTION    ENTRY
C                                ----------   --------    -----
C                                -            BIGSGN      BIGSGN
C                                SCATD        -           SCATD
C                                             -           SCATU
C ======================================================================
      DIMENSION X(1)
C ----------------------------------------------------------------------
C     FIND THE SIGN AND MAGNITUDE OF THE LARGEST ELEMENT OF X
C ----------------------------------------------------------------------
      NS=0
      NB10=NB*10
      XMAX=BIGSGN(NX,X)
      IABMAX=IABS(XMAX)
      MAXI=2**NB
```

```
C ------------------------------------------------------------------
C     SELECT DIRECTION OF THE BIT SHIFT
C ------------------------------------------------------------------
      IF(IABMAX-MAXI) 1000,3000,2000
C ------------------------------------------------------------------
C     SHIFT UP
C ------------------------------------------------------------------
1000  DO 10 I=1,NB10
      IARMAX=IARMAX*2
      IF(IABMAX.GT.MAXI) GOTO 3000
10    NS=NS+1
C ------------------------------------------------------------------
C     SHIFT DOWN
C ------------------------------------------------------------------
2000  DO 20 I=1,NR
      IARMAX=IARMAX/2
      NS=NS-1
20    IF(IARMAX.LE.MAXI) GO TO 3000
C ------------------------------------------------------------------
C     SCALE AND TRUNCATE X
C ------------------------------------------------------------------
3000  IF(NS.LT.0) CALL SCATD(NX,NS,X)
      IF(NS.GE.0) CALL SCATU(NX,NS,X)
      RETURN
      END
```

```
C =====================================================================
      FUNCTION BIGSGN(NX,X)
C =====================================================================
C          THIS FUNCTION WILL SEARCH A REAL OR COMPLEX VECTOR TO FIND
C            THE ELEMENT WHICH HAS THE LARGEST ABSOLUTE VALUE. THAT
C                   VALUE AND SIGN ARE RETURNED IN BIGSGN
C
C              PROGRAM          :  BIGSGN:S
C              ORIGINAL         :  JANUARY 27, 1981
C              REVISION         :  MARCH   11, 1981
C
C              PREPARED  BY     :  S. M. DANIEL  &  I. KERTESZ
C                                  RADAR SYSTEMS ANALYSIS GROUP
C                                  MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                  TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C     INPUT  :     X           -   VECTOR TO BE SEARCHED
C                  NX          -   NUMBER OF ELEMENTS IN X
C
C     OUTPUT :     BIGSGN      -   SIGN AND MAGNITUDE OF THE LARGEST
C                                  ELEMENT IN X
C ---------------------------------------------------------------------
C     ENTRY POINTS              :  BIGSGN
C
C     PROGRAMS CALLED           :  NONE
C =====================================================================
      DIMENSION X(1)
C ---------------------------------------------------------------------
C     FIND THE SIGN AND MAGNITUDE OF THE LARGEST ELEMENT OF X
C ---------------------------------------------------------------------
      ABMAX=0
      XMAX=0.
      DO 10 I=1,NX
      ABX=ABS(X(I))
      IF(ABMAX.GT.ABX) GOTO 10
      ABMAX=ABX
      XMAX=X(I)
   10 CONTINUE
      BIGSGN=XMAX
      RETURN
      END
```

```
C =================================================================
      SUBROUTINE BCREML
C =================================================================
C                     ESTIMATION OF THE ADAPTIVE WEIGHT VECTOR
C                                VIA BCR EMULATION
C
C                 PROGRAM        :  BCREML:S
C                 ORIGINAL       :  AUGUST  15, 1979
C                 REVISION       :  MARCH   25, 1981
C
C                 PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                   RADAR SYSTEMS ANALYSIS GROUP
C                                   MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                   TEMPE, ARIZONA  85282
C -----------------------------------------------------------------
C     INPUT          C           -  COVARIANCE OF AUXILIARY SIGNAL VECTOR
C                    B           -  CROSS-CORRELATION VECTOR BETWEEN
C                                   MAIN AND AUXILIARY SIGNALS
C                    NSX         -  MAXIMUM NUMBER OF SIGNAL SAMPLES
C                    NS          -  ACTUAL NUMBER OF SIGNAL SAMPLES
C                    NWX         -  MAXIMUM NUMBER OF WEIGHTS
C                    NPM1        -  NUMBER OF AUXILIARY PORTS
C                    KC          -  NUMBER OF BITS BY WHICH C WAS SHIFTED
C                    KB          -  NUMBER OF BITS BY WHICH B WAS SHIFTED
C                    NBITSP      -  NUMBER OF BITS IN THE PROCESSOR WORD
C                    NBITSW      -  NUMBER OF BITS IN A WORD OF W
C                    KM          -  MULTIPLIER TRUNCATION WITH OVERFLOW
C                    KM0         -  MULTIPLIER TRUNCATION WITHOUT OVERFLOW
C                    KD          -  DIVISION TABLE BIT-SHIFT
C                    KT1         -  1 - BIT TRUNCATIONS KR0,KP0,KW0
C                    KT3         -  3 - BIT TRUNCATIONS KRR0,KCP0,KPCP0
C                    LIM1        -  MAXIMUM ALLOWABLE DOWN-SHIFT
C                    LIM2        -  MAXIMUM ALLOWABLE UP-SHIFT
C                    LRRT        -  ALLOWED GLOBAL SCALE REDUCTION OF LRR
C                    ITERX       -  MAXIMUM NUMBER OF ITERATIONS
C
C     OUTPUT :       W           -  WEIGHTING VECTOR
C -----------------------------------------------------------------
C     ENTRY POINTS               :  BCREML
C
C     PROGRAMS CALLED            :  SUBROUTINE   FUNCTION    ENTRY
C                                   ----------   --------    -----
C                                   FORMAT       -           FORMI
```

```
C                                              -            FORMR
C                                              -            FORMC
C                                   SCALE      -            SCALE
C                                   ADDTB      -            ADDTB
C                                   LIMIT      -            LIMIT
C                                   UPDATE     -            UPDATE
C                                   -          ROUNDR       ROUNDR
C =====================================================================
      COMPLEX S0,S1,C,B,R,P,CP,W,X,ACP,AP,BP,CPCP,CRDR
      COMMON /BCR1/ S0(256),S1(256,20),NSX,NS,NBITS0,NBITS1,S0MAX,S1MAX
      COMMON /BCR2/ ISW(20),NWX,IW0,NPM1,ITERX,NBITSP,NBITSW
     -,KM,KM0,KD,KT1,KT3,LIM1,LIM2,LRRT
      COMMON /BCR3/ C(20,20),B(20),R(20),P(20),CP(20),X(400)
     -,W(20),ACP(20),AP(20),BP(20),KC,KB,LW
      DIMENSION LINE(30),PAGE(30)
      DATA LINE/' ---',29*'----'/, PAGE/'1---',29*'----'/
C ---------------------------------------------------------------------
C     INITIALIZATION
C ---------------------------------------------------------------------
      KWF=KB-KC
      NBW=NBITSW+1
      NBP=NBITSP+1
      KPW=NBW-NBP
      KR0=KT1
      KP0=KT1
      KW0=KT1
      KRR0=KT3
      KCP0=KT3
      KPCP0=KT3
      KS01=NBITS0-NBITS1
      IDOT=2**KM0
      INV=2**KD
      NPM12=2*NPM1
      NPSQ=NPM1**2
      LR=0
      LP=0
      ITER=0
      CALL FORMAT(PAGE,108)
      CALL FORMAT('                                   ADAPTIVE FIXED-POINT
     -ARITHMETIC OF BCR PROCESSOR',80)
      CALL FORMAT(LINE,108)
      CALL FORMC('   C                      ',X,NPSQ,1)
      CALL FORMI('   KC                     ',KC,1,0)
```

```
      CALL FORMC('   B                        !',B,NPM1,1)
      CALL FORMI('   KB                       !',KB,1,0)
      CALL FORMI('   KWF                      !',KWF,1,0)
      CALL FORMAT(LINE,108)
      CALL FORMAT('  INITIALIZATION',16)
      CALL FORMAT(LINE,108)
      DO 10 I=1,NPM1
      W(I)=0
      P(I)=B(I)
10    R(I)=B(I)
      CALL FORMC('   W                        !',W,NPM1,1)
      CALL FORMC('   P = R = B                !',W,0,1)
C ---------------------------------------------------------------
C     COMPUTE METRIC OF R
C ---------------------------------------------------------------
      CALL ADOTB(R,R,NPM1,CRDR,KM,0)
      CALL FORMC('   CRDR                     !',CRDR,1,1)
      RDR=REAL(CRDR)
      CALL SCALE(RDR,1,NBITSP,KRR)
      LRR=2*LR+KRR-KM
      KRRH=KRR+KRR0
      LRR0=LRR
      CALL FORMR('   RDR                      !',RDR,1,1)
      CALL FORMI('   KRRH                     !',KRRH,1,0)
      CALL FORMI('   KRR = KRRH - KRR0        !',KRR,1,0)
      CALL FORMI('   LRR = 2*LR + KRR - KM    !',LRR,1,0)
      CALL FORMAT(LINE,108)
C ---------------------------------------------------------------
C     BEGINNING OF ITERATION
C ---------------------------------------------------------------
1000  ITER=ITER+1
      CALL FORMAT(PAGE,108)
      CALL FORMI('   ITERATION  #             !',ITER,1,0)
      CALL FORMAT(LINE,108)
C ---------------------------------------------------------------
C     COMPUTATION OF CP
C ---------------------------------------------------------------
      DO 20 I=1,NPM1
20    CALL ADOTB(C(1,I),P,NPM1,CP(I),KM,0)
      CALL FORMC('   CP                       !',CP,NPM1,1)
      CALL SCALE(CP,NPM12,NBITSP,KCP)
      CALL LIMIT(CP,NPM12,KCP,KCP0,LIM1,LIM2)
      LCP=LP+KCP-KM
```

```
      KCPH=KCP+KCP0
      CALL FORMC('   CP LIMITED              !',CP,NPM1,1)
      CALL FORMI('   KCPH                    !',KCPH,1,0)
      CALL FORMI('   KCP = KCPH - KCP0       !',KCP,1,0)
      CALL FORMI('   LCP = LP + KCP - KM     !',LCP,1,0)
C ------------------------------------------------------------------
C     COMPUTATION OF PCP
C ------------------------------------------------------------------
      CALL ADOTB(P,CP,NPM1,CPCP,KM,0)
      CALL FORMC('   CPCP                    !',CPCP,1,1)
      PCP=REAL(CPCP)
      CALL SCALE(PCP,1,NBITSP,KPCP)
      CALL LIMIT(PCP,1,KPCP,KPCP0,0,KM0)
      LPCP=LP+LCP+KPCP-KM
      KPCPH=KPCP+KPCP0
      CALL FORMR('   PCP                     !',PCP,1,1)
      CALL FORMI('   KPCPH                   !',KPCPH,1,0)
      IF(KPCPH.LT.KM0) GOTO 2000
      CALL FORMAT(LINE,108)
      CALL FORMAT('   ***************************',28)
      CALL FORMAT('   *    SYSTEM INTERRUPT    *',28)
      CALL FORMAT('   *    DIVIDE PROTECTION   *',28)
      CALL FORMAT('   ***************************',28)
      CALL FORMI('   KPCPH                   !',KPCPH,1,0)
      CALL FORMI('   KM0                     !',KM0,1,0)
      CALL FORMAT(LINE,108)
      GO TO 5000
2000  CALL FORMI('   KPCP = KPCPH - KPCP0    !',KPCP,1,0)
      CALL FORMI('   LPCP=LP+LCP+KPCP-KM     !',LPCP,1,0)
C ------------------------------------------------------------------
C     COMPUTATION OF ALPHA
C ------------------------------------------------------------------
      IPCP=INV/PCP+.5
      PCPI=IPCP
      LPCPI=KD-LPCP
      CALL FORMR('   PCPI                    !',PCPI,1,1)
      CALL FORMI('   KPCPI                   !',KPCPI,1,0)
      CALL FORMI('   LPCPI = KD - LPCP       !',LPCPI,1,0)
      ALPHA=ROUNDR(RDR/IDOT,PCPI)
      CALL FORMR('   ALPHA                   !',ALPHA,1,1)
      CALL SCALF(ALPHA,1,NBITSP,KAL)
      LAL=LPR+LPCPI+KAL-KM0
      CALL FORMR('   ALPHA SCALED            !',ALPHA,1,1)
```

```
      CALL FORMI(:   KAL                     !',KAL,1,0)
      CALL FORMI('   LAL=LRR+LPCPI+KAL-KM0   !',LAL,1,0)
C ------------------------------------------------------------------
C     COMPUTATION OF AP AND ACP
C ------------------------------------------------------------------
      DO 30 I=1,NPM1
      ALPI=ALPHA/IDOT
      AP(I)=CMPLX(ROUNDR(ALPI,REAL(P(I))),ROUNDR(ALPI,AIMAG(P(I))))
   30 ACP(I)=CMPLX(ROUNDR(ALPI,REAL(CP(I))),ROUNDR(ALPI,AIMAG(CP(I))))
      CALL FORMC('   AP                      !',AP,NPM1,1)
      LAP=LAL+LP-KM0
      IF(ITER .EQ. 1) LW = LAP
      CALL FORMI('   LAP = LAL + LP - KM0    !',LAP,1,0)
C ------------------------------------------------------------------
C     UPDATE W
C ------------------------------------------------------------------
      CALL UPDATE(W,AP,W,NPM1,-1,KW,KW0,LW,LAP,LW,LIM1,NBP,IGO)
      KWH=KW+KW0
      IF(IGO.EQ.0) GOTO 5000
      CALL FORMC('   W                       !',W,NPM1,1)
      CALL SCALE(W,NPM12,NBITSP,KW)
      CALL LIMIT(W,NPM12,KW,KW0,LIM1,LIM2)
      LW=LW+KW
      KWH=KW+KW0
      CALL FORMC('   W SCALED                !',W,NPM1,1)
      CALL FORMI('   KWH                     !',KWH,1,0)
      CALL FORMI('   KW = KWH - KW0          !',KW,1,0)
      CALL FORMI('   LW = LW + KW            !',LW,1,0)
      CALL FORMC('   ACP                     !',ACP,NPM1,1)
      LACP=LAL+LCP-KM0
      CALL FORMI('   LACP = LAL + LCP - KM0  !',LACP,1,0)
C ------------------------------------------------------------------
C     UPDATF R
C ------------------------------------------------------------------
      IRDR=INV/RDR+0.5
      RDRI=IRDR
      LRRI=KD-LRR
      CALL FORMR('   RDRI                    !',RDRI,1,1)
      CALL FORMI('   LRRI = KD - LRR         !',LRRI,1,0)
      CALL UPDATE(R,ACP,R,NPM1,-1,KR,KR0,LR,LACP,LR,LIM1,NBP,IGO)
      KRH= KR+KR0
      IF(IGO.EQ.0) GOTO 5000
      CALL FORMC('   R                       !',R,NPM1,1)
```

```
      CALL SCALE(R,NPM12,NBITSP,KR)
      CALL LIMIT(R,NPM12,KR,KR0,LIM1,LIM2)
      LR=LR+KR
      KRH=KR+KR0
      CALL FORMC('   R SCALED                  !',R,NPM1,1)
      CALL FORMI('   KRH                       !',KRH,1,0)
      CALL FORMI('   KR = KRH - KR0            !',KR,1,0)
      CALL FORMI('   LR = LR + KR              !',LR,1,0)
C ----------------------------------------------------------------
C     COMPUTE METRIC OF R
C ----------------------------------------------------------------
      CALL ADOTB(R,R,NPM1,CRDR,KM,0)
      CALL FORMC('   CRDR                      !',CRDR,1,1)
      RDR=REAL(CRDR)
      CALL SCALE(RDR,1,NBITSP,KRR)
      CALL LIMIT(RDR,1,KRR,KRR0,0,KM0)
      LRR=2*LR+KRR-KM
      KRRH=KRR+KRR0
      LRRD=LRR-LRR0
      CALL FORMR('   RDR                       !',RDR,1,1)
      CALL FORMI('   KRRH                      !',KRRH,1,0)
      CALL FORMI('   KRR = KRRH - KRR0         !',KRR,1,0)
      CALL FORMI('   LRR = 2*LR + KRR - KM     !',LRR,1,0)
      CALL FORMI('   LRRD = LRR - LRR0         !',LRRD,1,0)
      IF(KRRH.LT.KM0) GOTO 3000
      CALL FORMAT(LINE,108)
      CALL FORMAT('  ***************************',28)
      CALL FORMAT('  *    SYSTEM INTERRUPT    *',28)
      CALL FORMAT('  *    DIVIDE PROTECTION   *',28)
      CALL FORMAT('  ***************************',28)
      CALL FORMI('   KRRH                      !',KRRH,1,0)
      CALL FORMI('   KM0                       !',KM0,1,0)
      CALL FORMAT(LINE,108)
      GOTO 5000
 3000 IF(LRRD.LT.LRRT) GOTO 4000
      CALL FORMAT(LINE,108)
      CALL FORMAT('  ***************************',28)
      CALL FORMAT('  *    SYSTEM INTERRUPT    *',28)
      CALL FORMAT('  *  GRADIENT CONVERGENCE  *',28)
      CALL FORMAT('  ***************************',28)
      CALL FORMI('   LRRD                      !',LRRD,1,0)
      CALL FORMAT(LINE,108)
      GOTO 5000
```

```
C ------------------------------------------------------------------
C     COMPUTE BETA
C ------------------------------------------------------------------
4000  BETA=ROUNDR(RDR/IDOT,RDRI)
      CALL FORMR('   BETA                      $',BETA,1,1)
      CALL SCALE(BETA,1,NBITSP,KBET)
      LBET=LRR+LRRI+KBET-KM0
      CALL FORMR('   BETA SCALED               $',BETA,1,1)
      CALL FORMI('   KBET                      $',KBET,1,0)
      CALL FORMI('   LBET=LRR+LRRI+KBET-KM0    $',LBET,1,0)
C ------------------------------------------------------------------
C     COMPUTE BP
C ------------------------------------------------------------------
      DO 40 I=1,NPM1
      BETAI=BETA/IDOT
40    BP(I)=CMPLX(ROUNDR(BETAI,REAL(P(I))),ROUNDR(BETAI,AIMAG(P(I))))
      CALL FORMC('   BP                        $',BP,NPM1,1)
      LBP=LBET+LP+KBP-KM0
      CALL FORMI('   LBP = LBET + LP - KM0     $',LBP,1,0)
C ------------------------------------------------------------------
C     UPDATE P
C ------------------------------------------------------------------
      CALL UPDATE(R,BP,P,NPM1,1,KR,KR0,LR,LBP,LP,LIM1,NBP,IGO)
      KPH=KP+KP0
      IF(IGO.EQ.0) GOTO 5000
      CALL FORMC('   P                         $',P,NPM1,1)
      CALL SCALE(P,NPM12,NBITSP,KP)
      CALL LIMIT(P,NPM12,KP,KP0,LIM1,LIM2)
      LP=LP+KP
      KPH=KP+KP0
      CALL FORMC('   P SCALED                  $',P,NPM1,1)
      CALL FORMI('   KPH                       $',KPH,1,0)
      CALL FORMI('   KP = KPH - KP0            $',KP,1,0)
      CALL FORMI('   LP = LP + KP              $',LP,1,0)
      CALL FORMAT(LINE,108)
      IF(ITER.LT.ITERX) GOTO 1000
      CALL FORMAT('   **************************',28)
      CALL FORMAT('   *     SYSTEM INTERRUPT    *',28)
      CALL FORMAT('   *       NORMAL EXIT       *',28)
      CALL FORMAT('   **************************',28)
      CALL FORMI('   ITERATION  #              $',ITER,1,0)
C ------------------------------------------------------------------
C     TERMINATION
```

```
C   ----------------------------------------------------------------------
5000  CALL FORMAT(PAGE,108)
      LW=LW+KWF+KPW
      CALL SCATD(NPM1*2,KPW,W)
      CALL FORMI('  FINAL WEIGHT VECTOR       ',I,0,0)
      CALL FORMAT(LINE,108)
      CALL FORMI('   NBW                     !',NBW,1,0)
      CALL FORMI('   KPW = NBW - NBP         !',KPW,1,0)
      CALL FORMI('   LW = LW + KWF + KPW     !',LW,1,0)
      CALL FORMC('   W  (TO COMBINER)        !',W,NPM1,1)
      FACTOR=2.0**(-LW-KS01)
      DO 60 I=1,NPM1
60    X(I)=W(I)*FACTOR
      CALL FORMC('   W  (FLOATING-POINT)     !',X,NPM1,0)
      CALL FORMAT(LINE,108)
      RETURN
      END
```

```
C =========================================================================
      SUBROUTINE ADOTB(A,B,N,ADB,KM,IC)
C =========================================================================
C          TWO'S COMPLEMENT DOT-PRODUCT OF COMPLEX VECTORS A AND B
C                        WITH CONJUGATE OPTION ON A
C                      INCLUDING MULTIPLIER TRUNCATION
C
C              PROGRAM        :  ADOTB:S
C              ORIGINAL       :  SEPTEMBER 15, 1979
C              REVISION       :  MARCH     19, 1981
C
C              PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C -------------------------------------------------------------------------
C     INPUT  :    A          -   VECTOR TO BE MULTIPLIED BY B
C                 B          -   VECTOR TO MULTIPLY A
C                 N          -   NUMBER OF ELEMENTS IN A AND B
C                 IC         -   CONJUGATE SELECTOR FOR VECTOR A
C                                 0     - USE CONJUGATE OF A
C                                 OTHER - USE A AS IS
C                 KM         -   MULTIPLIER TRUNCATION IN BITS
C
C     OUTPUT :    ADB        -   DOT PRODUCT OF SCALED A AND B
C -------------------------------------------------------------------------
C     ENTRY POINTS            :  ADOTB
C
C     PROGRAMS CALLED         :  SUBROUTINE   FUNCTION    ENTRY
C                                ----------   --------    -----
C                                -            ROUNDC      ROUNDC
C =========================================================================
      COMPLEX A,AI,B,ADB,ROUNDC
      DIMENSION A(1),B(1)
      ADB=0
      IDOT=2**KM
      DO 10 I=1,N
      AI=A(I)/IDOT
      IF(IC.EQ.0) AI=CONJG(AI)
   10 ADB=ADB+ROUNDC(AI,B(I))
      RETURN
      END
```

```
C ======================================================================
      FUNCTION ROUNDC(X,Y)
C ======================================================================
C  THIS  FUNCTION WILL MULTIPLY THE COMPLEX ARGUMENTS AND ROUND OFF
C      THE RESULT TO THE NEAREST TWO'S COMPLEMENT INTEGER
C
C           PROGRAM       :  ROUNDC:S
C           ORIGINAL      :  SEPTEMBER 15, 1979
C           REVISION      :  MARCH     11, 1981
C
C           PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                            RADAR SYSTEMS ANALYSIS GROUP
C                            MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                            TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     INPUT  :     X         -  COMPLEX NUMBER TO BE MULTIPLIED BY Y
C                  Y         -  COMPLEX NUMBER TO MULTIPLY X
C
C
C     OUTPUT :     ROUNDC    -  COMPLEX ROUNDED PRODUCT OF X AND Y
C ----------------------------------------------------------------------
C     ENTRY POINTS        :  ROUNDC
C
C     PROGRAMS CALLED     :  SUBROUTINE   FUNCTION    ENTRY
C                            ----------   --------    -----
C                            -            ROUNDR      ROUNDR
C ======================================================================
      COMPLEX X,Y,ROUNDC
      A=REAL(X)
      B=AIMAG(X)
      C=REAL(Y)
      D=AIMAG(Y)
      AC=ROUNDR(A,C)
      BD=ROUNDR(B,D)
      BC=ROUNDR(B,C)
      AD=ROUNDR(A,D)
      ROUNDC=CMPLX((AC-BD),(BC+AD))
      RETURN
      END
```

```
C ========================================================================
      FUNCTION ROUNDR(X,Y)
C ========================================================================
C          THIS FUNCTION WILL MULTIPLY THE ARGUMENTS AND ROUND OFF
C            THE RESULT TO THE NEARESST TWO'S COMPLEMENT INTEGER
C
C               PROGRAM         :  ROUNDR:S
C               ORIGINAL        :  SEPTEMBER 15, 1979
C               REVISION        :  MARCH     11, 1981
C
C               PREPARED  BY    :  S. M. DANIEL  &  I. KERTESZ
C                                  RADAR SYSTEMS ANALYSIS GROUP
C                                  MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                  TEMPE, ARIZONA  85282
C ------------------------------------------------------------------------
C     INPUT  :    X          -  REAL NUMBER TO BE MULTIPLIED BY Y
C                 Y          -  REAL NUMBER TO MULTIPLY X
C
C
C     OUTPUT :    ROUNDR     -  REAL ROUNDED PRODUCT OF X AND Y
C ------------------------------------------------------------------------
C     ENTRY POINTS          :  ROUNDR
C
C     PROGRAMS CALLED       :  NONE
C ========================================================================
      REAL X,Y,ROUNDR
      XY=X*Y+.5
      IXY=XY
      IF(XY.LT.0.) IXY=XY-1
      ROUNDR=IXY
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE LIMIT(X,NX,K,K0,LIM1,LIM2)
C ======================================================================
C                 APPLICATION OF HARDWARE SCALING LIMITS
C
C              PROGRAM        :  LIMIT:S
C              ORIGINAL       :  AUGUST  15, 1979
C              REVISION       :  MARCH   25, 1981
C
C              PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                RADAR SYSTEMS ANALYSIS GROUP
C                                MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     INPUT  :      X         -  INPUT VECTOR TO BE LIMITED
C                   NX        -  NUMBER OF ELEMENTS IN X
C                   K         -  INPUT AND OUTPUT BIT-SHIFT
C                   K0        -  INTRINSIC BIT-SHIFT
C                   LIM1      -  MAXIMUM ALLOWABLE DOWN-SHIFT
C                   LIM2      -  MAXIMUM ALLOWABLE UP-SHIFT
C
C     OUTPUT :      X         -  RETURNED LIMITED VECTOR
C ----------------------------------------------------------------------
C     ENTRY POINTS            :  LIMIT
C
C     PROGRAMS CALLED         :  SUBROUTINE   FUNCTION     ENTRY
C                                ----------   --------     -----
C                                SCATD           -         SCATD
C                                                -         SCATU
C ======================================================================
      DIMENSION X(1)
      IN=0
      ID=0
      KH=K+K0
      IF(KH.GE.LIM1) GOTO 1000
      K=LIM1-K0
      IN=LIM1-KH
 1000 IF(KH.LE.LIM2) GOTO 2000
      K=LIM2-K0
      ID=KH-LIM2
 2000 NS=IN-ID
      IF(NS.LT.0) CALL SCATD(NX,NS,X)
      IF(NS.GE.0) CALL SCATU(NX,NS,X)
```

```
44.          RETURN
45.          END
```

```
C ======================================================================
      SUBROUTINE UPDATE(X,Y,Z,NX,ISGN,KX,KX0,LX,LY,LZ,LIM1,LIM2,IGO)
C ======================================================================
C                 UPDATE VECTORS W, R, AND P WHILE EXERCISING
C                  SCALE EQUALIZATION WITHIN HARDWARE LIMITS
C
C                 PROGRAM       :  UPDATE:S
C                 ORIGINAL      :  AUGUST  15, 1979
C                 REVISION      :  MARCH   19, 1981
C
C                 PREPARED  BY  :  S. M. DANIEL  &  I. KERTESZ
C                                  RADAR SYSTEMS ANALYSIS GROUP
C                                  MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                  TEMPE, ARIZONA  85282
C ---------------------------------------------------------------------
C       INPUT  :      X         -  CURRENT VECTOR VALUE
C                     Y         -  UPDATING VECTOR INCREMENT
C                     NX        -  VECTOR DIMENSIONALITY
C                     ISGN      -  ADDITION/SUBTRACTION INDICATOR
C                                    1  :  X+Y
C                                   -1  :  X-Y
C                     KX        -  LOCAL SCALE OF VECTOR X
C                     KX0       -  INTRINSIC SCALE OF VECTOR X
C                     LX        -  GLOBAL SCALE OF VECTOR X
C                     LY        -  GLOBAL SCALE OF VECTOR Y
C                     LIM1      -  LOW BIT-SHIFT LIMIT
C                     LIM2      -  HIGH BIT-SHIFT LIMIT
C
C       OUTPUT :      Z         -  UPDATED VECTOR
C                     LZ        -  GLOBAL SCALE OF VECTOR Z
C                     IGO       -  RETURNED  GO/NO-GO CONDITION
C                                   1  :  NORMAL OPERATION
C                                   0  :  DYNAMIC RANGE INTERRUPT
C ---------------------------------------------------------------------
C       ENTRY POINTS            :  UPDATE
C
C       PROGRAMS CALLED         :  SUBROUTINE   FUNCTION     ENTRY
C                                  ----------   --------     -----
C                                  FORMAT          -         FORMI
C                                                  -         FORMC
C                                  SCATD           -         SCATD
C                                                  -         SCATU
C ======================================================================
```

```
      COMPLEX X(1),Y(1),Z(1)
      IGO=1
      LYX=LY-LX
      IX=0
      IY=0
      KXH=KX+KX0
      LIM1A=LIM1-KXH
      LIM2A=LIM2-LIM1
      CALL FORMC('   X                        :',X,NX,1,1)
      CALL FORMI('   LX                       :',LX,1,0)
      CALL FORMC('   Y                        :',Y,NX,1)
      CALL FORMI('   LY                       :',LY,1,0)
      CALL FORMI('   LYX                      :',LYX,1,0)
      IF(LYX.LT.LIM1A.OR.LYX.GT.LIM2A) GOTO 3000
      IF(LYX.LT.0.OR.LYX.GT.LIM2A) GOTO 1000
      IY=LYX
      LY=LY-LYX
      GOTO 2000
1000  IF(LYX.LT.LIM1A.OR.LYX.GE.0) GOTO 2000
      IX=-LYX
      KXH=KXH+LYX
      KX=KXH-KX0
      LX=LX+LYX
2000  LZ=LX
      NX2=NX*2
      NS=-IX
      IF(NS.LT.0) CALL SCATD(NX2,NS,X)
      IF(NS.GE.0) CALL SCATU(NX2,NS,X)
      NS=-IY
      IF(NS.LT.0) CALL SCATD(NX2,NS,Y)
      IF(NS.GE.0) CALL SCATU(NX2,NS,Y)
      CALL FORMC('   X                        :',X,NX,1)
      CALL FORMI('   KXH                      :',KXH,1,0)
      CALL FORMI('   KX = KXH - KX0           :',KX,1,0)
      CALL FORMI('   LX                       :',LX,1,0)
      CALL FORMC('   Y                        :',Y,NX,1)
      CALL FORMI('   LY                       :',LY,1,0)
      DO 20 I=1,NX
20    Z(I)=X(I)+ISGN*Y(I)
      CALL FORMC('   Z                        :',Z,NX,1)
      CALL FORMI('   LZ                       :',LZ,1,0)
      RETURN
3000  IGO=0
```

```
      CALL FORMI(' ***************************',I,0,0)
      CALL FORMI(' *     SYSTEM INTERRUPT    *',I,0,0)
      CALL FORMI(' * DYNAMIC RANGE PROTECTION*',I,0,0)
      CALL FORMI(' ***************************',I,0,0)
      CALL FORMI('   LYX                     :',LYX,1,0)
      CALL FORMI('   LIM1                    :',LIM1,1,0)
      CALL FORMI('   LIM2                    :',LIM2,1,0)
      RETURN
      END
```

```
C ======================================================================
      SUBROUTINE CSEML
C ======================================================================
C          CALCULATION OF COMBINED PORT SIGNALS AND THE ASSOCIATED
C                              POWER SUPPRESSION
C
C
C               PROGRAM        :  CSEML:S
C               ORIGINAL       :  AUGUST 15, 1979
C               REVISION       :  MARCH  25, 1981
C
C               PREPARED  BY   :  S. M. DANIEL  &  I. KERTESZ
C                                 RADAR SYSTEMS ANALYSIS GROUP
C                                 MOTOROLA GOVERNMENT ELECTRONICS DIV.
C                                 TEMPE, ARIZONA  85282
C ----------------------------------------------------------------------
C     ENTRY POINTS             :  CSEML
C
C     PROGRAMS CALLED          :  SUBROUTINE   FUNCTION     ENTRY
C                                 ----------   --------     -----
C                                 SCATD           -         SCATD
C                                                 -         SCATU
C                                 FORMAT          -         FORMAT
C                                                 -         FORMI
C                                                 -         FORMR
C                                                 -         FORMC
C ======================================================================
      COMPLEX S0,S1,C,B,R,P,CP,W,SC,ACP,AP,BP
      COMMON /BCR0/ IWS0,IWS1,IWCB,IDP(20)
      COMMON /BCR1/ S0(256),S1(256,20),NSX,NS,NBITS0,NBITS1,S0MAX,S1MAX
      COMMON /BCR2/ ISW(20),NWX,IW0,NPM1,ITERX,NBITSP,NBITSW
     -,KM,KM0,KD,KT1,KT3,LIM1,LIM2,LRRT
      COMMON /BCR3/ C(20,20),B(20),R(20),P(20),CP(20),SC(400)
     -,W(20),ACP(20),AP(20),BP(20),KC,KB,LW
      DIMENSION PAGE(30),LINE(30)
      DATA PAGE/'1---',29*'----'/,LINE/' ---',29*'----'/
C ----------------------------------------------------------------------
      P0=0
      PC=0
      NS2=2*NS
      DO 10 IS=1,NS
      SC(IS)=0
      DO 10 IP=1,NPM1
```

```
10    SC(IS)=SC(IS)+S1(IS,IP)*W(IP)
      IF(LW.GE.0) CALL SCATD(NS2,-LW,SC)
      IF(LW.LT.0) CALL SCATU(NS2,-LW,SC)
      DO 20 IS=1,NS
      SC(IS)=SC(IS)+S0(IS)
      P0=P0+CABS(S0(IS))**2
20    PC=PC+CABS(SC(IS))**2
      PCAN=100.
      IF(PC.NE.0) PCAN=10*ALOG10(P0/PC)
      CALL FORMAT(PAGE,108)
      CALL FORMI('  COMBINED PORT SIGNAL       ',I,0,0)
      CALL FORMAT(LINE,108)
      CALL FORMC('    SC                      ',SC,NS,1)
      CALL FORMAT(LINE,108)
      CALL FORMR('    SUPPRESSION (DB)        ',PCAN,1,5)
      CALL FORMAT(LINE,108)
      RETURN
      END
```

binary module has been created by compilation. A general form of a Job Control Language (JCL) program for doing this is shown in Table 4-4.

Table 4-4. JCL Program, JCL:B

```
1.000 !JOB 1269,DANIEL(8512),7,BLDG90
2.000 !LIMIT (TIME,1),(UO,10),(CO,16),(ACCOUNT)
3.000 !SET M:SI/NAME:S;IN;SAVE
4.000 !SET M:BO/NAME:B;OUT;SAVE
5.000 !FORTRAN LS,NS,BC,SI,BO
```

Note that any source file, FILE:S, may be converted to its binary version, FILE:B, by the name-substitution batch command:

!BATCH JCL:B 'NAME' = FILE

4.1.5 Executable Load Module

The next step toward program execution is the creation of an executable load module. This involves the linking of all binary modules which together contain all the subroutines and functions referenced in the complete program suggested by the tree diagram of Figure 4-1. For the present case, this can be accomplished through the execution of a JCL program such as JCLBCRM:BL, as shown in Table 4-5. It can be seen there that all the binary

Table 4-5, JCL Program, JCLBCRM:BL

```
 1.000 !JOB 1269,DANIEL(8512),7,BLDG90
 2.000 !LIMIT (TIME,1),(UO,2),(CO,16),(ACCOUNT)
 3.000 !.....JCLBCRM:BL.......................
 4.000 !PCL
 5.000 C      BCRMMAIN:B              OVER BCRM:B
 6.000 C         BCRMSET:B
 7.000 C            RW:B
 8.000 C               FORMAT:B
 9.000 C         BCRMEXEC:B
10.000 C            AGC:B
11.000 C               SCATD:B
12.000 C            CBEML:B
13.000 C               SCALE:B
14.000 C                  BIGSGN:B
15.000 C            BCREML:B
16.000 C               ADOTB:B
17.000 C                  ROUNDC:B
18.000 C                     ROUNDR:B
19.000 C               LIMIT:B
20.000 C               UPDATE:B
21.000 C            CSEML:B
22.000 !LYNX BCRM:B                   OVER BCRM:L
```

modules of BCRM are in the process of concatenation into one composite binary module, BCRM:B. After the computer system supplies all the system references, the linking takes place, and the resulting module is place into a file called BCRM:L. This file is the executable load module of program BCRM.

The load module can be executed with a "RUN" command. An example of this is the JCL program, JCL:X in Table 4-7. It assigns an input file (suffix:D), and an output file (suffix:O) to the executable load module (suffix:L) for the duration of the job. When JCL:X is batched, "BCRM" is substituted for "NAME". Thus BCRM:L will read input data from BCRM:D, and write output data into BCRM:O.

The load module BCRM:L may now be executed via a JCL program, JCL:X, given in Table 4-6.

Table 4-6. JCL Program, JCL:X

```
1 -     1.000 !JOB 1269,DANIEL(8512),7,BLDG90
2 -     2.000 !LIMIT (TIME,1),(UO,50),(CO,24),(ACCOUNT)
3 -     3.000 !SET F:105/NAME:D
4 -     4.000 !SET F:108/NAME:O
5 -     5.000 !RUN (LMN,NAME:L)
```

4.1.6 Output Data File - BCRM:O

The output data file, BCRM:O, produced by the BCR emulation program is listed under Table 4-7 that follows. It consists of two major parts. The first part is a reprint of the input data file, BCRM:D. This is a desirable convenience to the user and, of course, helps to completely identify the particular case executed.

The second part of BCRM:O is the actual emulation results obtained from the execution of BCRM:L. It gives a listing of all sampled port signals and quantities C and $\underline{b}$ according to available print options. Then, starting with a number of operational specifications including C and $\underline{b}$ data, an exhaustive numerical description of the adaptive fixed-point arithmetic of the BCR process follows.

Included, as shown is the initialization part consisting of loading initial values for $\underline{w}$, $\underline{r}$ and $\underline{p}$ into the Vector Storage RAM, setting a global scale reference for $\underline{r}$ and $\underline{p}$ and computing $\|\underline{r}\|^2$ with its global scale. Following subsequently, is the complete arithmetic activity of the iterative portions of the BCR process. The left-hand-side of the output is reserved for variable names and pertinent local and global scale equations. The right-hand-side includes the corresponding computed values as would be produced by a 16-bit BCR processor implementation.

Table 4-7. Output File of the BCR Emulation Program
Benchtest Example (See Figure 2-2, PM 8512-04)

```
------------------------------------------------------------------------------
                    DIGITAL ADAPTIVE ARRAY PROCESSING
                    ---------------------------------
                                  USING
                                  -----
                        BATCH COVARIANCE RELAXATION
                        ---------------------------
------------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWS0    -  OPTIONAL OUTPUT OF  S0                  :        1
  IWS1    -  OPTIONAL OUTPUT OF  S1                  :        0
  IWS0F   -  OPTIONAL OUTPUT OF  FIXED-POINT S0      :        1
  IWS1F   -  OPTIONAL OUTPUT OF  FIXED-POINT S1      :        1
  IWCB    -  OPTIONAL OUTPUT OF  C  AND  B           :        1

BCR PARAMETERS
--------------
  NSX     -  MAXIMUM NUMBER OF SIGNAL SAMPLES        :      256
  IW0     -  SIZE OF BAND DIAGONAL                   :        0
  NWX     -  MAXIMUM NUMBER OF WEIGHTS               :       20
  ISW     -  WEIGHT SELECTOR ARRAY                   :    1 1 1 1 0 0 0 0 0 0
                                                     :    0 0 0 0 0 0 0 0 0 0
  ITERX   -  MAXIMUM NUMBER OF ITERATIONS            :        4
------------------------------------------------------------------------------
  ITER    -  ACTUAL NUMBER OF ITERATIONS             :        X
------------------------------------------------------------------------------
EMULATION PARAMETERS
--------------------
  NB0     -  NUMBER OF BITS IN A WORD OF S0          :       10
  NB1     -  NUMBER OF BITS IN A WORD OF S1          :        8
  NBP     -  NUMBER OF BITS IN THE PROCESSOR WORD    :       16
  NBW     -  NUMBER OF BITS IN A WORD OF W OUTPUT    :       12
  KM      -  MULTIPLIER TRUNCATION WITH OVERFLOW     :       16
  KM0     -  MULTIPLIER TRUNCATION WITHOUT OVERFLOW  :       15
  KD      -  DIVISION TABLE BIT-SHIFT     (2*NBP-3)  :       29
  KT1     -  1 - BIT TRUNCATIONS KR0,KP0,KW0         :        1
  KT3     -  3 - BIT TRUNCATIONS KRR0,KCP0,KPCP0     :        3
  LIM1    -  LOW  BIT-SHIFT LIMIT AT SCALER #1       :       -7
  LIM2    -  HIGH BIT-SHIFT LIMIT AT SCALER #1       :        8
  LRRT    -  ALLOWED GLOBAL SCALE REDUCTION OF LRR   :       18
```

```
--------------------------------------------------------------------------------
                    SPECIFICATION OF SYSTEM PARAMETERS
                    ----------------------------------

--------------------------------------------------------------------------------
PRINT OPTIONS
-------------
  IWANT   -  MAIN ANTENNA ARRAY WEIGHTING            :          0
  IWRAP   -  RECEIVER AMPLITUDE AND PHASE            :          0
  IWRI    -  RECEIVER IMPULSE RESPONSE               :          0
  IWCAP   -  CHANNEL AMPLITUDE AND PHASE             :          0
  IWCI    -  CHANNEL IMPULSE RESPONSE                :          0
  IWSC    -  INDIVIDUAL CHANNEL SIGNALS              :          0

FILTER PARAMETERS
-----------------
  NPOL    -  NUMBER OF LOWPASS PROTOTYPE POLES       :          2
             POL(1)                                  :     -.70700   -.70700
             POL(2)                                  :     -.70700    .70700
  FRIF    -  FRACTIONAL BANDWIDTH AT FINAL IF        :      .10000
  FRRF    -  FRACTIONAL BANDWIDTH AT RF              :      .00100
  RADRNG  -  NORMALIZED RADIAN FREQUENCY RANGE       :     4.00000
  RADIN   -  NORMALIZED INITIAL RADIAN FREQUENCY     :    -2.00000
  NF      -  NUMBER OF FREQUENCY SAMPLES             :          32
  LPBP    -  LOWPASS/BANDPASS OPTION                 :          1
             0  :   LOWPASS
             1  : . BANDPASS

CHANNEL PARAMETERS
------------------
  NCHNLS  -  NUMBER OF CHANNELS PER PORT             :         20
  ISC     -  CHANNEL SELECTOR ARRAY                  :     1 1 0 0 0 0 0 0 0 0
                                                     :     0 0 0 0 0 0 0 0 0 0

CHANNEL  1:
  AN      -  AMPLITUDE OF NOISE SOURCE               :     1.00000
  IX0     -  INITIAL RANDU SETTING                   :          1
  N1      -  FIRST-TIME-ON SAMPLE NUMBER             :          1
  NB      -  BLINK DURATION IN SAMPLES               :      10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)       :    45.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS      :      .00000
```

```
CHANNEL   2:
  AN      -  AMPLITUDE OF NOISE SOURCE                     :      1.00000
  IXO     -  INITIAL RANDU SETTING                         :           11
  N1      -  FIRST-TIME-ON SAMPLE NUMBER                   :            1
  NB      -  BLINK DURATION IN SAMPLES                     :        10000
  TH      -  AZIMUTH ANGLES OF INCIDENCE (DEG)             :     10.00000
  CDEL    -  CHANNEL DELAY IN SAMPLE-TIME UNITS            :       .10000

PORT PARAMETERS
---------------
  D0      -  ANTENNA-ELEMENT SEPARATION FACTOR             :      1.00000
  NS      -  NUMBER OF SIGNAL SAMPLES                      :          128
  NPORTS  -  NUMBER OF PORTS                               :           21
  ISP     -  PORT SELECTOR ARRAY                           :      1 1 1 1 0 0 0 0 0 0
                                                           :      0 0 0 0 0 0 0 0 0 0

PORT      0:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                    :          255
  LOC1    -  LOCATION OF THE FIRST ELEMENT                 :            1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                    :      1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                       :       .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY            :       .00000

PORT      1:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                    :            1
  LOC1    -  LOCATION OF THE FIRST ELEMENT                 :            1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                    :      1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                       :       .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY            :       .00000

PORT      2:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                    :            1
  LOC1    -  LOCATION OF THE FIRST ELEMENT                 :            1
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                    :      1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                       :       .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY            :       .10000

PORT      3:
  NEL     -  NUMBER OF ANTENNA ELEMENTS                    :            1
  LOC1    -  LOCATION OF THE FIRST ELEMENT                 :          255
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR                    :      1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                       :       .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY            :       .00000
```

```
PORT      4:
  NEL     -  NUMBER OF ANTENNA ELEMENTS              :          1
  LOC1    -  LOCATION OF THE FIRST ELEMENT           :        255
  BWFCTR  -  BANDWIDTH TOLERANCE FACTOR              :    1.00000
  BWOFF   -  BANDWIDTH OFFSET FACTOR                 :     .00000
  PDEL    -  FRACTION OF MAXIMUM APERTURE DELAY      :     .10000
-----------------------------------------------------------------------------
```

PORT 0

RECEIVED BASEBAND PORT SIGNAL
NORMALIZATION CONSTANT : .359902E 00

.05464	.03818	-.27935	-.18506	-.80850	-.80400	-.96805	-1.00000
-.89117	-.93809	-.86581	-.91809	-.51389	-.71681	-.01191	-.37046
.51795	.07067	.69954	.48863	.33405	.37157	-.72323	-.56527
-.48070	-.23752	-.28359	.13504	-.05666	.10339	.17505	.00267
-.06350	-.11284	-.27573	-.25835	-.11408	.03863	.07952	.27616
-.00055	-.11476	.45160	.31795	-.25093	-.05661	-.53158	-.23006
.10718	-.05535	.29687	.03986	-.22163	-.13246	-.32173	-.22894
-.37581	.00988	-.03880	.09207	.23237	.18048	-.03938	.26499
-.60312	-.16596	-.22783	-.27798	.15059	.04556	.49784	.37388
.05924	.20007	-.67014	-.26716	-.51796	-.17555	.41518	.42498
.46477	.55190	-.24890	.00095	-.22332	-.22121	-.17883	-.20944
-.14506	-.08280	-.48174	-.33351	-.15840	-.23864	.03017	.12233
-.24552	.07159	.07142	.10982	.79052	.48894	.42138	.42662
-.56041	-.25491	-.64554	-.51348	.04592	.16574	.45397	.33689
-.15339	-.20463	-.60183	-.43150	-.23293	.15092	-.05293	.03788
.70177	.26965	.66569	.35406	.76548	.50369	.33144	.11207
.34198	.16070	-.22887	.04206	-.10226	.16876	.08978	.24863
.07344	-.10328	.46115	.36154	.51861	.34287	.17532	.33440
-.01579	.20464	.13438	.29128	-.17331	-.08766	.00878	.12494
-.21701	-.41771	.06459	-.23581	-.57297	-.59389	-.25658	-.38524
-.71101	-.39013	-.17867	-.31976	.16877	.45003	-.15970	.15444
.40744	.45485	.68224	.67182	.68668	.85575	-.08121	.25855
.30583	.30440	.19326	.26502	-.49589	-.22002	-.67978	-.41590
-.30489	-.53854	.35300	.05499	-.29986	.01299	-.08097	-.02180
.53392	.59253	-.08945	.13345	-.56345	-.37763	-.77421	-.36072
-.67038	-.15848	-.09812	.07619	-.07994	.11141	-.19309	-.00092
-.09846	-.05803	-.32363	-.11191	-.70204	-.51644	-.42421	-.15956
.35976	.29671	.55861	.31601	.34769	-.05135	.29570	.17155
-.02046	.16898	-.40331	-.40163	-.33261	-.31393	.02263	.05234
-.29163	-.15834	.20011	.26848	.21790	.33672	.14524	.09034
.12943	.14092	-.11299	.08524	.11257	.27772	.18866	.44657
.08273	.17843	.09462	.15594	-.23630	-.08516	.11599	-.09951

MAIN PORT SIGNAL

PORT	:	0							
SOMAX	:	.35990							
NBSO	:	10							
SO	:	27.0	19.0	-143.0	-94.0	-413.0	-411.0	-495.0	-512.0
		-456.0	-480.0	-443.0	-470.0	-263.0	-367.0	-6.0	-189.0
		265.0	36.0	358.0	250.0	171.0	190.0	-370.0	-289.0
		-246.0	-121.0	-145.0	69.0	-29.0	52.0	89.0	1.0
		-32.0	-57.0	-141.0	-132.0	-58.0	19.0	40.0	141.0
		.0	-58.0	231.0	162.0	-128.0	-28.0	-272.0	-117.0
		54.0	-28.0	151.0	20.0	-113.0	-67.0	-164.0	-117.0
		-192.0	5.0	-19.0	47.0	118.0	92.0	-20.0	135.0
		-308.0	-84.0	-116.0	-142.0	77.0	23.0	254.0	191.0
		30.0	102.0	-343.0	-136.0	-265.0	-89.0	212.0	217.0
		237.0	282.0	-127.0	.0	-114.0	-113.0	-91.0	-107.0
		-74.0	-42.0	-246.0	-170.0	-81.0	-122.0	15.0	62.0
		-125.0	36.0	36.0	56.0	404.0	250.0	215.0	218.0
		-286.0	-130.0	-330.0	-262.0	23.0	84.0	232.0	172.0
		-78.0	-104.0	-308.0	-220.0	-119.0	77.0	-27.0	19.0
		359.0	138.0	340.0	181.0	391.0	257.0	169.0	57.0
		175.0	82.0	-117.0	21.0	-52.0	86.0	45.0	127.0
		37.0	-52.0	236.0	185.0	265.0	175.0	89.0	171.0
		-8.0	104.0	68.0	149.0	-88.0	-44.0	4.0	63.0
		-111.0	-213.0	33.0	-120.0	-293.0	-304.0	-131.0	-197.0
		-364.0	-199.0	-91.0	-163.0	86.0	230.0	-81.0	79.0
		208.0	232.0	349.0	343.0	351.0	438.0	-41.0	132.0
		156.0	155.0	98.0	135.0	-253.0	-112.0	-348.0	-212.0
		-156.0	-275.0	180.0	28.0	-153.0	6.0	-41.0	-11.0
		273.0	303.0	-45.0	68.0	-288.0	-193.0	-396.0	-184.0
		-343.0	-81.0	-50.0	39.0	-40.0	57.0	-98.0	.0
		-50.0	-29.0	-165.0	-57.0	-359.0	-264.0	-217.0	-81.0
		184.0	151.0	286.0	161.0	178.0	-26.0	151.0	87.0
		-10.0	86.0	-206.0	-205.0	-170.0	-160.0	11.0	26.0
		-149.0	-81.0	102.0	137.0	111.0	172.0	74.0	46.0
		66.0	72.0	-57.0	43.0	57.0	142.0	96.0	228.0
		42.0	91.0	48.0	79.0	-120.0	-43.0	59.0	-50.0

AUXILIARY PORT SIGNAL

PORT	1							
S1MAX	.78027							
NBS1	8							
S1(IP)	3.0	2.0	-9.0	-17.0	-18.0	-83.0	-29.0	-100.0
	-21.0	-98.0	-24.0	-93.0	4.0	-83.0	37.0	-58.0
	69.0	-25.0	44.0	32.0	25.0	27.0	-37.0	-48.0
	-64.0	3.0	-61.0	42.0	-26.0	28.0	34.0	-15.0
	5.0	-16.0	-8.0	-27.0	-37.0	21.0	-13.0	38.0
	9.0	-14.0	32.0	21.0	-18.0	-2.0	-67.0	7.0
	21.0	-11.0	48.0	-20.0	-13.0	-11.0	-26.0	-15.0
	-66.0	31.0	-21.0	24.0	14.0	13.0	-34.0	42.0
	-71.0	15.0	-8.0	-23.0	13.0	.0	29.0	28.0
	-2.0	19.0	-72.0	1.0	-74.0	16.0	.0	52.0
	18.0	50.0	-36.0	17.0	-7.0	-19.0	-4.0	-21.0
	-9.0	-6.0	-31.0	-23.0	-4.0	-23.0	-8.0	14.0
	-48.0	30.0	-12.0	22.0	61.0	29.0	28.0	30.0
	-47.0	-8.0	-49.0	-34.0	-27.0	33.0	38.0	21.0
	17.0	-35.0	-53.0	-24.0	-61.0	44.0	-16.0	17.0
	76.0	-2.0	61.0	10.0	60.0	28.0	45.0	-9.0
	31.0	2.0	-35.0	16.0	-53.0	46.0	-4.0	31.0
	17.0	-17.0	25.0	29.0	40.0	20.0	-7.0	37.0
	-37.0	43.0	-7.0	36.0	-20.0	.0	-11.0	20.0
	22.0	-54.0	41.0	-47.0	-7.0	-65.0	-2.0	-44.0
	-53.0	-22.0	-10.0	-27.0	-22.0	56.0	-51.0	44.0
	1.0	53.0	17.0	70.0	10.0	89.0	-42.0	49.0
	.0	38.0	11.0	22.0	-48.0	-3.0	-53.0	-20.0
	10.0	-61.0	48.0	-20.0	-42.0	16.0	-26.0	13.0
	10.0	61.0	-15.0	20.0	-41.0	-23.0	-80.0	-6.0
	-97.0	28.0	-27.0	24.0	-24.0	22.0	-30.0	16.0
	-8.0	-2.0	-29.0	.0	-45.0	-39.0	-63.0	13.0
	15.0	30.0	57.0	9.0	65.0	-36.0	20.0	6.0
	-16.0	26.0	-6.0	-42.0	-25.0	-22.0	1.0	3.0
	-28.0	-5.0	-13.0	38.0	1.0	37.0	9.0	7.0
	5.0	11.0	-31.0	23.0	-24.0	43.0	-24.0	62.0
	-7.0	25.0	-3.0	18.0	-22.0	1.0	20.0	-18.0

AUXILIARY PORT SIGNAL

PORT	:	2							
S1MAX	:	.78027							
NBS1	:	8							
S1(IP)	:	-2.0	3.0	16.0	-8.0	81.0	-18.0	100.0	-29.0
		98.0	-21.0	93.0	-24.0	83.0	3.0	59.0	36.0
		26.0	69.0	-31.0	45.0	-29.0	25.0	48.0	-36.0
		-1.0	-64.0	-42.0	-60.0	-29.0	-28.0	14.0	34.0
		16.0	6.0	27.0	-7.0	-19.0	-36.0	-39.0	-14.0
		14.0	8.0	-20.0	32.0	2.0	-16.0	-7.0	-68.0
		10.0	18.0	21.0	49.0	10.0	-12.0	16.0	-25.0
		-30.0	-66.0	-24.0	-23.0	-13.0	14.0	-42.0	-33.0
		-17.0	-72.0	23.0	-9.0	1.0	13.0	-28.0	29.0
		-20.0	.0	-1.0	-71.0	-15.0	-75.0	-51.0	-1.0
		-51.0	20.0	-19.0	-36.0	19.0	-7.0	22.0	-4.0
		6.0	-8.0	23.0	-31.0	23.0	-5.0	-13.0	-7.0
		-30.0	-48.0	-22.0	-14.0	-29.0	60.0	-30.0	30.0
		6.0	-46.0	35.0	-49.0	-32.0	-28.0	-23.0	37.0
		34.0	19.0	26.0	-53.0	-43.0	-61.0	-18.0	-19.0
		2.0	75.0	-10.0	61.0	-29.0	60.0	8.0	46.0
		-1.0	32.0	-16.0	-33.0	-45.0	-54.0	-33.0	-5.0
		18.0	17.0	-29.0	25.0	-20.0	40.0	-37.0	-5.0
		-42.0	-38.0	-37.0	-7.0	.0	-19.0	-20.0	-11.0
		53.0	21.0	48.0	42.0	64.0	-7.0	46.0	-1.0
		21.0	-52.0	29.0	-12.0	-55.0	-20.0	-44.0	-53.0
		-52.0	1.0	-69.0	16.0	-90.0	11.0	-51.0	-42.0
		-38.0	-1.0	-22.0	12.0	3.0	-47.0	19.0	-54.0
		60.0	8.0	21.0	50.0	-16.0	-40.0	-13.0	-28.0
		-61.0	10.0	-22.0	-14.0	23.0	-41.0	6.0	-79.0
		-28.0	-98.0	-24.0	-29.0	-21.0	-23.0	-17.0	-31.0
		3.0	-8.0	.0	-29.0	39.0	-44.0	-11.0	-64.0
		-30.0	12.0	-10.0	57.0	35.0	65.0	-4.0	22.0
		-27.0	-17.0	42.0	-6.0	23.0	-25.0	-4.0	1.0
		6.0	-27.0	-37.0	-14.0	-38.0	1.0	-8.0	8.0
		-11.0	6.0	-23.0	-31.0	-42.0	-24.0	-62.0	-24.0
		-26.0	-7.0	-18.0	-2.0	-2.0	-22.0	19.0	19.0

AUXILIARY PORT SIGNAL

PORT	:	3							
S1MAX	:	.78027							
NBS1	:	8							
S1(IP)	:	4.0	4.0	-29.0	-31.0	-95.0	-111.0	-110.0	-127.0
		-107.0	-123.0	-101.0	-117.0	-79.0	-98.0	-39.0	-58.0
		6.0	-9.0	56.0	59.0	26.0	28.0	-61.0	-68.0
		-11.0	1.0	16.0	39.0	13.0	22.0	-5.0	-16.0
		-13.0	-17.0	-30.0	-34.0	16.0	29.0	22.0	33.0
		-7.0	-16.0	32.0	33.0	-12.0	-7.0	-15.0	-3.0
		-7.0	-17.0	.0	-11.0	-18.0	-16.0	-20.0	18.0
		7.0	28.0	11.0	18.0	21.0	21.0	29.0	47.0
		-18.0	-4.0	-26.0	-33.0	8.0	6.0	41.0	43.0
		14.0	21.0	-25.0	-12.0	-6.0	9.0	53.0	64.0
		52.0	62.0	.0	8.0	-24.0	-30.0	-21.0	-24.0
		-11.0	-10.0	-37.0	-37.0	-20.0	-26.0	13.0	22.0
		10.0	26.0	18.0	23.0	53.0	48.0	38.0	43.0
		-32.0	-27.0	-45.0	-46.0	27.0	40.0	27.0	24.0
		-32.0	-43.0	-35.0	-30.0	21.0	45.0	5.0	8.0
		27.0	11.0	38.0	30.0	50.0	47.0	6.0	-4.0
		19.0	15.0	3.0	15.0	32.0	51.0	17.0	24.0
		-5.0	-15.0	40.0	43.0	36.0	33.0	35.0	47.0
		31.0	45.0	26.0	35.0	-4.0	-3.0	10.0	17.0
		-51.0	-72.0	-28.0	-44.0	-73.0	-85.0	-35.0	-43.0
		-50.0	-43.0	-19.0	-25.0	46.0	69.0	24.0	41.0
		55.0	65.0	80.0	91.0	90.0	110.0	30.0	46.0
		40.0	45.0	20.0	24.0	-22.0	-15.0	-47.0	-43.0
		-56.0	-75.0	5.0	-3.0	-1.0	13.0	10.0	15.0
		65.0	79.0	5.0	11.0	-38.0	-38.0	-33.0	-18.0
		-6.0	17.0	10.0	18.0	13.0	23.0	.0	7.0
		-4.0	-3.0	-17.0	-11.0	-54.0	-55.0	-4.0	10.0
		33.0	36.0	26.0	17.0	-10.0	-32.0	24.0	25.0
		11.0	21.0	-47.0	-58.0	-25.0	-26.0	1.0	3.0
		-13.0	-9.0	38.0	49.0	31.0	39.0	11.0	9.0
		14.0	17.0	13.0	25.0	37.0	51.0	50.0	68.0
		19.0	23.0	18.0	22.0	-12.0	-9.0	-3.0	-13.0

AUXILIARY PORT SIGNAL

PORT	:	4							
SIMAX	:	.78027							
NBS1	:	8							
SI(IP)	:	-4.0	4.0	29.0	-28.0	110.0	-93.0	128.0	-110.0
		123.0	-107.0	117.0	-102.0	99.0	-80.0	59.0	-41.0
		10.0	5.0	-57.0	55.0	-31.0	28.0	67.0	-61.0
		1.0	-13.0	-39.0	16.0	-22.0	13.0	16.0	-4.0
		17.0	-13.0	34.0	-29.0	-27.0	14.0	-35.0	24.0
		16.0	-8.0	-32.0	32.0	6.0	-11.0	3.0	-16.0
		16.0	-7.0	12.0	.0	16.0	-17.0	19.0	-20.0
		-28.0	7.0	-19.0	11.0	-20.0	20.0	-47.0	29.0
		2.0	-17.0	34.0	-26.0	-5.0	7.0	-42.0	40.0
		-22.0	16.0	12.0	-25.0	-7.0	-7.0	-63.0	52.0
		-63.0	53.0	-9.0	1.0	30.0	-24.0	25.0	-21.0
		9.0	-11.0	36.0	-37.0	27.0	-21.0	-21.0	13.0
		-26.0	10.0	-22.0	18.0	-47.0	52.0	-44.0	39.0
		25.0	-31.0	48.0	-47.0	-39.0	25.0	-26.0	28.0
		41.0	-30.0	33.0	-36.0	-45.0	21.0	-9.0	5.0
		-11.0	27.0	-29.0	37.0	-47.0	51.0	3.0	7.0
		-14.0	19.0	-16.0	3.0	-49.0	30.0	-26.0	19.0
		16.0	-6.0	-43.0	39.0	-33.0	36.0	-47.0	35.0
		-45.0	30.0	-37.0	28.0	3.0	-5.0	-18.0	11.0
		70.0	-51.0	45.0	-29.0	84.0	-72.0	45.0	-37.0
		41.0	-48.0	29.0	-22.0	-69.0	47.0	-41.0	24.0
		-64.0	54.0	-91.0	79.0	-111.0	91.0	-47.0	31.0
		-45.0	39.0	-25.0	21.0	14.0	-22.0	42.0	-46.0
		76.0	-57.0	5.0	4.0	-14.0	.0	-13.0	8.0
		-78.0	65.0	-14.0	7.0	38.0	-38.0	19.0	-33.0
		-16.0	-7.0	-18.0	9.0	-23.0	13.0	-8.0	.0
		4.0	-4.0	10.0	-16.0	56.0	-54.0	-8.0	-6.0
		-36.0	32.0	-18.0	27.0	32.0	-10.0	-23.0	22.0
		-23.0	13.0	57.0	-47.0	28.0	-27.0	-4.0	1.0
		10.0	-14.0	-48.0	36.0	-41.0	32.0	-9.0	11.0
		-16.0	14.0	-24.0	13.0	-50.0	37.0	-69.0	50.0
		-24.0	19.0	-22.0	18.0	8.0	-11.0	15.0	-5.0

```
-----------------------------------------------------------------------------------------------
BATCH COVARIANCE MATRIX
-----------------------------------------------------------------------------------------------
 NBITSP            :        16
 KC                :        -4
 C                 :   20185.0        .0     -30.0   20198.0   14730.0   -8388.0    8407.0   14787.0
                         -30.0  -20199.0   20230.0        .0   -8365.0  -14698.0   14765.0   -8390.0
                       14730.0    8387.0   -8365.0   14697.0   25113.0        .0        .0   25113.0
                        8407.0  -14788.0   14765.0    8389.0        .0  -25114.0   25136.0        .0
-----------------------------------------------------------------------------------------------
FORCING VECTOR
-----------------------------------------------------------------------------------------------
 NBITSP            :        16
 KB                :        -6
 B                 :   22091.0   -3351.0   -3328.0  -22097.0   25913.0     274.0     325.0  -25968.0

-----------------------------------------------------------------------------------------------
```

ADAPTIVE FIXED-POINT ARITHMETIC OF RCR PROCESSOR

C	:	20185.0	.0	-30.0	20198.0	14730.0	-8388.0	8407.0	14787.0
		-30.0	-20199.0	20230.0	.0	-8365.0	-14698.0	14765.0	-8390.0
		14730.0	8387.0	-8365.0	14697.0	25113.0	.0	.0	25113.0
		8407.0	-14788.0	14765.0	8389.0	.0	-25114.0	25136.0	.0
KC	:	-4							
B	:	22091.0	-3351.0	-3328.0	-22697.0	25913.0	274.0	325.0	-25968.0
KB	:	-6							
KWF	:	-2							

INITIALIZATION

W	:	.0	.0	.0	.0	.0	.0	.0	.0
P = R = B	:								
CRDR	:	35776.0	.0						
RDR	:	17888.0							
KRRH	:	2							
KRR = KRRH - KRR0	:	-1							
LRR = 2*LR + KRR - KM	:	-17							

ITERATION #	:	1							
CP	:	25378.0	-8560.0	-8568.0	-25365.0	30656.0	4378.0	4387.0	-30708.0
CP LIMITED	:	25378.0	-8560.0	-8568.0	-25365.0	30656.0	4378.0	4387.0	-30708.0
KCPH	:	3							
KCP = KCPH - KCPO	:	0							
LCP = LP + KCP - KM	:	-16							
CPCP	:	42308.0	1.0						
PCP	:	21154.0							
KPCPH	:	2							
KPCP = KPCPH - KPCP0	:	-1							
LPCP=LP+LCP+KPCP-KM	:	-33							
PCPI	:	25379.0							
KPCPI	:	0							
LPCPI = KD - LPCP	:	62							
ALPHA	:	13854.0							
ALPHA SCALED	:	27708.0							
KAL	:	1							
LAL=LRR+LPCPI+KAL-KM0	:	31							
AP	:	18680.0	-2834.0	-2814.0	-18685.0	21912.0	232.0	275.0	-21958.0
LAP = LAL + LP - KM0	:	16							
X	:	.0	.0	.0	.0	.0	.0	.0	.0
LX	:	16							
Y	:	18680.0	-2834.0	-2814.0	-18685.0	21912.0	232.0	275.0	-21958.0
LY	:	16							
LYX	:	0							
X	:	.0	.0	.0	.0	.0	.0	.0	.0
KXH	:	1							
KX = KXH - KX0	:	0							
LX	:	16							
Y	:	18680.0	-2834.0	-2814.0	-18685.0	21912.0	232.0	275.0	-21958.0
LY	:	16							
Z	:	-18680.0	2834.0	2814.0	18685.0	-21912.0	-232.0	-275.0	21958.0
LZ	:	16							

W	:	-18680.0	2834.0	2814.0	18685.0	-21912.0	-232.0	-275.0	21958.0
W SCALED	:	-18680.0	2834.0	2814.0	18685.0	-21912.0	-232.0	-275.0	21958.0
KWH	:	1							
KW = KWH - KW0	:	0							
LW = LW + KW	:	16							
ACP	:	21459.0	-7238.0	-7245.0	-21448.0	25922.0	3702.0	3710.0	-25966.0
LACP = LAL + LCP - KM0	:	0							
RDRI	:	30013.0							
LRRI = KD - LRR	:	46							
X	:	22091.0	-3351.0	-3328.0	-22097.0	25913.0	274.0	325.0	-25968.0
LX	:	0							
Y	:	21459.0	-7238.0	-7245.0	-21448.0	25922.0	3702.0	3710.0	-25966.0
LY	:	0							
LYX	:	0							
X	:	22091.0	-3351.0	-3328.0	-22097.0	25913.0	274.0	325.0	-25968.0
KXH	:	1							
KX = KXH - KX0	:	0							
LX	:	0							
Y	:	21459.0	-7238.0	-7245.0	-21448.0	25922.0	3702.0	3710.0	-25966.0
LY	:	0							
Z	:	632.0	3887.0	3917.0	-649.0	-9.0	-3428.0	-3385.0	-2.0
LZ	:	0							
R	:	632.0	3887.0	3917.0	-649.0	-9.0	-3428.0	-3385.0	-2.0
R SCALED	:	5056.0	31096.0	31336.0	-5192.0	-72.0	-27424.0	-27080.0	-16.0
KRH	:	4							
KR = KRH - KR0	:	3							
LR = LR + KR	:	3							
CRDR	:	53205.0	.0						
RDR	:	26602.0							
KRRH	:	2							
KRR = KRRH - KRR0	:	-1							
LRR = 2*LR + KRR - KM	:	-11							
LRRD = LRR - LRR0	:	6							

```
BETA                      :   24365.0
BETA SCALED               :   24365.0
KBET                      :         0
LBET=LRR+LRRI+KBET-KMO    :        20
BP                        :   16426.0   -2492.0   -2475.0  -16430.0   19268.0     204.0     242.0  -19309.0

LBP = LBET + LP - KMO     :         5
X                         :    5056.0   31096.0   31336.0   -5192.0     -72.0  -27424.0  -27080.0     -16.0

LX                        :         3
Y                         :   16426.0   -2492.0   -2475.0  -16430.0   19268.0     204.0     242.0  -19309.0

LY                        :         5
LYX                       :         2
X                         :    5056.0   31096.0   31336.0   -5192.0     -72.0  -27424.0  -27080.0     -16.0

KXH                       :         4
KX = KXH - KXO            :         3
LX                        :         3
Y                         :    4106.0    -623.0    -619.0   -4108.0    4817.0      51.0      60.0   -4828.0

LY                        :         3
Z                         :    9162.0   30473.0   30717.0   -9300.0    4745.0  -27373.0  -27020.0   -4844.0

LZ                        :         3
P                         :    9162.0   30473.0   30717.0   -9300.0    4745.0  -27373.0  -27020.0   -4844.0

P SCALED                  :    9162.0   30473.0   30717.0   -9300.0    4745.0  -27373.0  -27020.0   -4844.0

KPH                       :         1
KP = KPH - KPO            :         0
LP = LP + KP              :         3
---------------------------------------------------------------------------------------------------------------
```

ITERATION #	2							
CP	864.0	5380.0	5418.0	-911.0	-2.0	-4745.0	-4690.0	3.0
CP LIMITED	3456.0	21520.0	21672.0	-3644.0	-8.0	-18980.0	-18760.0	12.0
KCPH	5							
KCP = KCPH - KCP0	2							
LCP = LP + KCP - KM	-11							
CPCP	36825.0	.0						
PCP	18412.0							
KPCPH	2							
KPCP = KPCPH -.KPCP0	-1							
LPCP=LP+LCP+KPCP-KM	-25							
PCPI	29159.0							
KPCPI	0							
LPCPI = KD - LPCP	54							
ALPHA	23672.0							
ALPHA SCALED	23672.0							
KAL	0							
LAL=LRR+LPCPI+KAL-KM0	28							
AP	6619.0	22014.0	22190.0	-6718.0	3428.0	-19775.0	-19520.0	-3499.0
LAP = LAL + LP - KM0	16							
X	-18680.0	2834.0	2814.0	18685.0	-21912.0	-232.0	-275.0	21958.0
LX	16							
Y	6619.0	22014.0	22190.0	-6718.0	3428.0	-19775.0	-19520.0	-3499.0
LY	16							
LYX	0							
X	-18680.0	2834.0	2814.0	18685.0	-21912.0	-232.0	-275.0	21958.0
KXH	1							
KX = KXH - KX0	0							
LX	16							
Y	6619.0	22014.0	22190.0	-6718.0	3428.0	-19775.0	-19520.0	-3499.0
LY	16							
Z	-25299.0	-19180.0	-19376.0	25403.0	-25340.0	19543.0	19245.0	25457.0
LZ	16							

W :	-25299.0	-19180.0	-19376.0	25403.0	-25340.0	19543.0	19245.0	25457.0
W SCALED :	-25299.0	-19180.0	-19376.0	25403.0	-25340.0	19543.0	19245.0	25457.0
KWH :	1							
KW = KWH - KW0 :	0							
LW = LW + KW :	16							
ACP :	2497.0	15546.0	15656.0	-2632.0	-6.0	-13711.0	-13552.0	9.0
LACP = LAL + LCP - KM0 :	2							
RDRI :	20182.0							
LRRI = KD - LRR :	40							
X :	5056.0	31096.0	31336.0	-5192.0	-72.0	-27424.0	-27080.0	-16.0
LX :	3							
Y :	2497.0	15546.0	15656.0	-2632.0	-6.0	-13711.0	-13552.0	9.0
LY :	2							
LYX :	-1							
X :	2528.0	15548.0	15668.0	-2596.0	-36.0	-13712.0	-13540.0	-8.0
KXH :	3							
KX = KXH - KX0 :	2							
LX :	2							
Y :	2497.0	15546.0	15656.0	-2632.0	-6.0	-13711.0	-13552.0	9.0
LY :	2							
Z :	31.0	2.0	12.0	36.0	-30.0	-1.0	12.0	-17.0
LZ :	2							
R :	31.0	2.0	12.0	36.0	-30.0	-1.0	12.0	-17.0
R SCALED :	3968.0	256.0	1536.0	4608.0	-3840.0	-128.0	1536.0	-2176.0
KRH :	8							
KR = KRH - KR0 :	7							
LR = LR + KR :	9							
CRDR :	934.0	.0						
RDR :	29888.0							
KRRH :	8							
KRR = KRRH - KRR0 :	5							
LRR = 2*LR + KRR - KM :	7							
LRRD = LRR - LRR0 :	24							

```
*************************
*    SYSTEM INTERRUPT    *
*  GRADIENT CONVERGENCE  *
*************************
LRRD                     :      24
```

```
FINAL WEIGHT VECTOR
--------------------------------------------------------------------------------------------------
NBW                      :      12
KPW = NBW - NBP          :      -4
LW = LW + KWF + KPW      :      10
W  (TO COMBINER)         :  -1582.0   -1199.0   -1211.0   1587.0   -1584.0   1221.0   1202.0   1591.0
W  (FLOATING-POINT)      :  -.38623   -.29272   -.29565   .38745   -.38672   .29810   .29346   .38843
```

COMBINED PORT SIGNAL

SC	:	.0	2.0	3.0	2.0	2.0	5.0	2.0	.0
		1.0	.0	3.0	-4.0	10.0	-2.0	5.0	-8.0
		-4.0	-5.0	-15.0	-1.0	6.0	16.0	-22.0	10.0
		-7.0	-5.0	-1.0	-2.0	29.0	-11.0	1.0	4.0
		-7.0	1.0	-10.0	4.0	-11.0	-9.0	20.0	2.0
		1.0	-3.0	4.0	-2.0	-30.0	11.0	8.0	-8.0
		29.0	-7.0	-21.0	4.0	-7.0	3.0	-16.0	1.0
		-4.0	-5.0	27.0	-7.0	-7.0	3.0	-19.0	6.0
		12.0	2.0	16.0	-9.0	-1.0	-6.0	4.0	.0
		-19.0	12.0	-14.0	7.0	3.0	-2.0	21.0	-14.0
		-14.0	11.0	5.0	3.0	8.0	-2.0	-7.0	-5.0
		-6.0	-1.0	-4.0	-1.0	5.0	-7.0	-22.0	1.0
		1.0	-1.0	18.0	-9.0	9.0	-6.0	-27.0	13.0
		.0	7.0	-6.0	-3.0	10.0	-13.0	23.0	3.0
		-20.0	17.0	-19.0	-5.0	.0	-4.0	32.0	-9.0
		10.0	-4.0	-12.0	3.0	4.0	1.0	-4.0	2.0
		-13.0	3.0	-21.0	10.0	6.0	-11.0	19.0	3.0
		-4.0	-3.0	2.0	-4.0	-2.0	1.0	-26.0	7.0
		7.0	-7.0	6.0	4.0	-7.0	.0	11.0	-3.0
		22.0	1.0	-14.0	-1.0	1.0	2.0	-17.0	-3.0
		-1.0	5.0	4.0	-16.0	-26.0	3.0	12.0	-6.0
		6.0	-4.0	-1.0	-5.0	-13.0	7.0	2.0	4.0
		18.0	-8.0	-8.0	12.0	-11.0	1.0	16.0	-5.0
		27.0	-9.0	-25.0	-1.0	-19.0	9.0	12.0	-10.0
		-2.0	1.0	1.0	12.0	-8.0	2.0	-19.0	.0
		5.0	-5.0	21.0	-3.0	-12.0	5.0	17.0	-3.0
		-11.0	-2.0	.0	4.0	-15.0	5.0	1.0	-9.0
		26.0	-9.0	8.0	5.0	.0	6.0	-31.0	2.0
		14.0	2.0	-5.0	6.0	-2.0	-10.0	4.0	4.0
		-13.0	-2.0	2.0	-11.0	3.0	4.0	9.0	.0
		-8.0	6.0	-6.0	-1.0	-3.0	-3.0	-1.0	-2.0
		9.0	.0	-10.0	3.0	7.0	3.0	.0	-5.0

SUPPRESSION (DB) : 25.03136

EXIT

4.2 BCR Processor Performance

The BCRM program described above is useful in evaluating the adaptive nulling performance of a practical BCR processor implementation employing 16-bit adaptive fixed-point arithmetic. As already mentioned, Table 4-7 includes the detailed arithmetic activity of the BCR processor for the case of the benchtest example. The BCR processor performance for this case and two other examples is compared to that of the 32-bit floating-point simulation performed via BCRS in Project Memorandum 8512-04.

4.2.1 Benchtest Example

The BCR processor performance for the benchtest example is summarized in Table 4-8 where it is contrasted to the actual simulation results reported

Table 4.8. BCR Processor Performance
Benchtest Example (See Figure 2-2, PM 8512-04)

ITERATION	EMULATION		SIMULATION
	$\|\|\underline{r}\|\|^2$	$\|\|\underline{r}\|\|^2/\|\|\underline{b}\|\|^2$ (db)	$\|\|\underline{r}\|\|^2/\|\|\underline{b}\|\|^2$ (db)
0	$17{,}888 \times 2^{17}$	0.00	0.00
1	$26{,}602 \times 2^{11}$	-16.34	-16.34
2	$29{,}888 \times 2^{-7}$	-70.01	-69.65
$P_c(\underline{w})/P$	(dB)	-25.03	-25.24

in Project Memorandum 8512-04. To be noted under the emulation results is a column containing the actual 16-bit fixed-point values of $||\underline{r}||^2$ at initialization and the two iterations that follow. These values may be read directly from Table 4-7. Since the initial weight-vector estimate, $\underline{w}^0$, is taken to be $\underline{0}$, $||\underline{b}||^2$, is identical to the initial value of $||\underline{r}||^2$, $17{,}88\overline{8} \times 2^{17}$. This fact was used to construct the second column showing the relative gradient metric $||\underline{r}||^2/||\underline{b}||^2$. When contrasted with the available simulation results, the agreement is impressive. What is more important is the actual combined power level achieved by the processor. Compared to that of a 32-bit floating-point simulation, it is degraded only by 0.21 dB. This relatively small adaptive nulling degradation was, in part, due to the adaptive fixed-point arithmetic employed by the BCR processor. Of course, part of the degradation is due to the reduced numerical resolution in the sampled signals.

It should be noted in Table 4-7 that the BCR processor terminated processing during the second iteration. In fact, a build-in system interrupt associated with the gradient convergence condition, $\|\underline{r}\|^2/\|\underline{b}\|^2 < 2^{-15}$, came into effect. Since $\underline{w}^0 = \underline{0}$, this meant that, at the second iteration, the global bit-shift, $\overline{LRR}$, of $\|\underline{r}^2\|^2$ exceeded that of the initial global bit-shift, LRR0, of $\|\underline{r}^0\|$ or $\|\underline{b}\|^2$ by more than 15. In fact, LRR-LRR0 took a value of 24.

Of interest here is the floating-point equivalent of the 12-bit version of the processor-estimated weight-vector and its comparison to the weight-vector estimated via floating-point estimation. This comparison is given in Table 4-9.

Table 4-9. Comparison of Simulation/Emulation Weight Vector Estimates Benchtest Example (See Figure 2-2, PM 8512-04)

WEIGHT-VECTOR COMPONENTS	$\underline{w}^s$	$\underline{w}^e$	$\Delta\underline{w} = \underline{w}^s - \underline{w}^e$
1	-0.37375 - j0.28405	-0.38623 - j0.29292	0.01248 + j0.00867
2	-0.28686 + j0.37356	-0.29565 + j0.38745	0.00879 - j0.01389
3	-0.37553 + j0.28835	-0.38671 + j0.29810	0.01118 - j0.00975
4	0.28409 + j0.37616	0.29345 + j0.38842	-0.00936 - j0.01226

$\underline{w}^s$: Weight Vector Estimate Via Simulation

$\underline{w}^e$: Weight Vector Estimate Via Emulation (12-bit Equivalent)

At first glance, it is surprising to notice the rather large deviation between the components of the two weight-vector estimates. However, even with the nearly 3% magnitude deviation, the adaptive performance of the BCR processor has not suffered. A probable reason why such a deviation would not give rise to a commensurate degradation in performance is that the covariance C is not full-rank. As such, a weight-vector deviation from $\underline{w}^s$ would not necessarily imply poor nulling.

4.2.2 Blinking Source Example

The next example chosen for emulation was the blinking source example discussed in Project Memorandum 8512-04. Its scenario and system description is given there in Figure 3-1. Its simulated BCR convergence characteristics may be found in Figure 3-1 of the same memorandum.

Figure 4-1 contrasts the emulated BCR convergence characteristics with those predicted via simulation. To be noted, immediately, is the degraded behavior of the relative gradient metric, $\|\underline{r}\|^2/\|\underline{b}\|^2$ in the case of the emulation. However, with a gradient convergence threshold of $\|\underline{r}\|^2/\|\underline{b}\|^2 < 2^{-15}$, the BCR processor will terminate processing at the second iteration. At that point the relative combined power level of -25.2 dB almost matches the corresponding simulation value. Motivated by this fact, the BCR

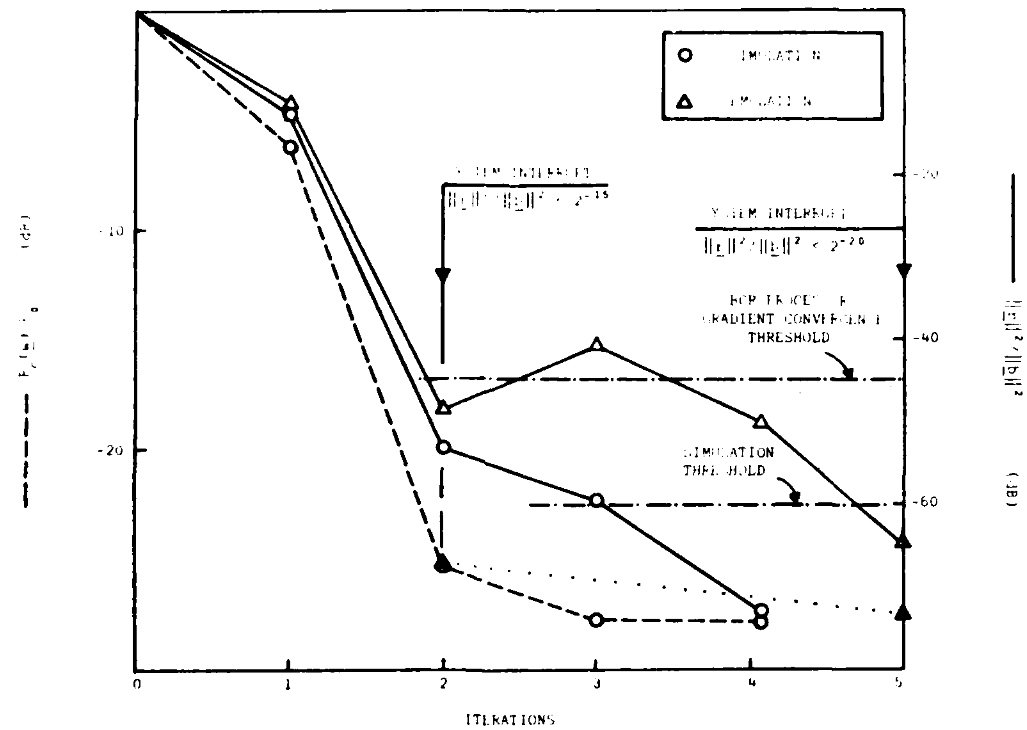


Figure 4-1. Simulation/Emulation Comparison BCR Convergence Characteristics Blinking Source Example (See Figure 3-1, PM 8512-04)

processor threshold was lowered to the simulation threshold of -60 dB. As shown, the BCR processor cross it in the 5th iteration, one more iteration than indicated in the simulation case. The combined power level achieved by the BCR processor in the 5th iteration is -27.50 dB compared to -27.79 dB in the 4th iteration of the simulation.

The present example serves to illustrate the behavior of BCR convergence with reduced numerical accuracy. The nontonic behavior of the gradient metric with the 34-bit floating-point arithmetic is not maintained with the 16-bit adaptive fixed-point BCR processor implementation. This is due to two fundamental reasons. First, in the emulation, the word lengths in the main and auxiliary signal samples are 10 and 8 respectively, certainly less accurate than the floating-point versions with five significant digits used in simulation. Second, the CG-based weight-estimation process would certainly give rise to a larger roundoff error with 16-bit arithmetic than with 32-bit floating point. Of course, a better comparison between emulation and simulation would be possible if the first of these possible causes of inaccuracy were to be eliminated. That is, if the simulation were to take the 16-bit C and $\underline{b}$ quantities and exercise the CG process with a 32-bit floating-point arithmetic, the only discrepancy in convergence characteristics of the BCR emulation would be truly confined to the CG-stage.

A final comment is in order with respect to this example. Based on the results, it is clear that the gradient convergence criterion, $\|\underline{r}\|^2/\|\underline{b}\|^2 < 2^{-20}$ was used. If the latter stopping condition were used, along with a maximum iteration count of 5, the indicated nulling level would be achieved. If only 4 iterations were allowed, the number of adaptive weight-vector components, then the nulling level would be somewhere between -25.21, at the second iteration, and -27.40 in the 5th. It would appear then that the stopping criterion for the BCR processor must be dictated first by the maximum number of iterations and then by the gradient convergence threshold.

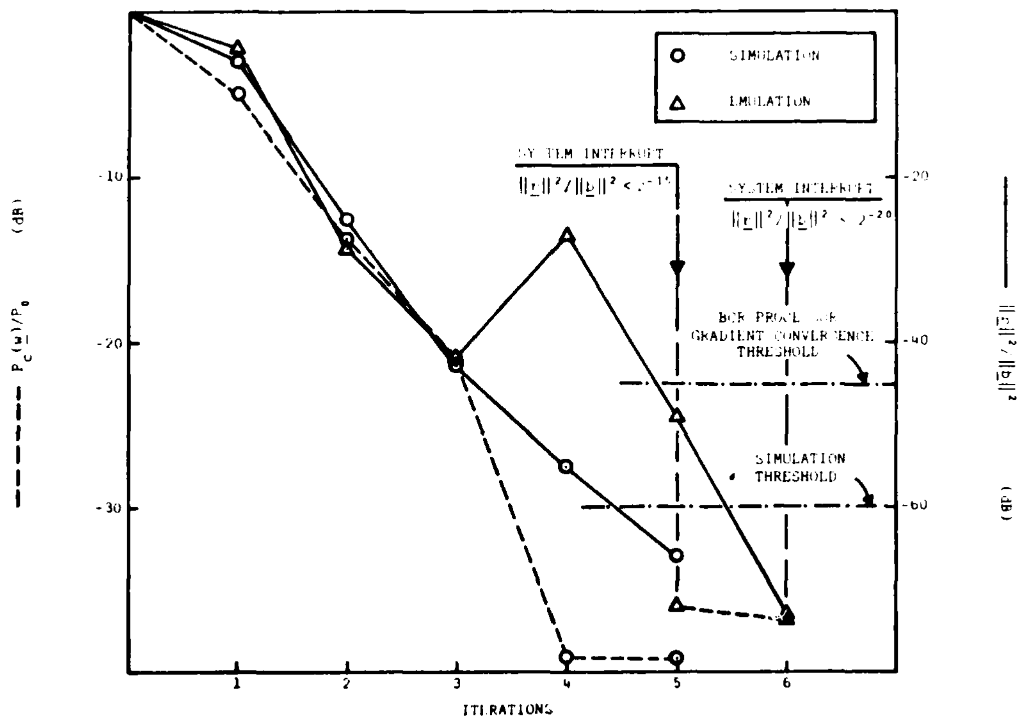


Figure 4-2. Simulation/Emulation Performance Comparison
BCR Convergence Characteristics
Wideband Source Example
(See Figure 3-10), PM 8512-04)

4.2.3 Wideband Example

The next example considered was the wideband source example whose scenario and system description may be found in Figure 3-10 of PM 8512-04. The BCR processor convergence characteristics for this example are contrasted to the simulation results in Figure 4-2.

As for the previous, Figure 4-2 includes the complete relative gradient profile for the six iterations needed to cross the lowest threshold of -60 dB, where $\|\underline{r}\|^2/\|\underline{b}\|^2 < 2^{-20}$. The combined power nulling level achieved is -36.67 dB, a degradation of less than 3.00 dB with respect to the simulation performance after five iterations.

Using the higher threshold, the BCR processor converged in five iterations. The achieved nulling level was about 3.50 dB worse than the simulation level. Significantly, the BCR processor nulling improved only by 0.5 dB between the fifth and sixth iterations.

5.0 CONCLUSIONS AND RECOMMENDATIONS

The implementation of a BCR adaptive processor has been presented in considerable detail and its performance has been evaluated. More specifically, a 4-port BCR adaptive processor block diagram has been described qualitatively and quantitatively. An exact representation of its 16-bit adaptive fixed-point arithmetic was incorporated into a computer emulation program in order to evaluate the numerical effectiveness of the mechanization. Three examples have been included to demonstrate the floating-point simulation. In each case, the BCR processor provided excellent numerical accuracy and, more importantly, achieved a nulling performance that compared well with that predicted via simulation.

Although a 4-port BCR processor was described, the reader can easily extend the design to a higher-dimensionality system. From the practical point of view, however, it may be undesirable to extend this design byond a certain dimensionality. For example, bus-widths that may be required will quickly become unwieldy. Optical fiber bussing may be incorporated to multiplex the parallel vector signals one component at a time, thus alleviating this problem.

A more desirable alternate design approach would be a serialized implementation which would be suitable for any desired dimensionality, within some practical limits. Accordingly, the CPU could be replaced with a single complex multiplier/accumulator design and all busses will be limited to a single complex word-width. This approach will allow flexibility of design and also take advantage of the emerging VHSIC technology.

MISSION
of
Rome Air Development Center

RADC plans and executes research, development, test and selected acquisition programs in support of Command, Control Communications and Intelligence (C^3I) activities. Technical and engineering support within areas of technical competence is provided to ESD Program Offices (POs) and other ESD elements. The principal technical mission areas are communications, electromagnetic guidance and control, surveillance of ground and aerospace objects, intelligence data collection and handling, information system technology, ionospheric propagation, solid state sciences, microwave physics and electronic reliability, maintainability and compatibility.

END

DATE

FILMED

3-82

DTIC